

# Economic Role of Population Density

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## **Abstract.**

The role of population density for economic activity was neglected in most part of economic theory. This paper is a review and extension of the previous works of the author in this field.

So far, densities did not become common economic variables in economic analysis, and two countries with different densities of population and infrastructure but similar in macroeconomic parameters are treated as similar. One of the presented models is about an influence of population density on infrastructure development, and later on country potential for economic growth (Yegorov 2005a).

Self-organization of production activity in space is elaboration of the ideas of von Thunen. When a city emerges as a center of industrial activity, it deforms the space, and agricultural land rent becomes a function of distance due to transport cost to bring the good to the market.

Population density plays an important role in harvesting societies, i.e. those that depend on agriculture and natural resources. Too high population density decreases the natural endowment per capita, but eases the development of infrastructure, leading to existence of an optimal population density for economic growth (Yegorov, 2009).

The trade-off between scale economies and transport costs leads to an optimal area served by a local monopolist. In the world with low population density competition might not even emerge because even monopolist can become bankrupt due to low demand density, especially in the environment of high transport costs. Such situation took place on the most part of Russian territory after liberalization that also lead to an increase in relative transport costs.

Population density also can play role for an optimal size of a country. While there are many other reasons (mostly historical), spatial structure is also important. Land area of a country is considered as some capital bringing rent from natural resource extraction. The length of a border requires protection efforts and thus is a first type of cost. Commuting with the capital is another type of cost, and here the population density also matters. All these 3 factors enter with different power, and optimization with respect to linear scale give different results (Yegorov, 2005b).

**Keywords:** population density, optimization, economic growth, transport costs.

**JEL Codes:** O18, Q10, R14, R40.

## 1. Introduction

Population density is an important factor influencing economic activity. Geographers use maps of population density but there is not sufficient theoretic research about its influence. This factor has not only economic but also geographic origin. This paper starts from empirical facts about population density and then moves to some models describing its influence analytically.

Due to both exogenous (geographical conditions) and endogenous (history dependence, see Arthur (1994)) factors population density has very huge variation over the globe, especially in rural areas. Variation of the population density across the globe is very high (see Fig. 1). However, apart from agricultural economics, there are very few models where it is taken into account. Urban economics deal with population density in cities. However, the variation of densities there is not so high. The highest population density is observed in the cities of Philippines and India (30 to 42 thousand people per sq. km)<sup>1</sup>. Big European cities have lower densities; Paris has 21.5 th., Barcelona 16 th., Moscow 10.5 th.<sup>2</sup> The cities in the USA are more dispersed and typically have lower population density than European, typically from 4 to 10 th.<sup>3</sup> Hence, we have global variation of urban population density of about 10.

The situation with rural (and country average) population density is completely different. As one can see from Fig.1, the highest population density (more than 500 people per sq. km) is observed in some regions of China and India, and also in some districts of Europe. Most of rural India and China have the population density between 100 and 500, while in Europe it is closer to 100. The USA has very heterogeneous districts, with typical rural density between 10 and 100 people per sq. km in the East, and below 10 in the West (excluding densely populated California). Latin America and Africa have densities comparable to the USA. There are several countries with very low population density (of just 1-2) on the major part of their territories. They include Canada, Australia, Mongolia and Siberia (Asian part of Russia).

There is no immediate correlation between the density and income per capita. Among low populated countries, Canada and Australia are rich, while Mongolia is poor. Russian GDP per capita has intermediate position despite possession of resource-rich Siberia. On the other hand, there are rich but densely populated countries (like Singapore and Monaco) and poor economies with high population density (like India).

This suggests that the influence of the population density on economic welfare is not straight forward. First of all, there may be different division into urban and rural areas. Almost all population of Australia lives in urban agglomerations, while natural resources of the rest of its territory are not extensively used. Canada has different pattern, with most of its economic activity concentrated along the border with the USA.

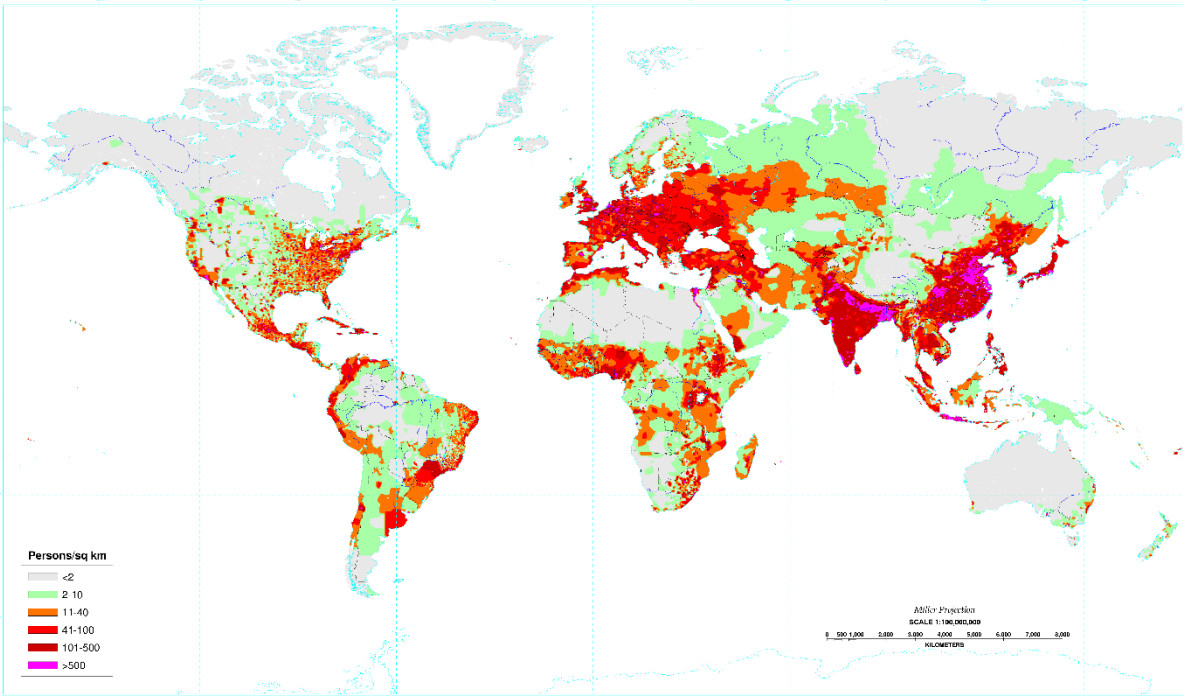
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<sup>1</sup> [http://en.wikipedia.org/wiki/List\\_of\\_cities\\_proper\\_by\\_population\\_density](http://en.wikipedia.org/wiki/List_of_cities_proper_by_population_density)

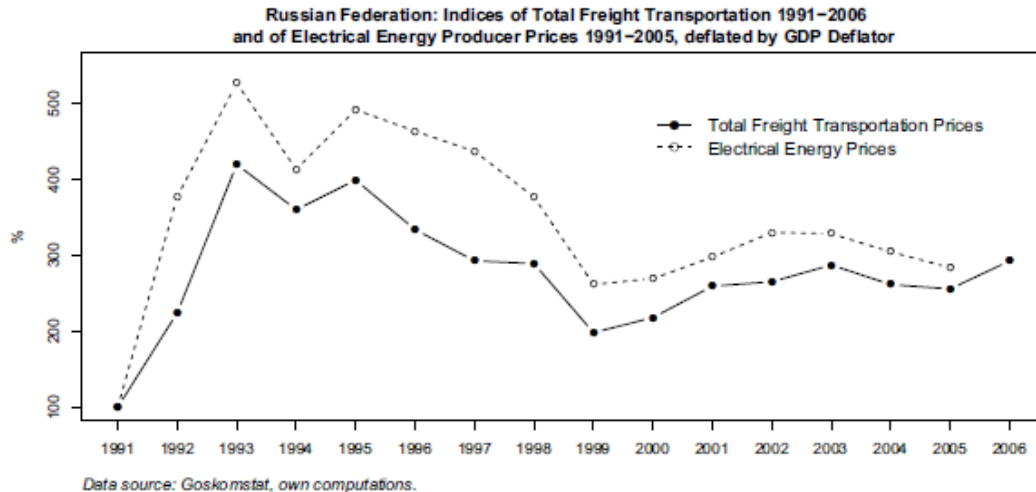
<sup>2</sup> [http://en.wikipedia.org/wiki/List\\_of\\_European\\_Union\\_cities\\_proper\\_by\\_population\\_density](http://en.wikipedia.org/wiki/List_of_European_Union_cities_proper_by_population_density)

<sup>3</sup> [http://en.wikipedia.org/wiki/List\\_of\\_United\\_States\\_cities\\_by\\_population\\_density](http://en.wikipedia.org/wiki/List_of_United_States_cities_by_population_density)

In the times of the USSR, Siberian renewable resources (like forest) were used at higher intensity than today. Now Russia is more concerned with extraction and export of non-renewable resources (especially oil and gas). This might be a strategic mistake leading to non-sustainable development, but it was also an outcome of transition to liberal principles in economics, that have also removed relatively low transport cost (implicit subsidy) that has prevailed in the Soviet period. The relative shock to transport and energy cost for domestic consumers in Russia has been investigated, for example, in Kauffmann (2007). He found that the relative price of transportation to the price index of other goods in Russia have risen by 300% in 1992 and after 2000 still stayed 2-3 times higher than in 1991 (end of USSR); see Fig. 2. Yegorov (2014) provides an analysis of other negative influences of this dynamics on Russian economy. In particular, after dissolution of the USSR air transport went in crisis with reduction of passenger flow from 90 to 21 million of passengers per year between 1990 and 1999.



**Fig.1.** Population density in 1994. Source: [http://en.wikipedia.org/wiki/Population\\_density](http://en.wikipedia.org/wiki/Population_density)



**Fig. 2.** Change in relative transport cost in Russia after 1991. Source: Kauffmann A. (2007)

The paper is organized as follows. Section 2 presents a review of theoretical models about the different economic roles of population density. Section 3 elaborates one of author’s model related to optimal size of a country. Section 4 presents a simple Hotelling-type model with 2 firms with fixed locations producing different goods. Section 5 studies policy implications for different countries. Section 6 concludes.

## 2. Theoretical Papers on the Influence of Population Density

Hotelling (1929) was probably the first researcher to introduce population density into theoretical economic literature. He put uniformly distributed consumers on a beach, which was interval  $[0, 1]$  in his model and allowed 2 firms to choose location and price. Further we was followed by researchers from regional science of 1930s: Chamberlin (1933), Losch, Christaller and others. All of them considered spatial competition of firms with some location and distributed consumers around. In those models population density could be one- or two-dimensional, but typically constant. Its role was to create spatial structure of the demand, and this is the first economic role of population density.

Urban economics also arrives to population density in cities. When many agents select some land for residence, the space becomes packed. If one allows for multi-story buildings, it is possible to put more people on the same territory. An introduction to this theory can be found in Fujita (1989).

Although land is an obvious factor of production, only agricultural economics nowadays deals with it. Macroeconomic theory typically considers only labour and capital as production factors. In the models with harvesting natural resources distributed in space it is important to account not only for land but also for transport costs. The accounting for all geographical details requires use of GIS. However, in the case of completely cultivated plain agricultural land occupies most of the territory and one can use Euclidean metrics to account for distances. Agricultural machines (like tractors) can move over field in any direction without roads.

The role of transport costs is especially pronounced in countries with low population density. Yegorov (2009) shows that monopoly may not survive in the environment of low population density just because it may not be able to cover transport costs to all consumers, or consumer demand will be too low in its location to cover its fixed costs.

In general, transport costs have to be accounted very carefully. They play a role completely different to other goods. The ratio of transport costs of other goods in Russia have increased by factor 3-4 in early 1990s after the transition that has included global prices for oil (fuel). After that the share of transport cost in the total cost to produce and deliver goods have grown up to 30-40%.

Clearly, this share depends on the composition of transported goods. It is clear that the share of transport cost in the total price for consumer will be low for diamonds and computers, but very high for raw materials and agricultural goods, especially when the distances are comparable. This effect was investigated both theoretically empirically in Grohall & Yegorov (2011).

The paper of Yegorov (2005a) contains the general introduction for the role of population density and presents some models of the author. It starts from the description of two-dimensional Hotelling model where demand continuity can be proved for any functional form of bounded population density in two-dimensional space with Euclidean metrics. It also introduces a concept of field emerging from demand density of consumers that influences the location choice of a firm in heterogeneous space. Some theorems about survival of monopolistic firms given the demand density and fixed costs are formulated.

Next model is about the influence of population density on the output of economy that specializes on harvesting of distributed resources. It assumes a spatial set of small firms (or farms) that use Cobb-Douglas technology for production using labour and land and the transportation network with transport costs linear in distance that collects this output in one hub and then exports it. It is shown that there exists an optimal population density (for given transport cost and other parameters) that maximizes the profit of transport-export firm. In the case of too low population density the resource endowment per worker is high but the maintenance of transport network and total transport costs (given that the physical output from the unit of land is lower in this case due to Cobb-Douglas assumption) becomes too high. In an opposite case of too high population density profits are reduced due to too little land used by one worker. There exists an optimal population density for any particular economic activity based on harvesting of natural resource.

Yegorov (2009) addresses some other aspects of the influence of population density. Possibility of non-survival for a monopoly in low-populated areas was considered there. A simple model with counterbalance between returns from scale economies with additional transport costs to serve spatially larger area was considered there. This paper present a similar model but for two firms.

Another important aspect considered there is whether population density has an influence on cooperative and non-cooperative behavior. Typically huge projects (building castles or churches) require cooperation, while non-cooperative behavior (and possibility of cheating)

emerges in small projects. It is argued that low population density is more favourable for cooperation.

Yegorov (2014) presents a model of coexistence of urban areas (doing manufacturing) and rural areas (doing agriculture) with a possibility of migration. The focus is on the influence of price shocks on energy (or food) on such equilibrium allocation of population in space. Further the model is applied for the case of Russia.

### 3. Optimal Country Size

Yegorov (2005b) addresses the question of optimal country size. Both population density and resource density  $A$  determine resource endowment per capita. An abstract country is assumed to be of square shape (of size  $a$ ) with uniform population density. There are two main costs in this model. The first is the cost of border protection, obviously linear in spatial parameter  $a$ . The second cost is related with the link between the center and population (like trips of government to the region and commuting of citizens with the center of power). For the whole country this cost is proportional to the population density,  $\rho$ , and the third power of scale parameter,  $a$ . Thus, country's surplus is

$$\Pi = A a^2 - 4 a - c t \rho a^3. \quad (1)$$

Here  $A$  is resource density,  $t$  is unit distance transport cost, while  $c$  is a constant accounting for frequency of trips to the capital. Then the country border is  $4a$ , and the unit cost of protecting one unit of border distance is normalized to one. The optimality can be viewed from two perspectives. In the case of dictatorship this is the total country's surplus, while in the case of democracy it is per capita surplus, given by  $\pi = \Pi/N$ , where the total population  $N = \rho a^2$ . The optimal spatial scale for a democracy is given by the formula

$$a^* = 2 (c t \rho)^{-1/2}. \quad (2)$$

Both partial derivatives have negative sign:

$$\partial a^* / \partial \rho = - (c t)^{-1/2} \rho^{-3/2} < 0, \quad \partial a^* / \partial t = - (c \rho)^{-1/2} t^{-3/2} < 0. \quad (3)$$

This means that an increase in transport costs will make optimal country size smaller. The necessity to have higher commuting cost will counterbalance the benefits of scale economies in border protection. The resource density  $A$  does not play a role here, because resource endowment per capita does not depend on country size.

Interestingly, under the assumptions of the model all democracies have the same total population  $N = \rho a^2$ , but may differ in population densities.

Consider now the model (1) in the case of dictatorship. Formal differentiation w.r.t. scale parameter  $a$  gives optimal size:

$$a^{**} = [A + (A^2 - 12ct\rho)^{1/2}] / (3ct\rho), \quad (4)$$

It is easy to calculate partial derivatives. The optimal country size again becomes smaller, if transport cost grows:

$$\partial a^{**}/\partial t = - [A+(A^2-12ct\rho)^{1/2}]/(3ct^2\rho) - 2/t(A^2-12ct\rho)^{1/2} < 0. \quad (5)$$

Since population density and transport cost enter the formula (4) symmetrically, the corresponding partial derivative is again negative:  $\partial a^{**}/\partial \rho < 0$ . But the partial derivative with respect to resource density is now positive:

$$\partial a^{**}/\partial A = [1+(1-12B)^{-1/2}]/(3ct\rho) > 0, \quad B \equiv ct\rho/A^2, \quad 0 < B < 1/12. \quad (6)$$

Now it is possible to calculate optimal population N:

$$N^* = [1 + (1-12B)^{1/2}] 2B/(9ct) - 3/(3ct). \quad (7)$$

For small B,  $\partial N^*/\partial B > 0$ . For fixed transport costs and population density, higher B means lower resource density A per capita. This means that a dictator optimally tries to expand the size of its country (empire) also in the terms of population when additional territories have lower resource endowment per capita.

One may think here about the several historical examples. British Empire has lower resource and population density in both Australia and Canada than in UK. The same is true about American expansion to the West in the 19th century and Russian expansion to Siberia in the 16<sup>th</sup> century. Technological development have led to lower transport costs, making a<sup>\*\*</sup> larger (see formula (5)), and this was another reason for expansion.

#### 4. A Hotelling Type Model with Two types of Goods and Dispersed Consumers

The goal of this simple model is to show how the distance between consumer and two producers (with fixed and different locations) influences their substitution between 2 consumed goods and how population density influences profitability of those producers (for fixed prices). One may think also about provision of 2 services. Theater and stadium can be located in different edges of a city, and consumers choose to visit more that place which is located closer.

Consider a set of consumers (indexed by  $0 < z < 1$ ) uniformly distributed along the interval  $[0, 1]$ , like in the model of Hotelling (1929). Suppose that they demand two goods that are produced in the border points: good X in  $z=0$ , while good Y in  $z=1$ . Let all consumers have the same income normalized to one and Cobb-Douglas preferences with  $a=1/2$  for goods X and Y:

$$U(z) = X^{1/2}(z) Y^{1/2}(z). \quad (8)$$

The reason to produce goods in those points might be in scale economies, favoring concentrated production. Alternatively we may think about access to two services (like theater and stadium) located in different points. Suppose that the producer prices for both goods X, Y (or services) are normalized to one. One may think about regulated monopoly that produces at the cost level or is allowed for fixed rate of profit. The total price will also include transport cost of each consumer to this market, like in Hotelling's model. Let  $\tau$  be the transport cost per unit of distance. Then the budget constraint of consumer will be:

$$(1+\tau z)X(z)+(1+\tau-\tau z) Y(z) = 1. \quad (9)$$

As we know from Cobb-Douglas optimization, in this case consumer will spend the same amount of money for both goods. Then

$$(1+\tau z)X(z) = (1+\tau-\tau z) Y(z).$$

Hence, we will have the following choice for consumer z:

$$X(z)=(2(1+\tau z))^{-1}, Y(z)=(2(1+\tau(1-z)))^{-1}. \quad (10)$$

Overall demand for good X can be obtained after integration:

$$D(X) = \ln (1+\tau)/2\tau. \quad (11)$$

If the population density over unit interval is not one, but  $\rho$ , then this demand can be multiplied by corresponding factor. Here we have two effects. First, low population density makes the total market for good (or service X) smaller, and thus undermines the role of scale economies. Second, the increase in transport cost  $\tau$  makes overall demand lower.

Formally, a firm has fixed cost F and some price p for its output which is determined by competition with the firms producing substitutes. Hence, this firm might be a local monopolist, like it is assumed in the model of monopolistic competition by Chamberlin (1933) and further elaborated by Beckmann and Thisse (1986). In this simple model its maximal profit (if all market is captured) is given by

$$\Pi = p \rho \ln (1+\tau)/2\tau - F. \quad (12)$$

It is obvious that  $\partial\Pi/\partial\rho >0$  and  $\partial\Pi/\partial\tau <0$ . Hence, even a monopolist may be bankrupt if either population density reaches some minimally acceptable value or when transport cost exceeds some maximally acceptable value.

This example shows that in low populated areas business has to compete not only with rival firms but also with the environment of low demand. Some minimal population is required for a particular service in a city. That is why small villages may lack some types of services (like theater, hospital, etc) and this adds to disutility for living there. Depopulation of remote villages can also happen when nobody wants to finance a road to it. Some countries have special state programs for building such an infrastructure.



## 5. Policy Implications

The policy implication is straightforward. In a country with low population density all efforts should be done to reduce unit distance transport cost  $\tau$ . Lower transport costs not only increase consumer utility but also increase the positive role of scale economies for producers.

Here we have a basic paradox which is still little understood by theorists. On one hand, classical microeconomics assumes costless and symmetric access of all consumers to the market. On the other hand, even in the environment of full competition among producers and distributors, transport costs makes them asymmetric. Moreover, its larger values strengthens this effect. In cities this effect is less pronounced, but it becomes extremely important in the economies with low population density. And it is even more important if the substantial fraction of transported goods have low ratio of value to weight.

That is why there should be an effort of regulator to create more competition among transporters. Still, even bringing their profits to zero will not eliminate the cost of transport. This transport cost has three main components (that are of about the same fraction of total costs): fuel cost, labour cost and depreciation of vehicle. *Ceteris paribus*, in the environment of competitive firms, labor markets and vehicle producer the cost of fuel makes the difference. That is why economies are very sensitive to the shocks in oil price.

On one hand, we have an increasing role of services and industry in GDP of most countries. Still, resource extraction and harvesting will remain an essential part of the global economy. Czech (2013) argues that resources occupy the basic trophic level of economy, and thus agriculture and harvesting has a role similar to plants in global ecosystem. Currently low value of agriculture in GDP gives a misperception of its importance for the global economy.

Those discussions may have little importance for densely populated countries. That is why it is important to focus on the countries with low population densities. Three major examples include Australia, Canada and Eastern Russia (Siberia). While those territories are close to 10 million square kilometers each (for a comparison, all EU28 occupies just 4.4 million sq.km<sup>4</sup>), the corresponding populations are about 20 to 30 million people. This gives the average population density of 2-3 people per sq. km. If we exclude urban population, the rural density will drop to 1 or below.

Australia has clustering of its population in mega-cities and thus faces a problem of access to natural resources dispersed over its territory. Canada has relatively more rural population, but still low development of infrastructure just 100 km away from the border with USA where most of its population is concentrated.

Siberia is an example of relatively developed infrastructure. Norilsk is an industrial city located at 69 degree of NL. Trans-Siberian railway crosses Siberia from East to West. There is still lack of roads and railroads in this region for an efficient resource exploitation. Most of them have been built in the time of USSR, when transport was subsidized and labour force was relatively cheap. The shock in transport costs from 1991 have caused substantial

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<sup>4</sup> [http://en.wikipedia.org/wiki/Geography\\_of\\_the\\_European\\_Union](http://en.wikipedia.org/wiki/Geography_of_the_European_Union)

depopulation of Siberia, while rural areas of it suffered even more. This is an obstacle for efficient harvesting of its renewable resources. Market forces can do nothing with it, and smart policy of transport subsidies and more competition have to be implemented to force transport firms operate at cost level and to make this cost minimally possible. Otherwise, markets will become disintegrated.

## 6. Conclusions

This paper presents a survey of economic role of population density. Also, it presents two simple models. The first elaborates one of previous works of the author by presenting comparative statics for optimal country size. The second shows the role of low population density on performance of firms.

Dispersed population over space makes it impossible to bring all producers and consumers to the same market without paying transport costs. Those costs become especially important in rural areas and for low price goods, where they form a significant share of the price of final product.

While industrial organization literature provided a lot of studies related to spatial competition of firms, since recent time there were few studies about the problem of firm survival in an environment with low population density. A consequence of non-survival can lead to abandonment of some services in low populated areas and out-migration of people from there. This makes spatial structure of the population even more heterogeneous, with overpopulated cities and underpopulated rural area. Such process takes place not only in countries with low or medium income (like Russia in 1990s) but also in rich economies like Australia.

Population density plays very high role for countries specializing on resource extraction and agriculture. While too high population density can make them poor due to low resource endowment per capita, low population density has a problem of too high per capita cost of building and maintaining infrastructure to collect and bring its resources to the market. This is one of the problem of Russia. After transition to market and acceptance of global prices for oil in domestic markets, transportation became relatively expensive. This has perturbed not only communication between people living in different regions but also made the harvesting of renewable resources (forestry, fishing, etc) in Siberia a problem that cannot be solved by the market.

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