Skill-Biased Share-Altering Technical Change in Spatial General Equilibrium

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Abstract. The paper consider the skill-biased "share-altering" technical change hypothesis in a spatial general equilibrium model where skilled and unskilled individual may exhibit different preferences for local amenities. A main novelty –both for labour and urban economics- is that, under this hypothesis, skill-biased technical change can be readily represented by simple Cobb-Douglas production functions, rather than CES technologies. We then analyse the local labour markets equilibrium, where the adoption of new technologies may require an adequate proportion of skilled workers.

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I. INTRODUCTION

During the last two decades much attention has been devoted to types of technical change that are biased towards skilled labour: see, e.g., Acemoglu (2002) and Per Krusell et al. (2000). Recently, Beaudry et al. (2010) have shown that skill-intensive PC technologies have been adopted in cities where highly educated workers were abundant. At the same time, new developments in urban economics have claimed that high levels of local human capital may attract even more educated workers, as in the “rise of the skilled city” story: see Glaeser (2008) for an overview. In particular, Berry and Glaeser (2005) have found that demand for high skilled workers has been rising in initially high skill cities. Also, for what it concerns education levels, there has been increased sorting across metropolitan areas.

This paper makes two main contributions. The first novelty, mainly technical, is that - in a model with two types of workers (skilled and unskilled) - skill-biased technical change can be given a Cobb-Douglas, rather than a standard CES, representation.[1] This step exploits the “share-altering” technical change hypothesis put forward by Seater (2005), Seater and Peretto (2008) and Zuleta (2008). The idea is rather simple: consistently with observation, most modern innovations seem to raise the share parameter of skilled workers while keeping constant the *total* labour share. We show that, under the skill-biased share-altering hypothesis, we are able to replicate the results about endogenous skill-biased technology adoption that Beaudry et al. (2010) derive from a traditional CES approach. Second, we cast the share-altering hypothesis into a spatial Roback-type framework where firms and workers (be them skilled or unskilled) are free to move across locations. As in the basic spatial framework introduced by Glaeser (2008), we model both preferences and technology as Cobb-Douglas functions. However, when Glaeser proceeds to investigate local skill-premia, he drops the Cobb-Douglas specification in favour of a CES technology. We argue that this step, under the share-altering hypothesis, is unnecessary.

The second main contribution of the paper has to do with the implications from spatial equilibrium. We show that new, share-altering technologies can be profitable only when there is an adequate local proportion of skilled workers.[2] For this reason, areas endowed with amenities which are particularly attractive to the educated, are also likely to satisfy the requirements for the adoption of such new technologies. This is a distinctive mechanism from traditional agglomeration externalities. Areas that benefit from the adoption of share-altering skilled-biased technologies will also exhibit higher wages and rents. Finally, the skill-mix tends to increase further in areas where such technologies are adopted: in other words, the model predicts a disproportionate inflow of skilled workers relative to unskilled workers.

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[1] Traditionally, Cobb-Douglas technologies have been considered unfit to represent skill-bias as emphasized, for example, by Acemoglu (2002, p.785-786).

[2] This implication is similar to those obtained by the matching model in Acemoglu (1996), where investment depends on the average skill level of the workforce. In Berry and Glaeser (2005), a higher the number of educated residents will generate more skilled entrepreneurs who hire skilled workers. In this perspective, the initial level of city skills crucially determine the future level of local skill demand. Our implication however is closest to Beaudry et al. (2010).

The paper is organized as follows. Section II.1 describes the basic Roback model, where skilled and unskilled individuals exhibit heterogeneous preferences foro local amenities. Then, in Section II.2, we discuss the implications of share-altering technical change. Section III concludes.

II. THE MODEL

We consider a standard general equilibrium model, where firms and workers are perfectly mobile across areas: see Roback (1982,1988). The economy is composed of two areas, Area 1 and Area 2, which are endowed with different characteristics, affecting both local productivity and residents' "quality of life". In each area, firms produce an homogeneous good by using two types of labour, skilled and unskilled, and "land". The good is traded competitively across areas. Workers earn a wage and consume both the produced good and "land". For simplicity, the supply of "land" in each area is taken to be fixed and landowners are absentee. Since firms are assumed perfectly mobile between areas, profits will be equalized across the economy. Similarly, when mobility costs are absent, workers' utility will be perfectly equalized across areas.

For what it concerns individuals' utility, we assume that each area possesses some local characteristics that affect the quality of life of both skilled and unskilled individuals. However, we postulate that there are other local features that affect the utility of skilled individuals only.[3]

We start by describing the features of spatial general equilibrium (which has become the standard analytical tool for the analysis of local labour markets: see Moretti 2008,2010) in the absence of technical change.

1. The basic framework

The local supplies of skilled labour, unskilled labour and land are given, respectively, by $\left\{n_c^s, n_c^u, \bar{\ell}_c\right\}$, with $c = \{1,2\}$. We first illustrate firms' optimal behavior and, then, we look at skilled and unskilled workers, so to characterize the equilibrium in the two areas.

Firms. Firms in area $c = \{1,2\}$ produce an homogeneous good by using land, L_c, and both skilled and unskilled labour, respectively $\left\{N_c{}^s, N_c{}^u\right\}$, with a Cobb-Douglas technology characterized by constant returns to scale:

$$Y_c = A_Y(Q_c) \cdot L_c^{1-\alpha-\beta} \cdot \left(N_c^s\right)^\alpha \cdot \left(N_c^u\right)^\beta \qquad (1)$$

where $\alpha + \beta \in (0,1)$. The term $A_Y(Q_c)$ denotes the impact of the vector Q_c of local characteristics on firm's productivity. We postulate that the elements of Q_c, q_{ic}, are measured in a way such that $\partial A_Y / \partial q_{ic} \geq 0$.[4] Respectively, $\{r_c, p_c\}$ denote the local price of land (rent) and the price of the traded good.

[3] This assumption is consistent, for example, with the findings in Carlino and Saiz (2008) for the US, and Dalmazzo and de Blasio (2010) for Italy, and is exploited also in some model extensions of Glaeser (2008).

[4] The vector of local characteristics affecting firms' productivity, Q_c, does not necessarily coincide with local characteristics affecting residents' utility.

In what follows, we will assume that $p_1 = p_2 = 1$. The wage received by a skilled worker in area *c* is denoted by w_c^s, while the unskilled wage is equal to w_c^u.

A competitive firm located in *c* will equate price to marginal cost, which is equivalent to:

$$\vartheta \cdot \left[\frac{A_Y(Q_c)}{r_c^{1-\alpha-\beta} \left(w_c^s\right)^{\alpha} \left(w_c^u\right)^{\beta}} \right] = 1, \tag{2}$$

where $\vartheta \equiv (1-\alpha-\beta)^{1-\alpha-\beta} \alpha^{\alpha} \beta^{\beta}$. Since firms are perfectly mobile across areas, it must hold that:

$$\left(\frac{r_1}{r_2}\right) = \left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right)^{\frac{1}{1-\alpha-\beta}} \cdot \left(\frac{w_2^{\,s}}{w_1^{\,s}}\right)^{\frac{\alpha}{1-\alpha-\beta}} \cdot \left(\frac{w_2^{\,u}}{w_1^{\,u}}\right)^{\frac{\beta}{1-\alpha-\beta}} \tag{3}$$

Skilled workers. Skilled workers living in area *c* maximize the utility function

$$U_c^s = A_U(X_c) \cdot B_U(Z_c) \cdot L_c^{1-\mu} \cdot Y_c^{\mu} \tag{4}$$

subject to the following budget constraint:

$$r_c \cdot L_c + Y_c = w_c^{\,s} \tag{5}$$

This skilled worker's utility function includes an "amenity" term $A_U(X_c)$, non-decreasing in the vector of local characteristics X_c, which is common to unskilled utility (see below). However, skilled utility also includes an additional "amenity" term $B_U(Z_c) \geq 1$, non-decreasing in Z_c. The vector Z_c denotes some additional territorial characteristics that are valuable to skilled individuals, but irrelevant to the welfare of the unskilled. The optimal choice of the consumption bundle generates an indirect utility for a skilled resident in area $c = \{1,2\}$ given by:

$$v_c^s = \eta \cdot A_U(X_c) \cdot B_U(Z_c) \cdot \frac{w_c^s}{r_c^{1-\mu}}, \qquad c = \{1,2\} \tag{6}$$

where $\eta \equiv (1-\mu)^{1-\mu} \mu^{\mu}$. A skilled worker is indifferent whether to migrate or not whenever the condition $v_1^{\,s} = v_2^{\,s} = \bar{v}^s$ holds. Thus, it follows that:

$$\left(\frac{w_1^s}{w_2^s}\right) = \frac{A_U(X_2)}{A_U(X_1)} \cdot \frac{B_U(Z_2)}{B_U(Z_1)} \cdot \left(\frac{r_1}{r_2}\right)^{1-\mu} \tag{7}$$

Unskilled workers. An unskilled worker in area *c* receives a wage equal to w_c^u. By maximizing utility[5] subject to $r_c \cdot L_c + Y_c = w_c^u$, one obtains the expression of the indirect utility of an unskilled worker who resides in region *c*:

$$v_c^u = \eta \cdot A_U(X_c) \cdot \frac{w_c^u}{r_c^{1-\mu}}, \qquad c = \{1,2\} \tag{8}$$

Free mobility of unskilled individuals implies that $v_1^u = v_2^u = \bar{v}^u$. Then, the following equilibrium condition must hold:

$$\left(\frac{w_1^u}{w_2^u}\right) = \frac{A_U(X_2)}{A_U(X_1)} \cdot \left(\frac{r_1}{r_2}\right)^{1-\mu} \tag{9}$$

Expression (9) shows that the relative wage among the unskilled depends on the rent ratio between areas.

Equilibrium in the absence of technical change. The relative rent ratio, the relative wage ratios among skilled workers and unskilled workers are obtained by solving the system given by equations (2), (7) and (9). Substituting (7) and (9) into (3) and rearranging yields:

$$\log\left(\frac{r_1}{r_2}\right) = \frac{1}{1-\mu(\alpha+\beta)}\left\{\log\frac{A_Y(Q_1)}{A_Y(Q_2)} + (\alpha+\beta)\cdot\log\frac{A_U(X_1)}{A_U(X_2)} + \alpha\cdot\log\frac{B_U(Z_1)}{B_U(Z_2)}\right\}. \tag{10}$$

Then, substituting (10) into (7) and (9) gives, respectively:

$$\log\left(\frac{w_1^s}{w_2^s}\right) = \frac{1}{1-\mu(\alpha+\beta)}\left\{(1-\mu)\cdot\log\frac{A_Y(Q_1)}{A_Y(Q_2)} - (1-\alpha-\beta)\cdot\log\frac{A_U(X_1)}{A_U(X_2)} - (1-\alpha-\mu\beta)\cdot\log\frac{B_U(Z_1)}{B_U(Z_2)}\right\} \tag{11}$$

[5] Unskilled utility has the same structure as skilled utility (4), except for the absence of the amenity term $B_U(Z_c)$.

and

$$\log\left(\frac{w_1^u}{w_2^u}\right) = \frac{1}{1-\mu(\alpha+\beta)}\left\{(1-\mu)\cdot\log\frac{A_Y(Q_1)}{A_Y(Q_2)} - (1-\alpha-\beta)\cdot\log\frac{A_U(X_1)}{A_U(X_2)} + \alpha(1-\mu)\cdot\log\frac{B_U(Z_1)}{B_U(Z_2)}\right\} \quad (12)$$

These equilibrium expressions have standard interpretations. Equation (10) shows that local characteristics that increase productivity and welfare in Area 1 have a positive effect on rents, relative to Area 2. However, expression (11) emphasizes that relative abundance of local amenities in Area 1 reduces the relative skilled wage in this region. Finally, equation (12) shows that, if Area 1 is relatively richer in amenities that are mostly appreciated by the skilled, Z_1, the unskilled wage in Area 1 tends to be relatively higher. This occurs because such specific amenities attract skilled workers to Area 1 and, for this reason, unskilled workers become more productive.

Relative population sizes across areas. The equilibrium derived above characterizes the relative prices (rents, wages) across the economy. We now derive the relative equilibrium sizes of skilled and unskilled populations in the two areas. Similarly to Roback (1988), the procedure to determine the equilibrium populations builds on the market-clearing conditions in the markets for skilled and unskilled labour in each area, respectively,

$$n_c^s = N_c^s, \quad n_c^u = N_c^u, \quad c = \{1,2\} \quad (13)$$

and in the market for land: we leave the details to Appendix A.1. In equilibrium, the proportion of skilled workers across areas is given by:

$$\log\left(\frac{n_1^s}{n_2^s}\right) = \log\left(\frac{\bar{\ell}_1}{\bar{\ell}_2}\right) + [1-\mu(\alpha+\beta)]^{-1}\left\{\mu\cdot\log\left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right) + \log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + (1-\mu\beta)\cdot\log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right)\right\} (14)$$

Expression (14) shows that skilled workers will tend to locate in Area 1 when productivity and both types of amenities (X_1, Z_1) in Area 1 are high relative to Area 2. Thus, local characteristics that enhance productivity and welfare are central factors in attracting skilled workers.

Similar calculations show that the proportion of unskilled workers across areas is equal to:

$$\log\left(\frac{n_1^u}{n_2^u}\right) = \log\left(\frac{\bar{\ell}_1}{\bar{\ell}_2}\right) + [1-\mu(\alpha+\beta)]^{-1}\left\{\mu\cdot\log\left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right) + \log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + \alpha\mu\cdot\log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right)\right\} \quad (15)$$

Again, higher local productivity (due to Q_1) and general amenities (due to X_1) in Area 1 will bias the location of unskilled workers toward that area. Notice that abundance of local amenities that specifically attract *skilled* individuals (Z_1) will also tend to increase the location of unskilled workers in Area 1. When more skilled workers locate in Area 1, the local productivity of *unskilled* workers will increase, and their wage rises.

Finally, since it holds that $\log\left(\frac{n_1^s}{n_2^s}\right)-\log\left(\frac{n_1^u}{n_2^u}\right)=\log\left(\frac{n_1^s}{n_1^u}\right)-\log\left(\frac{n_2^s}{n_2^u}\right)$, equations (14) and (15) can be used to characterize the difference in the skill mix across areas, given by the following expression:

$$\log\left(\frac{n_1^s}{n_1^u}\right)-\log\left(\frac{n_2^s}{n_2^u}\right)=\log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right) \tag{16}$$

Expression (16) shows that differences in the local proportion between skilled and unskilled workers depend on differences in amenities that are *specific to the tastes of the skilled*, such as those included in vector Z_c.[6] This is a relevant implication of our model. Indeed, Area 1 will have a higher ratio of skilled vs. unskilled individuals only if it is endowed, relative to Area 2, with characteristics that are particularly appreciated by the educated. Consequently, since the local wage-ratio (in the absence of share-altering technical change) is given by $\frac{w_c^s}{w_c^u}=\frac{\alpha}{\beta}\cdot\frac{n_c^u}{n_c^s}$, when Area 1 is richer in skills, i.e. $\frac{n_1^s}{n_1^u}>\frac{n_2^s}{n_2^s}$, it will also exhibit a *lower* skill premium relative to Area 2, that is, $\frac{w_1^s}{w_1^u}<\frac{w_2^s}{w_2^u}$. These implications are summarized by the following:

Result 1. Skilled labor is cheaper in areas that are relatively rich in amenities which are particularly attractive to the educated.

As argued in what follows, the local capacity to attract skilled individuals may be crucial for the implementation of skill-biased technologies.

[6] In fact, in this Cobb-Douglas model, local general amenities (X_c) and productivity advantages (Q_c) affect skilled and unskilled individuals in the *same* way and, thus, they are unable to affect the local skill mix.

2. Skill-biased share-altering technical change.

Technological change has been most often represented as a shift in Total Factor Productivity, the term denoted by $A_Y(Q_c)$ in our model. In the last decades, however, the demand for skilled workers has grown faster than the pool of educated workers. This observation has motivated a large body of literature, especially in labour economics, to investigate the so-called "skilled-biased technical change".[7] In terms of production theory, technical change is skill-biased if it increases the marginal productivity of skilled workers relative to other factors: see Acemoglu (2002, p.785). Typically, representations of skill-biased technical change rely on CES production functions, as for example in Acemoglu (2002), Krusell *et al*. (2000) and Beaudry et al. (2010).[8]

Some recent literature, however, has asked what happens when technological change is represented as a change in the Cobb-Douglas share parameters. In particular, Seater (2005), Peretto and Seater (2008), and Zuleta (2008), observing the historical fall in the share of raw labour in the US, together with the stability of the share going to labour income, have explored some implications of the "share-altering" technical change hypothesis with Cobb-Douglas production functions. Such a representation of technological change is very convenient in our Cobb-Douglas spatial model, since we can explore the impact of an increase in skilled workers' share that is *exactly matched* by a decrease in the unskilled share, so that the *overall* income share of labour remains unchanged. In particular, by referring to the Cobb-Douglas technology (1), we suppose that – at date *t*=0 - a new, share-altering, technology becomes available. The new technology is such that the share of skilled labour α increases by $\Delta \geq 0$, while the share of unskilled labour β is reduced by the same amount, so that the *total* labour share, $\alpha+\beta$, remains constant. Share-altering technical change is thus associated with the following production function:[9]

$$Y_c = A_Y(Q_c)\cdot L_c^{1-\alpha-\beta}\cdot\left(N_c^s\right)^{\alpha+\Delta}\cdot\left(N_c^u\right)^{\beta-\Delta} \tag{17}$$

The following result, an immediate application of the Envelope Theorem[10], holds true:

Result 2. Each individual firm will find it profitable to adopt the share-altering technology (17) when it holds that $\frac{\partial Y_c}{\partial \Delta} = Y_c \cdot \log\left(\frac{N_c^s}{N_c^u}\right) > 0$. *Under local labour market clearing, this inequality is satisfied when Area c is sufficiently rich in skills, that is, when it holds that:*

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[7] Skill-biased technologies are a relatively recent phenomenon. As noticed in Berry and Glaeser (2005), Ford's automobile mass production involved large numbers of unskilled workers.

[8] For an argument about the unsuitability of Cobb-Douglas production functions, see Acemoglu (2003, p.3).

[9] Notice that the production function (17) implies that the ratio between the marginal productivity of skilled labour and the marginal productivity of other factors (i.e., unskilled labour, land) is increasing in Δ. Thus, this form of technical change is consistent with Acemoglu's (2002) definition of skill-biased technical change.

[10] See Appendix A.2 for details of the proof.

$$\frac{N_c^s}{N_c^u} = \frac{n_c^s}{n_c^u} > 1. \tag{18}$$

Result 2 has also a "dual" representation in terms of factor prices. Since it initially holds that $\frac{w_c^s}{w_c^u} = \frac{\alpha}{\beta} \cdot \frac{n_c^u}{n_c^s}$, condition (18) implies the following:

Corollary 1. Each individual firm will find it profitable to adopt the share-altering technology (17) when the skill-premium $\frac{w_c^s}{w_c^u}$ *is lower than* $\frac{\alpha}{\beta}$.

This Corollary has an immediate explanation. When the new technology becomes available at date *t*=0, an area rich in skills (where the ratio $\frac{n_c^s}{n_c^u}$ is high) is characterized by a relatively *low* skill-premium. Thus, the presence of cheap skilled labour in the local labour market may make it convenient to adopt skilled biased technologies: see also Beaudry et al. (2010).

Result 2 has a remarkable implication. Suppose that in Area 1 the ratio between skilled and unskilled individuals is greater than 1 while, in Area 2, is less than 1. Recall that, as emphasized by Result 1, this requires that Area 1 is relatively better endowed with those amenities that are particularly attractive to skilled individuals. Then, firms locating in Area 1 will find it profitable to implement the new technology (17), with $\Delta > 0$, while firms locating in Area 2 will stick to the "old" technology, given by (1).

In what follows, we give two sets of results deriving from skilled-biased share-altering technical change, one pertaining to the size of the skill-premium *within* each area, the other related to relative prices and populations *across* areas in spatial general equilibrium.

Implications for local skill-premia. When at date *t*=0 condition (18) is respected only in Area 1, Corollary 1 suggests that the local skill-premium is lower than the one in Area 2. Then, once adopted, the share-altering technology will have a *direct* positive impact on the local wage-premium: by taking as *given the local skill-mix* $\frac{n_1^s}{n_1^u}$, the ratio $\frac{w_1^s}{w_1^u} = \frac{\alpha + \Delta}{\beta - \Delta} \cdot \frac{n_1^u}{n_1^s}$ is increasing in Δ. Thus, areas where the skilled-biased technology is adopted exhibit – at least, initially - an increase in the local skill-premium.

It is immediate to show, however, that the direct impact on the skill-premium caused by $\Delta > 0$ must be entirely compensated by re-adjustments in the local skill-mix caused by migrations over time. The proof for this claim, as in Glaeser (2008), goes as follows. Recall that the indirect utility of a skilled worker is equal to $\overline{v}^{s} = \eta \cdot A_U(X_c) \cdot B_U(Z_c) \cdot \left(w_c^s / r_c^{1-\mu}\right)$, where $\overline{v}^{s}$ denotes the "reservation utility" of skilled individuals across the economy (there is free-mobility). Similarly, the indirect utility of an unskilled worker is given by $\overline{v}^{u} = \eta \cdot A_U(X_c) \cdot \left(w_c^u / r_c^{1-\mu}\right)$, where $\overline{v}^{u}$ denotes the "reservation utility" of the unskilled in the economy. Thus, in equilibrium, the wage-premium in Area c is given by:

$$\frac{w_c^s}{w_c^u} = \left(\frac{1}{B_U(Z_c)}\right) \cdot \frac{\overline{v}^{s}}{\overline{v}^{u}} \tag{19}$$

Expression (19) shows that the skilled-unskilled wage-gap depends on amenities that affect skilled utility, and it does *not* depend on technological factors, such as Δ. Thus, as will be confirmed in what follows, there must occur re-adjustments in the skill-mix across areas which exactly compensate for the direct effect generated by the adoption of the new technology.

The following statement, which replicates most results obtained under the CES approach by Beaudry et al. (2010), summarizes the conclusions obtained so far:

Result 3 (Skill-premia). The adoption of a skill-biased share-altering technology has a positive effect on the local skill-premium that is, at most, temporary. Re-adjustments in the local skill-mix will entirely compensate the initial positive effect, taking the local skill-premium back to its pre-adoption level.

We next consider the spatial general equilibrium implications of share-altering technical change.

Implications for spatial general equilibrium. We now explore the effects of share-altering technological change localized *only* in Area 1 on relative local prices and populations across the two regions. To this purpose, we will evaluate the results for an *initially* given skill-ratio, N_1^s / N_1^u, set equal to the constant $\Sigma_0 > 1$.[11]

Since condition (18) is satisfied in Area 1, but not in Area 2, firms in the former region adopt the share-altering innovation, while firms in the latter one continue to use the old technology. Under perfect competition in tradable good production, price (the *numeraire*) equals marginal cost, implying that firms locating in Area 1 will respect the following condition:

$$\vartheta'\left[\frac{A_Y(Q_1)}{r_1^{1-\alpha-\beta} \cdot \left(w_1^s\right)^{\alpha+\Delta} \cdot \left(w_1^u\right)^{\beta-\Delta}}\right] = 1\,, \tag{20}$$

[11] Moreover, derivatives in comparative statics results will be calculated by setting $\Delta \approx 0$, that is, starting with the same production function in both areas.

where $\vartheta' \equiv (1-\alpha-\beta)^{1-\alpha-\beta}(\alpha+\Delta)^{\alpha+\Delta}(\beta-\Delta)^{\beta-\Delta}$. For competitive firms locating in Area 2, the condition:

$$\vartheta \cdot \left[\frac{A_Y(Q_2)}{r_2^{1-\alpha-\beta} \cdot (w_2^s)^{\alpha} \cdot (w_2^u)^{\beta}} \right] = 1 \qquad (21)$$

will continue to hold. Free mobility implies that, in equilibrium, firms must make zero profit no matter where they choose to locate. Thus, by combining (20) and (21), one obtains:

$$\frac{r_1}{r_2} = \left[\left(\frac{(\alpha+\Delta)^{\alpha+\Delta}(\beta-\Delta)^{\beta-\Delta}}{\alpha^{\alpha}\beta^{\beta}} \right) \cdot \left(\frac{A_Y(Q_1)}{A_Y(Q_2)} \right) \cdot \left(\frac{w_2^s}{w_1^s} \right)^{\alpha} \cdot \left(\frac{w_2^u}{w_1^u} \right)^{\beta} \cdot \left(\frac{w_1^u}{w_1^s} \right)^{\Delta} \right]^{\frac{1}{1-\alpha-\beta}} \qquad (22)$$

By substituting equations (7) and (9) into (22), one obtains the equilibrium rent-ratio between Area 1 and Area 2:

$$\log\left(\frac{r_1}{r_2}\right) = \frac{1}{1-\mu(\alpha+\beta)} \left\{ \begin{array}{l} \left[(\alpha+\Delta)\cdot\log(\alpha+\Delta) + (\beta-\Delta)\cdot\log(\beta-\Delta) - \log\left(\alpha^{\alpha}\beta^{\beta}\right) \right] + \Delta\cdot\log\left(\frac{w^u}{w_1^s}\right) + \\ + \log\left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right) + (\alpha+\beta)\cdot\log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + \alpha\cdot\log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right) \end{array} \right\} \qquad (23)$$

where profit-maximization implies that $\left(\frac{w_1^u}{w_1^s}\right) = \frac{\beta-\Delta}{\alpha+\Delta}\left(\frac{N_1^s}{N_1^u}\right)$. Differentiating (23) with respect to Δ, and evaluating the result for $\Delta \approx 0$ and $\frac{N_1^s}{N_1^u} = \Sigma_0 > 1$, one obtains:

$$\left.\frac{d\log(r_1/r_2)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} = \left(\frac{1}{1-\mu(\alpha+\beta)}\right)\cdot\log\Sigma_0 \qquad (24)$$

Since $\Sigma_0 > 1$, the sign of expression (24) is positive. Thus, localized skill-biased technical change will increase rents in Area 1 relative to Area 2.

Consider now the impact of the change in the skilled share on the skilled wage-ratio across areas. By exploiting (7), differentiating with respect to Δ, and evaluating the result for $\Delta \approx 0$ and $\frac{N_1^s}{N_1^u} = \Sigma_0 > 1$, one obtains that:

$$\left.\frac{d\log\left(w_1^s/w_2^s\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=(1-\mu)\cdot\left.\frac{d\log\left(r_1/r_2\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\left(\frac{1-\mu}{1-\mu(\alpha+\beta)}\right)\cdot\log\Sigma_0>0 \tag{25}$$

Similarly, one can use (9) to assess the impact of share-altering change on relative unskilled wages. It turns out that the effect is the same as for skilled wages:

$$\left.\frac{d\log\left(w_1^u/w_2^u\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=(1-\mu)\cdot\left.\frac{d\log\left(r_1/r_2\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\left(\frac{1-\mu}{1-\mu(\alpha+\beta)}\right)\cdot\log\Sigma_0>0 \tag{26}$$

Finally, we analyse what happens to equilibrium populations in the two areas when share-altering technological change occurs. As shown in Appendix A.3, the change in the ratio between skilled populations in Area 1 and 2 is given by:

$$\left.\frac{d\log\left(n_1^s/n_2^s\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\frac{1}{\alpha}+\frac{\mu}{1-\mu(\alpha+\beta)}\cdot\log\Sigma_0>0 \tag{27}$$

Thus, a localised skill-biased technological change will generate a relative increase in the skilled population of that area. The opposite result generally holds for the unskilled. As shown in Appendix A.3, share-altering technical change has the following effect on the ratio of unskilled populations:

$$\left.\frac{d\log\left(n_1^u/n_2^u\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\frac{-1}{\beta}+\frac{\mu}{1-\mu(\alpha+\beta)}\cdot\log\Sigma_0 \tag{28}$$

The sign of expression (28) is ambiguous in principle. However, plausible values of the parameters imply that such a type of technological change will reduce the relative size of the unskilled population in Area 1.[12]

Expressions (27) and (28) can be exploited to find the effect of share-altering technical progress on the relative *skill mix* of the two areas.[13] Thus,

[12] With μ=2/3 and α=β=1/3, expression (28) is negative when $\log\Sigma_0<2$; when instead α=4/9 and β=2/9, (28) is negative when $\log\Sigma_0<\frac{13}{4}$. Thus, if $(\log\Sigma_0)$>0 is not implausibly large, this derivative will have a negative sign. The parameterization of factor-shares is consistent with Mankiw et al. (1992).

[13] Indeed, it holds that $d\left[\log\left(n_1^s/n_2^s\right)-\log\left(n_1^u/n_2^u\right)\right]/d\Delta=d\left[\log\left(n_1^s/n_1^u\right)-\log\left(n_2^s/n_2^u\right)\right]/d\Delta$.

$$\left.\frac{d\left[\log\left(n_1^s / n_1^u\right) - \log\left(n_2^s / n_2^u\right)\right]}{d\Delta}\right|_{\Delta\approx 0, \Sigma_0 > 1} = \frac{1}{\alpha} + \frac{1}{\beta} > 0 \tag{29}$$

Expression (29) suggests that skilled-biased share-altering technical change localized in Area 1 will lead to an increase in the skilled-unskilled ratio in that region. This conclusion has a relevant implication. When Area 1 can adopt skilled-biased share-altering technologies (i.e., when condition (18) is satisfied), later on it will be ready to adopt additional technological advances of the same kind. By contrast, if the innovation could not be profitably adopted in Area 2, this region will remain stuck with the old technology also in the future. This implies that output will grow in Area 1, while Area 2 stagnates.[14]

The spatial general equilibrium implications are summarized by the following:

Result 4 (Relative prices and population sizes across areas). When skill-biased share-altering technical innovation is introduced in Area 1 but not in Area 2, then Area 1 will exhibit: (i) an increase in its relative skilled wage; (ii) an increase in its relative unskilled wage; (iii) an increase in relative rents; (iv) an increase in its skilled population; (v) an ambiguous effect on unskilled population, relative to Area 2. Further, (vi) an increase in its skill-mix, the ratio between skilled and unskilled workers.

Thus, our model predicts that the regional skill-mix will be driven mainly by two factors: skill-biased local amenities (i.e., amenities that mostly affect the utility of educated workers; see equation 16), and skill-biased technologies.

[14] See also Seater (2005).

IV. CONCLUSIONS

The hypothesis of share-altering technical change generates two main results. The first one, mostly technical, is that one can represent skill-biased technical change even with Cobb-Douglas technologies. Indeed, we have shown how this hypothesis can replicate results that are commonly obtained under CES production functions, as for the spatial general equilibrium analysis by Glaeser (2008), and the labor-market analysis of skilled-biased technological change in Beaudry et al. (2010).

The second main contribution of this framework is related to the spatial general equilibrium implications of skilled-biased share-altering technical change. In particular, the model draws some specific conclusions about path-dependency in regional development, which would also apply to the CES framework in Beaudry et al. (2010). Only areas that are sufficiently rich in human capital will be ready to adopt share-altering technological advancements. After such changes, human capital tends to move more and more from areas that exhibit a relatively poor skill mix, to areas that are already rich in educated workers. As a result, there will be further polarization in the composition of the local labour force across the economy. This conclusion is consistent with the empirical results reported in Berry and Glaeser (2005) and Glaeser and Gottlieb (2008): areas which are rich in human capital (a pre-condition for the adoption of skilled-biased technologies, here) will attract a disproportionate number of skilled workers.

The model raises some policy questions. The existence of relevant non-linear effects of local skills, documented in Glaeser and Gottlieb (2008), may suggest that there can be returns from pushing skilled workers into *already* skilled areas. However, this would mean to subsidize areas that are rich in human capital, which seems inequitable and improper, because skilled people tend to move towards skilled places even without government aid. Still, our approach suggests that in an economy characterized by a low average level of education, it may be desirable to concentrate human capital in few specific places to get sort of areas of "excellence"[15] which allow for the adoption of techniques that would otherwise be unprofitable. In our model, this kind of intervention can be implemented by subsidizing local amenities that prove to be particularly attractive to educated individuals.

[15] Examples of such policies in the US are given by Glaeser and Gottlieb (2008, p.224-25).

APPENDIX

A.1. Derivation of relative population sizes in equilibrium.

Profit maximization for firms located in area c implies that the demand for skilled labour N_c^s, unskilled labour N_c^u, and land L_c are given, respectively, by:

$$N_c^s = \frac{\alpha \cdot Y_c}{w_c^s}, \quad N_c^u = \frac{\beta \cdot Y_c}{w_c^u}, \quad L_c = \frac{(1-\alpha-\beta)\cdot Y_c}{r_c} \qquad \text{(A.1)}$$

In equilibrium, skilled labour demand N_c^s must be equal to its local supply n_c^s. Also, unskilled labour demand N_c^u must be equal to local unskilled supply, n_c^u. Finally, the local supply of land, $\bar{\ell}_c$, must be equal to the total demand for land, which is given by the sum of land demanded by firms (as from A.1), plus the land demanded by the skilled workers, equal to $n_c^s \cdot (1-\mu)\cdot \frac{w_c^s}{r_c}$, plus the land demanded by the unskilled workers, $n_c^u \cdot (1-\mu)\cdot \frac{w_c^u}{r_c}$. Thus, the following three equations constitute a system in $\{Y_c, n_c^s, (1-u_c)\cdot n_c^u\}$, for any given price vector $\{r_c, w_c^s, w^u\}$:

$$n_c^s = \frac{\alpha \cdot Y_c}{w_c^s} \qquad \text{(A.2)}$$

$$n_c^u = \frac{\beta \cdot Y_c}{w_c^u} \qquad \text{(A.3)}$$

$$\bar{\ell}_c = \frac{1}{r_c}\left\{(1-\alpha-\beta)\cdot Y_c + (1-\mu)\cdot n_c^s \cdot w_c^s + (1-\mu)\cdot n_c^u \cdot w_c^u\right\} \qquad \text{(A.4)}$$

Using (A.2) and (A.3) to substitute $\{n_c^s \cdot w_c^s, n_c^u \cdot w_c^u\}$ away in (A.4), one obtains:

$$Y_c = \frac{\bar{\ell}_c \cdot r_c}{1-\mu(\alpha+\beta)} \qquad \text{(A.5)}$$

which can be substituted back into (A.2) and (A.3) to obtain:

$$n_c^s = \frac{\alpha}{w_c^s}\left[\frac{\bar{\ell}_c \cdot r_c}{1-\mu(\alpha+\beta)}\right], \quad c = 1,2 \tag{A.6}$$

$$n_c^u = \frac{\beta}{w_c^u}\left[\frac{\bar{\ell}_c \cdot r_c}{1-\mu(\alpha+\beta)}\right], \quad c = 1,2 \tag{A.7}$$

Thus, using (A.6), the relative population size of skilled individuals across areas will be given by:

$$\frac{n_1^s}{n_2^s} = \frac{\bar{\ell}_1}{\bar{\ell}_2} \cdot \frac{r_1}{r_2} \cdot \frac{w_2^s}{w_1^s} \tag{A.8}$$

Taking logs of (A.8) and using (10) and (11), one obtains equation (14) in the text.

Similarly, using (A.7), the relative population size of the unskilled individuals across areas is given by:

$$\frac{n_1^u}{n_2^u} = \frac{\bar{\ell}_1}{\bar{\ell}_2} \cdot \frac{r_1}{r_2} \cdot \frac{w_2^u}{w_1^u} \tag{A.9}$$

Again, taking logs of (A.9) and using (10) and (12), one obtains equation (15) in the text.

A.2. Proof of Result 2.

Share-altering technical change, summarized by $\Delta > 0$, will be adopted by local firms when it has a positive impact on profit, given by $\pi_c = Y_c - r_c \cdot L_c - w_c^s \cdot N_c^s - w_c^u \cdot N_c^u$. By Envelope Theorem, it holds that:

$$\frac{d\pi_c}{d\Delta} = \frac{\partial Y_c}{\partial \Delta} + \left[\frac{\partial Y_c}{\partial L_c} - r_c\right]\frac{dL_c}{d\Delta} + \left[\frac{\partial Y_c}{\partial N_c^s} - w_c^s\right]\frac{dN_c^s}{d\Delta} + \left[\frac{\partial Y_c}{\partial N_c^u} - w_c^u\right]\frac{dN_c^u}{d\Delta} = \frac{\partial Y_c}{\partial \Delta};$$

Thus, if condition (18) holds true, profit is increasing in Δ, making the share-altering technology convenient to adopt.

A.3. Populations and share-altering technological change.

We first analyze the impact of skill-biased share-altering technological change on the relative size of the skilled population. It is immediate to show that, in Area 1, skilled population is now given by:

$$n_1^s = \frac{\alpha+\Delta}{w_1^s}\left[\frac{\bar{\ell}_1 \cdot r_1}{1-\mu(\alpha+\beta)}\right], \tag{A.10}$$

while for Area 2 equation (A.6) still holds. Hence, with share-altering technical change, the skilled-population ratio is given by:

$$\frac{n_1^s}{n_2^s} = \frac{\alpha+\Delta}{\alpha} \cdot \frac{\bar{\ell}_1}{\bar{\ell}_2} \cdot \frac{r_1}{r_2} \cdot \frac{w_2^s}{w_1^s}. \tag{A.11}$$

Taking the logs of (A.11), differentiating with respect to Δ, and calculating the resulting expression for Δ≈0, one obtains:

$$\left.\frac{d\log\left(n_1^s / n_2^s\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} = \left(\frac{1}{\alpha}\right) + \left.\frac{d\log\left(r_1 / r_2\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} - \left.\frac{d\log\left(w_1^s / w_2^s\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} = \frac{1}{\alpha} + \frac{\mu}{1-\beta-\alpha\mu} \cdot \log\Sigma_0 > 0 \tag{A.12}$$

which is expression (27) in the text.

Unskilled population in Area 1 is given by

$$n_1^u = \frac{\beta-\Delta}{w_1^u}\left[\frac{\bar{\ell}_1 \cdot r_1}{1-\mu(\alpha+\beta)}\right], \tag{A.13}$$

while for Area 2 equation (A.7) still holds. Hence, the unskilled population ratio is equal to:

$$\frac{n_1^u}{n_2^u} = \frac{\beta-\Delta}{\beta} \cdot \frac{\bar{\ell}_1}{\bar{\ell}_2} \cdot \frac{r_1}{r_2} \cdot \frac{w_2^u}{w_1^u}. \tag{A.14}$$

Differentiating the log of (A.14) with respect to Δ and calculating the result for Δ≈0, one obtains:

$$\left.\frac{d\log\left(n_1^u / n_2^u\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} = \left(\frac{-1}{\beta}\right) + \left.\frac{d\log\left(r_1 / r_2\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} - \left.\frac{d\log\left(w_1^u / w_2^u\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} = \frac{-1}{\beta} + \frac{\mu}{1-\beta-\alpha\mu} \cdot \log\Sigma_0 \tag{A.15}$$

which is expression (28) in the text.

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