# Modelling intra-regional geographic mobility in a rural setting *

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**Abstract**

Large-scale models are often used in the urban planning context to model the effects of, for instance, a change in land-use policies or transportation infrastructure. This class of models accounts for factors such as the spatial distribution of jobs and workers, commuting flows, housing markets, modal choice and so on. One criticism of such models is their complexity, computational demands and data requirements. In this paper, we develop a model which shares certain features with large-scale models, but which is appropriate for studying development at the intra-regional level in a rural setting. The rural setting means that not all of the traditional features of a large-scale model are relevant, and these can therefore be omitted. This allows us to create a simple model which still captures the most relevant effects of large-scale models.

## Caveat

This is a very preliminary version of a paper we are currently working on. We plan to further develop the model, applications and policy implications. Having said that, the rest of this paper outlines our general ideas, approach and the sorts of results we get. We ask that it is not quoted.

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# 1 Introduction

The modelling of commuting and migration flows has long been of interest to researchers, and has spawned a vast literature. One of the reasons for this is that these forms of mobility are relevant for many different areas of regional policy. To understand regional labour markets we must understand how and why the supply of labour will adjust itself in response to shocks. Local government must understand what causes a region to grow as well as what causes it to decline in order to forecast demand for public services. Transport planners also need to understand commuting behaviour in order to provide appropriate infrastructure. Such a list could go on ad infinitum. What is clear is that understanding migration and commuting is important.

It has been obvious for some time that the concepts of commuting and migration are related (Hägerstrand, 1973; Roseman, 1971; Reitsma and Vergoossen, 1988; Zax, 1994). What is not always clear is exactly how. In one sense, it is possible to conceive of then as substitutes. People can shorten their commutes by changing their residential location. However, it could be argued that they are compliments. People may choose where they want to live and then commute accordingly. Zax (1994) addresses the related question of when a move should be considered 'migration' and discusses the implications this would have for any analysis involving the concept and its relationship to commuting.

One concept which links these two forms of mobility is accessibility. In his seminal paper, Hansen (1959) defines accessibility as 'the potential of opportunities for interaction'. This interaction could be motivated by diverse reasons. Hansen focuses particularly on travel for work, shopping and social reasons, noting that this accounted for almost 80% of total personal travel. The potential for interaction offered by a region is likely to be an attractive attribute for a zone to possess, and is also likely to affect the patterns of spatial interaction which we observe for that zone. The fact that accessibility is valuable has been demonstrated in the literature. For example, Osland and Thorsen (2008) show that accessibility capitalises into house prices. This indicates that people are willing to pay a premium to locate in an area with high accessibility. Eliasson et al. (2003) study the effect of accessibility on commuting and migration behaviour. They find that migration from accessible regions is quite low. They interpret this as people being unwilling to erelinquish a residence in an accessible location. At the same time, accessibility is found to have a positive effect on commuting flows. Accessiblity therefore would seem to be important in determining residential location patterns as well as spatial interaction.

Despite the acknowledgement that the concepts of commuting and migration are linked, most empirical studies usually ignore the interdependency (Eliasson et al., 2003). Deficiencies in analytical and econometric techniques were cited by van Ommeren et al. (1999) as possible reasons for the lack of a simultaneous treatment. It does not take long to find examples of studies dealing only with commuting or migration. In some cases, this may be permissible. Migration studies which are based on highly aggregated spatial units with little commuting between them could reasonably argue that there is no need to include commuting in any model formulation. However, care must still be taken. Green et al. (1999) documents the tendency for long-distance commuting to act as a substitute for migration in the UK. Sandow (2008) studies commuting in Sweden, and finds some workers with a daily commute of over 200 km.

While ignoring commuting may be permissible in a study of migration, there converse is seldom true. Migration, or at least relocation, can take place at any spatial scale. One popular way to model commuting flows is through the use of gravity models (Sen and Smith, 1995). This class of models is well known and has been applied extensively to both commuting and migration. With regard to commuting, one of the assumptions of the doubly-constrained model is that the location of workers and jobs remains fixed. While this may be a reasonable short-run assumption it is unlikely to hold in the medium to long term. In fact, some infrastructure projects may even have the aim of influencing residential location patterns. In such cases, a model which cannot predict or explain these patterns is of limited use.

The failure to account for the interdependency between commuting and migration can have important implications for policy. One particular policy challenge which will be addressed in this paper is rural depopulation. This has long been a policy challenge for the Nordic countries, where low-density regions have experienced fairly strong depopulation (Håkansson, 2000; Hjort and Malmberg, 2006). In particular, we are interesting in analysing whether innovation in the transportation network can help a peripheral area to retain its existing population or, indeed, to grow. Partridge et al. (2010) note that access to urban employment through commuting can be key source of population retention and growth for some rural areas. This presents a perfect example of the relationships between commuting, migration and accessibility.

On a related theme, Renkow (2007) studies who fills new jobs when they are introduced into an area. It is important to understand this if policy markers are attempting to stimulate growth by introducing employment into an area. Renkow (2007) finds that in metropolitan areas most new jobs (60-70%) are filled by commuters rather than locals. The finding for rural areas is somewhat different. In rural areas, an increase in the demand for labour tends to be met largely with a fall in the out-commuting rate. If the policy of increasing employment had been aimed at inducing population growth, Renkow (2007) notes, it would have largely failed. Only thourough understanding the relationships between commuting, migration and accessibility can we formulate effective regional policy.

In this paper, our aim is to construct a model which allows us to simultaneously account for commuting, migration, spatial structure and accessibility. The model will allow us to analyse how changes in one of these variables affect the others. The high degree of non-linearities which are captured by our model can generate seemingly counter-intuitive results. Understanding the reasons for these results is crucial for policy makers. The paper is structured as follows. Section 2 outlines how we model employment and migration. The procedure for modelling the geographic distribution of this employment is presented in Section 4. Section 5 explains how we model commuting flows while Section 6 explains how we deal with all of these factors simultaneously. Section 7 presents the results of a number of numerical experiments which show the sorts of insights which are provided by the model. Some concluding remarks and policy implications are given in Section 8.

## 2 An economic base model for a multi-zonal region

We begin the construction of our model by defining how employment will be treated. The central idea we use is that of economic base theory. For a textbook treatment of this theory see Treyz and Reaume (1993). In essence, the total employment in a region is decomposed into two sectors: local and basic. The level of local sector activity is determined by demand arising from within the study area. Conversely, activity in the basic sector is determined by factors unrelated to intra-regional demand. Using this theory, the regional growth process can be conceptualised as follows. An increase in activity in the basic sector causes a rise in labour demand. This attracts labour to the region and increases the demand for goods and services produced in the local sector. This creates further demand for labour and a positive growth cycle is initiated.

Decomposing employment into the local and basic sectors plays an important role in our model. We begin by defining total employment with the following identity:

$$E(i) \equiv E_b(i) + E_l(i) \tag{1}$$

$E(i) =$ Total employment in zone $i$

$E_b(i) =$ Basic sector employment in zone $i$

$E_l(i) =$ Local sector employment in zone $i$

In our model, we pay little attention to the basic sector industries. Our primary focus is the distribution of workers and employment amongst different zones within a region rather than the development of the region as a whole.

For the moment, we make the grossly simplifying assumption that:

$\boldsymbol{M} = [m_{ij}] =$ An exogenously given commuting matrix

$m_{ij} =$ The probability that a worker lives in zone $i$ and and works in zone $j$

$\boldsymbol{L} =$ A vector which represents the economically active population in the different zones

By definition, the number of workers who have accepted a job offer in a particular zone must be the same as the total employment in that zone:

$$\boldsymbol{ME} = \boldsymbol{L} \tag{2}$$

The next step in the modelling process relates to the distribution of local sector employment, which reflects people's shopping behaviour. The most simple hypothesis is that local sector employment is proportional to the population in the zone.

$$E_l(i) = kL(i) \quad \forall i \tag{3}$$

This assumption about people's shopping behaviour is too crude for a region divided into small zones. In general, we can introduce:

$C$ = $[c_{ij}]$ = An exogenously given shopping matrix

$c_{ij}$ = Number of workplaces in zone $i$ which are supported by shopping from residents from zone $j$

We will look later at what happens when we relax this assumption. For the moment, it follows that the geographic distribution of of employment in the local sector is given by:

$$\boldsymbol{E_l} = \boldsymbol{CL} \tag{4}$$

Given that the inverse of the matrix $\boldsymbol{(I - MC)}$ exists, it follows from Equations (1), (2) and (4) that:

$$\boldsymbol{L} = \boldsymbol{M}(\boldsymbol{I} - \boldsymbol{CM})^{-1}\boldsymbol{E_b} \tag{5}$$

According to this model, the economically active population of a zone is given by:

1. Employment in the basic-sector ($\boldsymbol{E_b}$)
2. The given commuting patterns ($\boldsymbol{M}$)
3. The population's geographic shopping behaviour ($\boldsymbol{C}$)

Employment in the local sector is determined by the same factors:

$$\boldsymbol{E_l} = \boldsymbol{CL} = \boldsymbol{CM(I - CM)}^{-1}\boldsymbol{E_b} \tag{6}$$

# 3 Patterns of migration and population

## 3.1 A model where residential preferences are represented with a parameter

In Nævdal et al. (1996), migration probabilities are modelled based on the characteristics of the geography. They began by introducing a symmetric matrix $\boldsymbol{Q} = \{\boldsymbol{Q}\}_{i,j=1}^{N}$ where all the elements of $\boldsymbol{Q}_{ij} \geq 0, i, j = 1, 2, ..., n$ are dependent on the characteristics of the geography. In particular, we let $\boldsymbol{Q}$ be defined such that the probability matrix, $\boldsymbol{M} = \{P_{i,j=1}\}_{ij}^{N}$, is given by:

$$P_{ij} = \frac{\boldsymbol{Q}_{ij}}{\sum_{k,k\neq j} \boldsymbol{Q}_{kj}} \qquad i,j = 1,2,...,N \tag{7}$$

In analyses of spatial interaction, it is normal to assume that the interaction between different zones depends on characteristics which are symmetric between zones. This applies, for example, to Euclidean distance $d_{ij}$. For a connected network, there exists a unique equilibrium condition for the Markov chain for such a symmetric connected transition matrix. There must therefore also exist a unique solution with the property that $\boldsymbol{ML} = \boldsymbol{L}$, where $\boldsymbol{L}$ represents the population pattern in the region. Nævdal et al. (1996) show that this equilibrium condition is given by eigenvector:

$$\boldsymbol{L} = \begin{bmatrix} \frac{\sum_{i,i\neq 1} Q_{i1}}{1-P_{11}} \\ . \\ . \\ . \\ \frac{\sum_{i,i\neq 1} Q_{iN}}{1-P_{NN}} \end{bmatrix} \tag{8}$$

The introduction of the coefficients $Q_{ij}$ may seem somewhat random. However, Nævdal et al. (1996) show that any assumption about these coefficients can be interpreted as an assumption about migration flows in the equilibrium state, i.e. a constant, and without loss of generality we can assume that $T_{ij} = Q_{ij}$.

In the specification of an operational model, we take the diagonal elements as given. We use the notation $\alpha_i = 1 - P_{ii} \neq 0, i = 1,2,...,N$, i.e. where $\alpha_i$ gives the probability that a person will not stay in zone $i$ within the given time-frame. When it comes to migration between different zones, we first introduce a search strategy where a person evaluates destinations successively outwards over the network. The person will move to the first place where the conditions are 'satisfactory'. Options further out in the network will then not be evaluated. We now introduce a simplifying assumption of constant absorption, defined by the absorption parameter $s$:

$$s^n = \frac{\text{Probability of moving to } (n+1) \text{ neighbour}}{\text{Probability of moving to } n}, \qquad n = 1,...,N$$

This absorption effect can be explained with a starting point in search theory, and forms the basis in the theory of intervening opportunities. The transition between the $n^{th}$ neighbour is proportional to $s^n$, i.e. the probability of moving decreases as the worker evaluates alternatives which lie progressively further out in the network.

Another central hypothesis within regional economics is that distance limits spatial interaction. Seen together, we can say that the the migration flows between zones $i$ and $j$ will be proportional to $\frac{s^n}{d_{ij}^{\beta}}$ where $d_{ij}$ is the distance between the zones, while $\beta$ is a distance deterrence parameter. The symmetric matrix which is derived from this procedure can be normalised into a probability matrix , with the equilibrium condition for this matrix given by the eigenvector $\boldsymbol{L}$ given in Equation (5). The resulting probability matrix is rather complicated. The concept is illustrated with a transition matrix for the three-node system in Figure 1.

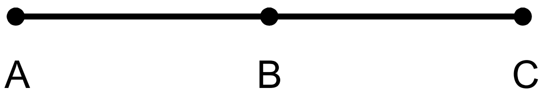


Figure 1: A linear three-node network.

$$\begin{bmatrix} 1-\alpha_1 & \frac{s\alpha_2}{d_{21}^{\beta}}\left(\frac{s}{d_{12}^{\beta}}+\frac{s}{d_{32}^{\beta}}\right) & \frac{s^2\alpha_3}{(d_{12}+d_{23})^{\beta}}\left(\frac{s}{d_{23}^{\beta}}+\frac{s^2}{(d_{12}+d_{23})^{\beta}}\right) \\ \frac{s\alpha_1}{d_{21}^{\beta}}\left(\frac{s}{d_{21}^{\beta}}+\frac{s^2}{(d_{21}+d_{32})^{\beta}}\right) & 1-\alpha_2 & \frac{s\alpha_3}{d_{23}}\left(\frac{s}{d_{23}^{\beta}}+\frac{s^2}{(d_{12}+d_{23})^{\beta}}\right) \\ \frac{s^2\alpha_1}{(d_{21}+d_{32})^{\beta}}\left(\frac{s}{d_{21}^{\beta}}+\frac{s^2}{(d_{21}+d_{32})^{\beta}}\right) & \frac{s\alpha_2}{\left(\frac{s}{d_{21}^{\beta}}+\frac{s}{d_{32}^{\beta}}\right)d_{32}^{\beta}} & 1-\alpha_3 \end{bmatrix}$$

### 3.2 Endogenous probability of staying in a zone

The model which has been outlined contains information on the process for every zone in the network. The model consists of four central parameters:

1. The absorption parameter $s$

2. The step parameter $n$

3. The distance deterrence parameter $\beta$

4. The parameters for residential preference $\alpha_i$

We will now give some consideration to the parameter $\alpha_i$, which governs the decision on whether a worker stays in their current zone or migrates. Nævdal et al. (1996) showed, with reference to fixed point theorem, that the equilibrium solution given by Equation (5) also applies to state-dependent transitions; $M(L)$. When changes are made only to the diagonal elements in the matrix, i.e. when symmetry is preserved, they showed that all of the results from their earlier work still applied.

The model formulation which follows is based on a hypothesis that the probability of remaining in zone $i$, $(1-\alpha_i)$, is positively related to the labour market accessibility of the zone. This is consistent with the findings from Swedish microdata of Lundholm (2010) and Eliasson et al. (2003), while Van Ham and Hooimeijer (2009) finds a similar result for the Netherlands. The explanation is that labour market accessibility allows greater flexibility, and can generally be seen as a favourable attribute for a residential location.

It is easy to find examples of sparsely populated rural areas where unemployment is close to zero and out-migration of the working age population is high (McArthur et al., 2010). For example, in western Norway many of the most rural areas have no unemployment to speak of while the centres of economic activity have the highest rates in the region. At the same time, these rural areas are experiencing persistent migration towards these regional centres. An important reason for this is that the probability of finding an appropriate job in a peripheral area is low, and known, causing workers to migrate out of the region or drop out of the labour force. The point is that when labour market accessibility is below some critical level, it is the local balance between the demand and supply of labour which determines the probabilities of staying in a zone. This effect will be incorporated into our model when we define the relationship between the tendency to migrate and labour market accessibility.

One candidate for an accessibility measure would be the so-called Hansen (1959) measure. However, we choose to use a measure for generalised distance. We let the generalised distance from zone $i$ be given by:

$$d_i = \sum_{j \neq i} \frac{W_j}{\sum_{k \neq i} W_k} d_{ij} \tag{9}$$

where $W_j$ is the weight which is assigned to zone $j$. One possibility is that $W_j = E_j, j = 1, 2, ..., N$. $d_i$ is then defined as the average Euclidean distance to the different employment opportunities in the system. However, imagine a zone which has many close neighbours and at the same time a high average distance to all of the other zones. In such a situation, the favourable position of that zone would not be reflected through the measure of generalised distance. We will therefore use a different measure based on the following logistic expression:

$$D(x) = \frac{1}{1 + e^{-k(x-x_0)}} \qquad x_0 = \frac{1}{2}(d_0 + d_\infty), k = \frac{2\log(\frac{1}{\mu} - 1)}{d_\infty - d_0} \tag{10}$$

Here, $d_\infty$ is the upper limit for how far people, as a rule, are willing to commute on a daily basis, $d_0$ is the lower limit (internal distance) where people are insensitive to further decreases in distance while $\mu$ captures friction effects in the system; which may be due to lags in the migration process etc. If the overall friction is set to $\mu = 0.05$, this means that the function will fall to 5% of its value outwith the range where $d_0 \leq x \leq d_\infty$. Here, the values of $x_0$ and $k$ are given so that the satisfy the conditions $D(d_0) = \mu$ and $(1 - D(d_\infty)) = \mu$. This function places a relatively high weight on destinations which lie within a short distance from the residential location. Glenn et al. (2004) give a microeconomic and geometric justification for the use of such a function.

We therefore let all nodes in the system be weighted by this logistic function:

$$W_j = E_j(1 - D(d_{ij})) \tag{11}$$

In addition, we add a variable which defines employment opportunities in the labour market i.e. that we measure the number of work places as a proportion of the total number of job seekers in each potential destination, $\frac{E_j}{L_j}$. This captures the competition for jobs (Liu and Zhu, 2004; Shen, 1998).

$$W_j = E_j(1 - D(d_{ij}))\frac{E_j}{L_j} \tag{12}$$

The generalised distance measure $d_i$ which we have developed is now included in the diagonal elements of the migration matrix:

$$\alpha_i = \alpha(L_i) + D(d_i)\max\{\rho\left(\frac{L_i - E_i}{L_i}\right), 0\} \tag{13}$$

We include two other terms in the diagonal elements of the migration matrix. The first term is only used for high values of $d_i$, and reflects the net supply of labour, $(L_i - E_i)$, within a zone.

For peripheral zones, it is this which drives the migration flows; $\rho$ is a parameter which reflects how quickly the zone moves towards a situation with a balance in the labour market, $L_i = E_i$. For $E_i > L_i$, there is no reason to suppose that out-migration from a zone will be larger than that which is given by the first term of the function. This first term in Equation (13) is determined by factors other than those relating to the labour market. We will treat the migration probabilities as interdependent:

$$\alpha_i(L_i) = \begin{cases} \alpha_0 & \text{if } \; L > L_0 \\ 1 + \frac{\alpha_0 - 1}{L_0} \cdot L & \text{if } \; L < L_0 \end{cases}$$

This simple function thus accounts for the fact that the probability of migrating from zone $i$ is state-dependent. Out migration is a constant equal to $\alpha_0$ when $L$ is above the threshold, but increases when the population is lower than $L_0$. The rationale for this is that when the population is below some critical level a zone will begin to lose amenities such as schools, shops etc.

# 4 The geographic distribution of employment

The assumption of an exogenously given shopping matrix $C$ is obviously unreasonable. There are good reasons to believe that patterns of shopping are influenced by the spatial structure of the region and the transportation network. Gjestland et al. (2006) develop a model for the distribution of local-sector activities within a region. For a whole region it is reasonable to assume proportionality:

$$E_l(r) = \sum_i^n E_l(i) = b \sum_i^n L(i) = b \cdot L(r) \qquad b > 0 \tag{14}$$

where $b$ is the proportion parameter and $n$ is the number of zones.

We take as a starting point for the theory $\frac{E_l}{L}$, which we define as the number of shop-employees per resident. In this way, we derive a relationship which is independent of how residential patterns are in the actual region. As a first simplifying assumption, which can be relaxed relatively easily, we assume that the region has one centre, and discuss how $\frac{E_l}{L}$ varies systematically with distance to this centre.

Some types of local-sector activity will largely be concentrated in the centre. This reflects the benefits of agglomeration. For example, one can argue that administrative services often locate in the centre, and that this gives rise to agglomeration benefits which in turn attract more activity to the centre. Businesses often choose to locate in the same area because from consumers' often perceive it to be beneficial if they can satisfy their demand for several goods and services with one shopping trip.

In the description of the centre we account for the fact that activities demand space, and therefore the pure geometric centre must have some geographic extent. However, as shown by Fu (2007), many of the benefits from agglomeration decay sharply with distance from the centre. We measure agglomeration with the number of retail jobs per resident, and let the agglomeration forces, $\frac{E_l}{L}(agg)$, be represented by the following function of Euclidean distance:

$$\frac{E_l}{L}(\text{centre}) = \frac{E_l(c)}{L(c)} e^{-\frac{d_{ic}^2}{2\sigma^2}} \tag{15}$$

There are therefore 2 parameters which determine the strength of agglomeration benefits in and around the centre:

1. $\frac{E_l(c)}{L(c)}$ regulates the level of agglomeration in the centre
2. $\sigma$ represents the geographic extent of the centre

We expect $\frac{E_l(c)}{L(c)}$ to vary between different centres. For example, it can be argued that the importance of a centre will be a decreasing function of an average distance, $d_c$, to potential customers outside of the centre. This average distance is defined by:

$$d_c = \sum_i \frac{W_i}{\sum_k W_k} d_{ic} \tag{16}$$

$$W_i = L_i(1 - D(d_{ci})) \tag{17}$$

The weights are now determined by the potential market which exists at different distances from the centre. In our model, we let the dominance of the centre be explained by:

$$\frac{E_l(c)}{L(c)} = \frac{E_l(r)}{L(r)}(1 + a(1 - D(d_c))) \tag{18}$$

From this, it follows that $\frac{E_l(c)}{L(c)} \to \frac{E_l(r)}{L(r)}$ when $d_c \to \infty$. When $d_c \to 0$, $\frac{E_l(c)}{L(c)} \to (1+a)\frac{E_l(r)}{L(r)}$, which therefore represents a maximum for the agglomeration forces of the centre. For given values of $d_c$, the parameter $a$ represents a measure for agglomeration i.e. it determines the maximum value of the normally-distributed function.

We develop these ideas by recognising that due to economies of scale, transportation costs and agglomeration benefits allow firms in a central location to offer goods and services at a lower price than firms located in more peripheral locations. A consumer will weigh the benefit of lower costs in the centre against the costs of reaching the centre. Transportation costs seen in this way provide an incentive for firms to decentralise in order to cater for local demand.

This trade-off between transport costs and potential price savings plays a central role in Gjestland et al. (2006). They begin with a simple situation where consumers demand only one good and transport costs are proportional. They then move towards a steadily more realistic situation regarding the distribution of price savings, product range, shopping frequency and the valuation of time. The more realism added to these conditions, the closer they come to a smooth, concave function for the frequency of shopping locally. For full details, see to Gjestland et al. (2006). Here, we conclude that we have a relatively strong theoretical base for working with a concave, increasing function of Euclidean distance:

$$\frac{E_l}{L}(distance) = \frac{E_l(r)}{L(r)}(1 - e^{-\gamma d_{ic}}) \tag{19}$$

Note that $\frac{E_l}{L}(distance) \to \frac{E_l(r)}{L(r)}$ when $d_{ic} \to \infty$

As in Weberian industrial location theory, location patterns are the result of the net effect of different forces:

$$\frac{E_l}{L} = \frac{E_l}{L}(\text{centre}) + \frac{E_l}{L}(\text{distance}) = \frac{E_l(c)}{L(c)} e^{-\frac{d_{ic}^2}{2\sigma^2}} + \frac{E_l(r)}{L(r)}(1 - e^{-\gamma d_{ic}}) \tag{20}$$

There are therefore four parameters determining the distribution of local-sector activity: $a$, $\frac{E_l(r)}{L(r)}$, $\sigma$ and $\gamma$.

The parameter $\gamma$ is determined based on the balancing requirements of the region we study. For a discretely defined geography split into zones we have specifically:

$$\sum_i \frac{E_l}{L}(i) \cdot L(i) = E_l(r) \tag{21}$$

Seen graphically, the parameter $\gamma$ determines how quickly the function for $\frac{E_l}{K}$ flattens out against $\frac{E_l(r)}{L(r)}$ in Figure 2. The higher the value of $\gamma$, the quicker the curve flattens out. The graph represents the net effect of the forces we model.

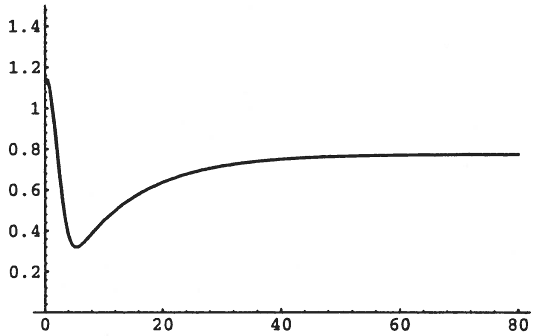


Figure 2: The variation in $\frac{E_l}{L}$ as the distance from the centre increases.

The curve for $\frac{E_l}{L}$ shown in Figure 2 has a clear intuitive appeal. For example, a significant proportion of the shopping trips emanating from a suburb will, as a rule, be directed towards the regional centre. For zones which lie a high distance from centres, virtually all shopping will take place within the zone i.e. that $\frac{E_l(i)}{L(i)} = \frac{E_l(r)}{L(r)}$. It is also possible to account for situations where there exists more than one centre. However, we do not discuss this point here.

## 5 The calculation of commuting flows

The next stage in the modelling process is to define how commuting flows between the nodes in the system are determined. The model used in this paper belongs to the gravity modelling tradition. For a general discussion of this tradition, see Erlander and Stewart (1990) or Sen and Smith (1995). In a gravity model, it is assumed that spatial interaction is explained by the distance between an origin and a destination, and by two aggregate measures: one to account for the generativity of origins and the other to address the attraction of destinations. In studies of journeys to work, we typically define the generativity of origins by the number of workers, while we usually measure the attraction of destinations by total employment.

In this paper, we will be using a doubly-constrained version of the gravity model. This means that we introduce a set of balancing constraints, ensuring that the column sums of the predicted commuting flow matrix equal the total number of jobs at the corresponding destinations, and that each row sum equals the number of workers residing in the corresponding zone. Hence, the model is based on the assumption of a given spatial distribution of jobs, and a given spatial residential pattern. It is well known that a doubly-constrained gravity model is equivalent to the multinomial logit model, see Anas (1983) for details. This means that the model can be derived from random utility theory. The formulation we use is given below:

$$T_{ij} = A_i O_i B_j D_j e^{(-\beta d_{ij})} \tag{22}$$

$$A_i = \left[ \sum_j B_j D_j e^{(-\beta d_{ij})} \right]^{-1} \tag{23}$$

$$B_j = \left[ \sum_i A_i O_i e^{(-\beta d_{ij})} \right]^{-1} \tag{24}$$

Here:

$T_{ij}$ is the number of commuters from origin $i$ to destination $j$

$O_i$ is the observed number of commuting trips originating from zone $i$

$D_j$ is the observed number of commuting trips terminating in zone $j$

$d_{ij}$ is the travel time from origin $i$ to destination $j$

$A_i$ and $B_j$ are the balancing factors which ensure the fulfilment of the marginal total constraints; $\sum_j T_{ij} = O_i$ and $\sum_i T_{ij} = D_j$. Consequently, this doubly-constrained model specification is constructed for a pure trip distribution problem.

# 6 A simultaneous model for the geographical distribution of residential and workplace location, and for migration and commuting flows

To initiate the iterative process, we begin with more or less random random initial values for employment and population ($E_0 (= E_{l0} + E_{b0})$ and $L_0$). These values are fed into the migration matrix M, which is then iterated until we find the fixed point[1] $L_1$, which represents the equilibrium solution for population (workers) i.e. that $M^{l_1} L_1 = L_1$. After this, $L_1$ is used as an input into the part of the model dealing with local-sector employment. This model gives updated values for $E_l$, which then gives new input into the migration matrix and so on, in an iterative process towards equilibrium for the geographic distribution of population and employment. When we have predicted the new zonal distribution of population and employment, the commuting matrix is updated according to the model outlined in Section 5.

## 7 A numerical example

In this section we will outline several numerical experiments which show how our model works and the sorts of insights it provides. All of our experiments will be based on the geography depicted in Figure 3. Node B is the centre in this geography, Node D a suburban zone, Node A represents the periphery, while Nodes E and C lie within reasonable commuting distance of the centre.

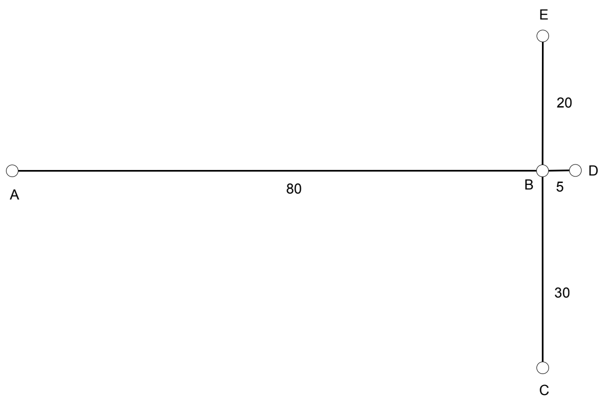


Figure 3: A 5-node network.

For illustrative purposes, we use the following parameter values:

- Migration flows: $s = 1$ and $\beta = 0.3$
- The logistic distance function: $d_0 = 5$, $d_\infty = 70$ and $\mu = 0.05$
- The probability of staying in a zone: $\alpha_0 = 0.05$, $\rho = 1$ and $L_0 = 500$

[1] A continuous mapping of a convex and compact quantity into itself always has a fixed point, according to Brouwer's fixed point theorem, which applies to n-dimensional space.

- The geographic distribution of local-sector activities: $\sigma = 2$ and $a = 1$

Table 1 presents the equilibrium solution corresponding to these parameter values and the initial values we used. Note that $E_l = E^0 - E_b$.

Table 1: The equilibrium spatial distribution of employment and population for the initial situation.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| Basic sector employment, $E_b$ | 250 | 1000 | 300 | 300 | 400 |
| Initial total employment, $E^0$ | 1000 | 6000 | 1000 | 1000 | 1000 |
| Initial population, $L^0$ | 1000 | 6000 | 1000 | 1000 | 1000 |
| Equilibrium local sector employment, $E_l$ | 1358 | 3530 | 1157 | 554 | 1152 |
| Equilibrium population, $L$ | 1754 | 3116 | 1612 | 1715 | 1803 |

## 7.1 Bifurcation

Generally, it would not be expected that the choice of initial values would affect the equilibrium solution generated. However, within our modelling framework, this can happen. Assume, for example, that the initial values for population and employment in zone A are much lower than in Table 1: $E_l^0(A) = L^0(A) = 50$, i.e. that $E^0 = 300$. To keep employment in the system as a whole constant, the jobs removed from Zone A have to be redistributed amongst the other zones in the system. It turns out that it does not matter how these jobs are redistributed. In Table 2 we can see that the equilibrium solution gives much lower population and employment in Zone A, while the activity level is higher in the other zones. The redistribution of local sector activities is particularly concentrated in the central Zone B, while the population changes are smoothly distributed between the other zones.

Table 2: The spatial equilibrium solution for $E_l^0(A) = L^0(A) = 50$.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| Basic sector employment, $E_b$ | 250 | 1000 | 300 | 300 | 400 |
| Equilibrium local sector employment, $\mathrm{E}_l$ | 143 | 4384 | 1286 | 649 | 1288 |
| Equilibrium population, L | 185 | 3864 | 1813 | 2081 | 2057 |

The explanation for the change in the character of the equilibrium solution is that the population level has fallen below a threshold value, such that the probability of migrating out of the zone increases. This results in reduced demand for locally produced goods, and reduces local-sector employment, which increases out-migration and so on. This also initiates a multiplier process towards an equilibrium solution where there is a very low level of activity in Zone A. This is particularly likely to occur in peripheral zones, where reduced employment makes the zone markedly less attractive as a residential location. This is especially true since commuting opportunities are limited. We see no corresponding effect in any of the other zones in our example.

There are strong non-linear effects driving the system towards this new equilibrium. With the simulation presented in our numerical example, we find that the equilibrium solution presented in Table 2 is generated for all initial values of local-sector employment lower than 317. For *all* other situations, the equilibrium solution presented in Table 1 pertains. Local sector employment of 317 therefore represents a *bifurcation* point.

Our simulation also shows that the equilibrium is entirely independent of which values are used for the total initial population. The bifurcation point is determined solely by the initial distribution of employment. So long as the total initial employment in Zone A is lower than 317, the economy will arrive at an equilibrium with low population and employment in that zone. Note the tendency that employment tends to be more geographically concentrated than population, irrespective of the initial situation.

The bifurcation point is dependent on the distance between zones A and B. Assume, for example, that the distance between these zones is reduced by 14 km due to an improvement in the transportation network. In this case, we will not find a bifurcation point. The equilibrium solution will be the same irrespective of the initial values for population and employment.

Figure 4 illustrates how the equilibrium solution for Zone A varies with distance between zones A and B for high initial levels in Zone A.

- When the distance is short, Zone A takes on the role of a suburb, with low employment and high population. Many of these workers commute.

- When the distance increases, Zone A becomes less attractive for commuters. Up until a distance of around 45 km, this effect dominates the effect that more people will want to shop locally, thus increasing local employment. For distances between 45 km and around 80 km, it is the effect of local employment which dominates the population development. For distances over 85 km, both population and local employment decline.

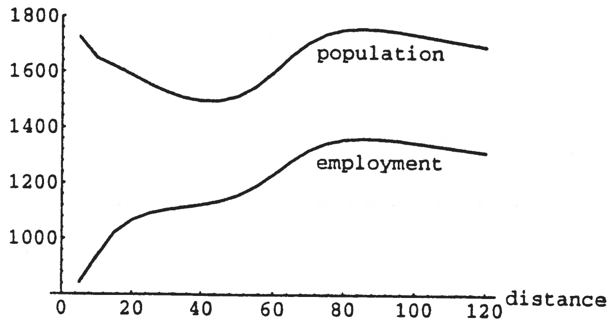


Figure 4: The response of employment and population in Zone A to changes in $d_{AB}$.

- The imbalance between the supply and demand for labour in Zone A, $L(A) - E(A)$, is largest with a short distance, and at first quickly decreases as the distance increases.

- In our example the 'optimal' for Zone A itself is to be located around 85 km from the central zone; optimal in the sense that it maximises population and employment levels. This is not necessarily optimal from a social welfare perspective for the residents in the zone however. Note that innovation in the transport network can lead to a decline in population and employment.

Figure 5 shows that the effect of distance is more dramatic when we begin with low initial values for employment in Zone A. For distances over 66 km from the centre, we pass the bifurcation point. Note that innovations in the transport network *can* lead the zone to a more favourable equilibrium.

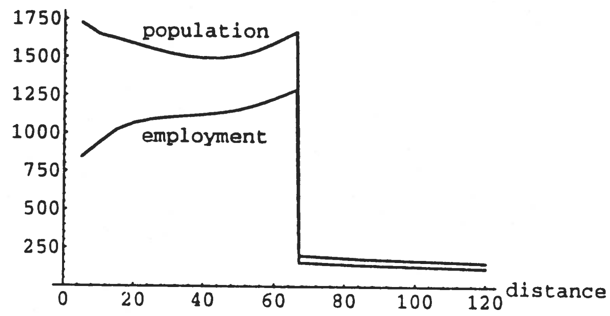


Figure 5: The response of employment and population in Zone A to changes in $d_{AB}$ for a situation with low initial employment in Zone A.

## 7.2 The effect of changes in basic sector employment

For the next experiment, we let the basic sector employment increase by 500 for each zone sequentially, at the same time as holding overall employment constant with a corresponding proportional decrease in the other zones.

Table 3: The impact on population and local-sector employment when $E_b$ is increased in one zone at a time. Aggregate basic sector employment is unchanged, through proportional reductions in basic-sector employment in the other zones.

| | | A | B | C | D | E |
|---|---|---|---|---|---|---|
| $\Delta E_b(A) = 500$ | $E_l$ | 1509 | 3555 | 1068 | 522 | 996 |
| | $L$ | 1975 | 3177 | 1527 | 1705 | 1616 |
| $\Delta E_b(B) = 500$ | $E_l$ | 1346 | 3884 | 960 | 566 | 995 |
| | $L$ | 1739 | 3420 | 1365 | 1863 | 1613 |
| $\Delta E_b(C) = 500$ | $E_l$ | 1279 | 3432 | 1636 | 455 | 951 |
| | $L$ | 1652 | 3037 | 2311 | 1473 | 1526 |
| $\Delta E_b(D) = 500$ | $E_l$ | 1271 | 3434 | 1095 | 787 | 1165 |
| | $L$ | 1640 | 3027 | 1475 | 2145 | 1712 |
| $\Delta E_b(E) = 500$ | $E_l$ | 1265 | 3409 | 995 | 497 | 1584 |
| | $L$ | 1634 | 3014 | 1380 | 1515 | 2485 |

When we compare the results to those in Table 1 we can see that:

- Increased basic-sector employment in Zone A has a positive effect on Zone B, while zones C and E experience the most negative outcomes. It is this redistribution which has the smallest effect on the regional distribution of population and employment.
- A stronger concentration of basic-sector employment in Zone B has a positive effect on the suburb Zone D. Zone A is affected only marginally while zones C and E are big losers from the change.
- Increased basic-sector employment in either Zone C or E will primarily benefit the zone itself at the cost of all the other zones. It is mainly the other zone in the corresponding position which loses, while the effect is weaker for the central zone and the peripheral zone.

## 7.3 The effect of a change in the transport network

As a final experiment, we will predict the effect on commuting flows of decreasing the distance between zones A and B from 80 km to 50 km following a change in the transport network. We distinguish between the following three situations:

The existing road network:

$$\begin{bmatrix} 1583 & 171 & 0 & 0 & 0 \\ 14 & 1744 & 391 & 438 & 529 \\ 0 & 841 & 686 & 53 & 32 \\ 8 & 728 & 314 & 203 & 462 \\ 1 & 1046 & 66 & 161 & 529 \end{bmatrix} \tag{25}$$

The new road network with fixed population and employment patterns:

$$\begin{bmatrix} 1315 & 439 & 0 & 0 & 0 \\ 145 & 1824 & 339 & 353 & 455 \\ 6 & 750 & 693 & 99 & 64 \\ 120 & 553 & 339 & 208 & 494 \\ 21 & 964 & 86 & 194 & 538 \end{bmatrix} \tag{26}$$

The new road network with variable location patterns:

$$\begin{bmatrix} 1142 & 370 & 0 & 0 & 0 \\ 134 & 1947 & 352 & 368 & 471 \\ 5 & 787 & 704 & 91 & 58 \\ 107 & 586 & 339 & 209 & 492 \\ 17 & 1006 & 82 & 186 & 547 \end{bmatrix} \tag{27}$$

The results show that:

- When we do not account for relocation effects, commuting from Zone A increases by 268, while commuting into Zone A increases by 269. Total commuting on this link therefore increases by 537.
- When we account for relocation effects, commuting from Zone A increases by 199, while commuting into Zone A increases by 263. Total commuting on the link therefore increases by 462.

When we account for the fact that actors adjust their location to account for the new situation, we would perhaps expect a further increase in commuting. However, we predict that aggregate commuting will fall by a little over 10% as a result of the relocation effects. The situation *can* be like that seen in Figure 5, where the change in the transport network took Zone A past the bifurcation point, giving strong growth. In such a case, the model would of course predict a much stronger increase in commuting when we account for relocation effects.

Figure 6 shows how aggregate commuting depends on the distance between zones A and B. Total commuting is measured both in the form of the number of commuters and the total commuter-kilometres. In Figure 6, we present both of these measures using indices, where 100 corresponds to $d_{AB} = 80$. The actual numbers for $d_{AB} = 80$ are 5255 of the 10000 workers commuting between the different zones, while the average commuting distance per worker is 23.4 km.

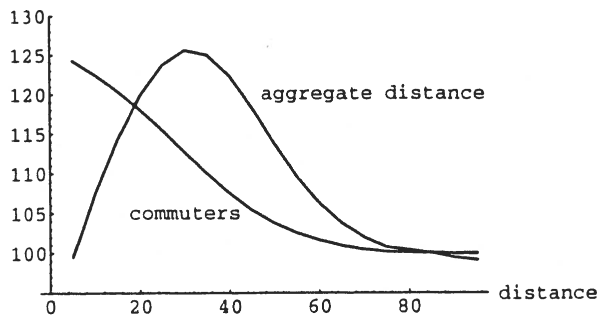


Figure 6: Indices for aggregate commuting distance and total number of commuters as the distance between Zone A and Zone B is systematically varied. The indices are equal to 100 for $d_{AB} = 80$

Figure 6 also shows that the number of commuters increase markedly as the distance $d_{AB}$ is reduced. The increase is strongest when the distance is reduced from a starting point below 50 km. The total number of commuter-kilometres is highest for a distance of around 30 km. For distances shorter than this, the number of commuters rises, but the average commuting distance decreases sharply. The net effect is to reduce aggregate commuter-kilometres. For distance reductions with a starting point higher than 50 km, the aggregate number of commuter-kilometres will increase i.e. the effect of a larger number of commuters will dominate.

# 8 Conclusion

The main aim of this paper was to generate a model which could simultaneously deal with commuting and migration. It was important that spatial structure was adequately captured in the model. Changes to the distribution of workplaces or transportation network affect zones differently depending on where the zone is located. More specifically, The accessibility of a zone will determine whether we see changes in population or commuting. This has been found empirically in the literature. We believe our model was successful in capturing this effect. We showed with our numerical experiments that the results of changing the transportation network or the employment in a particular zone was heavily dependent on the spatial configuration of the zones.

From a policy perspective, we were particularly interesting in building a modelling framework which could be used to examine the issue of rural depopulation. The model is suitable for such purposes. Of particular interest was the non-linear effects inherent in our model. We showed that there exist certain critical value, or bifurcation points, above and below which we observe different equilibrium solutions. This has important implications for policy analysis. For instance, we showed that in one of our experiments, reducing the distance between two zones can actually lead to a fall in commuting. This highlights the importance of accounting for relocation effects rather than assuming a fixed distribution of residences and workplaces. Once again, we showed that the effect on commuting would depend on the spatial structure.

While we have focused largely on the fortunes of rural regions, the modelling framework has numerous other applications. Consider once more the finding regarding the upgrading of infrastructure and the fall in commuting. It seems intuitive to assume that building more roads will increase the flow of traffic. However, as we have shown, changing residential patterns do not guarantee that this will be the case. If we were interested in the environmental consequences of a change to the transport infrastructure, such knowledge is very useful. It is also useful to be able to analyse such effects in a framework which allows us model the impact on regional development. Indeed, these two concepts should not be separated if strategies for sustainable development are to be devised; sustainable both from a community and environmental perspective.

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