NEIGHBORHOOD EFFECTS IN THE SOLOW MODEL WITH SPATIAL EXTERNALITIES

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Abstract

Recent theoretical and empirical work generally often focus on the interdependence of nations and regions underlying that the economy of one country or region is not independent of the economies of others. However, these models generally ignores the impact of location and neighborhood in explaining growth. This paper presents an augmented Solow model that includes spatial externalities and spatial interdependence among economies. I obtain a spatial econometric reduce form which allows testing the effects of the rate of saving and the rate of population growth on income per capita. Finally, I compare the estimated and observed distributions using a contrefactual analysis.

1 Introduction

The convergence of European regions has been largely discussed in the empiric literarure during the last decade. Two observations are often emphasized. First, the convergence rate among European regions appears to be very slow in the extensive samples considered (Barro and Sala-i-Martin 1991, 1995, Armstrong 1995, Sala-i-Martin 1996a, 1996b). Second, the tools used in the regional science literature show that the geographical distribution of European per capita GDP is highly clustered and so characterized by a strong evidence of global and local autocorrelation (Armstrong 1995, Ertur et al. 2004, López-Bazo et al. 1999 and Le Gallo and Ertur 2003). Many over studies show also that an evidence of global and local spatial autocorrelation as Rey and Montouri (1999) for US State data on per capita income throughout the period 1929-1994, Ying (2000) for growth rates of production in the Chinese provinces since the late seventies, and Conley and Ligon (2002) who develop an empirical approach that explicitly allows for interdependence among countries, and they underline the importance of cross-country spillovers in explaining growth using an international dataset.

Another empirical studies show aslo the importance of geography in the diffusion of knowledge and R&D as Keller (2002) who suggests that the international diffusion of technology is geographically localized, in the sense that the productivity effects of R&D decline with the geographic distance between countries. Audretsch and Feldman (1996), Jaffe (1989), Acs et al. (1992, 1994), Feldman (1994a, b) and Anselin et al. (1997) have identified the existence of spatially-mediated knowledge spillovers of R&D or academic research effects.

Therefore, this paper presents a spatially augmented Solow model that includes technological interdependence among regions in the structural model in order to take into account this global and local spatial autocorrelation and these neighborhood and locational effects on growth and convergence. Thus, I consider the Solow model (Solow 1956, Swan 1956) with physical capital externalities suggested by Romer (1986), Krugman (1991a, b) and Grossman and Helpman (1991), among others, who have focused on the role that spillovers of economic knowledge accross agents and firms play in generating increasing returns and ultimately economic growth. I add also spatial externalities in the model in order to take into account spatial knowledge spillovers and technological interdependence between regions.

More specifically, in Section 2, I suppose that the technical progress depends on the stock of physical capital per worker, which represents the stock of knowledge as in Romer (1986), in the home region and depends on the stock of knowledge in the neighboring regions which spills on the technical progress of the home region so as the regions are geographically close. This model leads to an equation for the steady state income level as well as a spatial conditional convergence equation. In Section 3, I present the database and the spatial weight matrix which is used to model spatial connections between all regions in the sample. In Section 4, I estimate the effects of investment rate, population growth and location on the real income per worker at steady state using a spatial econometric specification. I also estimate the magnitude of physical capital externalities at steady state which is usually not identified in the literature. In Section 5, I assess the role played by technological interdependence in growth and convergence processes. For this, I estimate a spatial version of the conditional convergence equation which leads to a convergence speed close to 2% as generally found in the literature. In the Section 6, I follow Di Nardo, Fortin and Lamorieux (1996) and Desdoigts (2002), looking at the implied distribution in order to analyse the distribution of income per worker would have if look like if regions had been characterized by technological interdependence as well as by different initial levels of income per worker after having controlled for differences in steady state. This allows us to focus on contrefactual dynamics of the european income distribution implied by the spatially augmented Solow model. I augment this methodology using the Moran scatterplot in order to take into account the

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spatial nature of income distribution. Finally, Section 7 concludes.

2 A spatially augmented Solow model

2.1 Production function and spatial externalities

In this section, I develop a neoclassical growth model with physical capital externalities and spatial externalities which implies a technological interdependence in Europe between the N regions denoted by $i = 1, ..., N$.

Let us consider an aggregate Cobb-Douglas production function exhibiting constants returns to scale in labor and reproducible physical capital of the form, in region i at time t:

$$Y_i(t) = A_i(t) K_i^\alpha(t) L_i^{1-\alpha}(t) \tag{1}$$

with the standards notations: $Y_i(t)$ the output, $K_i(t)$ the level of reproducible physical capital, $L_i(t)$ the level of labor and $A_i(t)$ the aggregate level of technology:

$$A_i(t) = \Omega(t) k_i^\phi(t) \prod_{j \neq i}^{N} A_j^{\gamma w_{ij}}(t) \tag{2}$$

The function describing the aggregate level of technology $A_i(t)$ of any region i depends on three terms. First, as in the Solow model, I suppose that a part of technological progress is exogenous and identical to all regions: $\Omega(t) = \Omega(0) e^{gt}$ where g is its constant rate of growth. Second, I suppose that each region's aggregate level of technology increases with the aggregate level of physical capital per worker $k_i(t) = K_i(t)/L_i(t)$ available in that region[1].The parameter ϕ, with $0 < \phi < 1$, describe the strength of internal externalities generated by the physical capital accumulation. Therefore, I have followed Arrow's (1962) and Romer's (1986) treatment of knowledge spillover from capital investment and assumed that each unit of capital investment not only increases the stock of physical capital but also increases the level of the technology for all firms in the economy through knowledge spillover. However, there is no reason to constrain these externalities within the barriers of the economy. In fact, we can suppose that the external effect of knowledge embodied in capital in place in one region extends across its border but does so with diminished intensity because of the physical distance for instance. This idea is modeled by the third term in the function (2). The particular functional form I assumed for this term in a region i, is a geometrically weighted average of the stock of knowledge of its neighbors denoted by j. The degree of international technological interdependence or the level of spatial externalities is describe by γ, with $0 < \gamma < 1$. This parameter is assumed identical for each region but the net effect of these spatial externalities on the level of productivity of the firms in a region i depends on the relative spatial connectivity between this region and its neighbors. I represent the technological interdependence between a region i and all the regions belonging to its neighborhood by the connectivity parameters w_{ij}, for $j = 1, ..., N$ and $j \neq i$. I assume that these parameters are non negative, non stochastic and finite; we have $0 \leq w_{ij} \leq 1$ and $w_{ij} = 0$ if $i = j$. I also assume that $\sum_{j \neq i}^{N} w_{ij} = 1$ for $i = 1, ..., N$.[2] The more a given region

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[1] I suppose that all knowledge is embodied in physical capital per worker and not in the level of capital in order to avoid the scale effects (Jones, 1995).

[2] This hypothesis allows us to assume a relative spatial connectivity between all regions in order to underline the importance of the geographical neighborhood for economic growth. Moreover, it allows us to avoid spatial scale effects and then explosive growth.

i is connected to its neighbors, the higher w_{ij} is and the more region i benefits from spatial externalities.

This international technological interdependence implies that regions cannot be analysed in separation but must be analysed as an interdependent system. For this, rewrite function (2) in matrix form:

$$A = \Omega + \phi k + \gamma W A \tag{3}$$

with A the $(N \times 1)$ vector of the logarithms of the level of technology, k the $(N \times 1)$ vector of the logarithms of the aggregate level of physical capital per worker and W the $(N \times N)$ Markov-matrix with parameter w_{ij}. We can resolve (3) for A, if $\gamma \neq 0$ and if $1/\gamma$ is not an eigenvalue of W^3:

$$A = (I - \gamma W)^{-1} \Omega + \phi (I - \gamma W)^{-1} k \tag{4}$$

we can develop (4), if $|\gamma| < 1$, and regroup terms to obtain:

$$A = \frac{1}{1 - \gamma} \Omega + \phi k + \phi \sum_{r=1}^{\infty} \gamma^r W^{(r)} k \tag{5}$$

where $W^{(r)}$ is the matrix W to the power of r. For a region i, we have:

$$A_i(t) = \Omega^{\frac{1}{1-\gamma}}(t) k_i^{\phi}(t) \prod_{j=1}^{N} k_j^{\phi \sum_{r=1}^{\infty} \gamma^r w_{ij}^{(r)}}(t) \tag{6}$$

The level of technology in a region i depends on its own level of physical capital per worker and on the level of physical capital per worker in its neighborhood. Replacing (6) in the production function (1) written per worker, we have finally:

$$y_i(t) = \Omega^{\frac{1}{1-\gamma}}(t) k_i^{u_{ii}}(t) \prod_{j \neq i}^{N} k_j^{u_{ij}}(t) \tag{7}$$

with:

$$u_{ii} = \alpha + \phi \left(1 + \sum_{r=1}^{\infty} \gamma^r w_{ii}^{(r)} \right) \tag{8}$$

and:

$$u_{ij} = \phi \sum_{r=1}^{\infty} \gamma^r w_{ij}^{(r)} \tag{9}$$

with $w_{ij}^{(r)}$ the element of the line i and the column j of the matrix W to the power of r, and $y_i(t) = Y_i(t)/L_i(t)$ the level of output per worker. This model implies spatial heterogeneity in the parameters of the production function. However, we can note that if there is no physical capital externalities, that is $\phi = 0$, we have $u_{ii} = \alpha$ and $u_{ij} = 0$, and then the production function is written as usually. This link between physical capital externalities and the heterogeneity in the parameters of the production function is very close

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[3]Actually $(I - \gamma W)^{-1}$ exists if and only if $|I - \gamma W| \neq 0$. This condition is equivalent to: $|\gamma| |W - (1/\gamma) I| \neq 0$ where $|\gamma| \neq 0$ and $|W - (1/\gamma) I| \neq 0$.

to models with threshold effects due to these externalities studied by Azariadis and Drazen (1990) for example.

Finally, we can evaluate the social elasticity of income per worker in a region i with respect to all physical capital. In fact, from equation (7), it can be seen that when region i increases its own stock of physical capital per worker, it obtains a social return of u_{ii}, whereas this return increases to $u_{ii} + \sum_{j \neq i}^{N} u_{ij} = \alpha + \frac{\phi}{1-\gamma}$ if all regions simultaneously increase their stocks of physical capital per worker. In order to warrant the local convergence and then avoid explosive or endogenous growth, we suppose that there is decreasing social return: $\alpha + \frac{\phi}{1-\gamma} < 1$.[4]

2.2 Capital accumulation and steady state

As in the textbook Solow model, I assume that a constant fraction of output s_i is saved and that the labor exogenously grows at the rate n_i for a region i. I suppose also a constant and identical annual rate of depreciation of physical capital for all regions, denoted by δ. The evolution of output per worker in the region i is governed by the fundamental dynamic equation of Solow:

$$\dot{k}_i(t) = s_i y_i(t) - (n_i + \delta) k_i(t) \tag{10}$$

where the dot on a variable represents its derivative with respect to time. Since the production function per worker is caracterised by decreasing returns, equation (10) implies that the physical capital-output ratio of region i, for $i = 1, ..., N$, is constant and converges to a balanced growth rate defined by $\dot{k}_i(t) / k_i(t) = g$, or:

$$\left[\frac{k_i}{y_i} \right]^* = \frac{s_i}{n_i + g + \delta} \tag{11}$$

or in other words:

$$k_i^* = \Omega^{\frac{1}{(1-\gamma)(1-u_{ii})}}(t) \left(\frac{s_i}{n_i + g + \delta} \right)^{\frac{1}{1-u_{ii}}} \prod_{j \neq i}^{N} k_j^{\frac{u_{ij}}{1-u_{ii}}}(t) \tag{12}$$

As the production technology is characterized by externalities across regions, we can observe how the physical capital per worker at steady state depends on the usual technological and preference parameters but also on physical capital per worker intensity in neighboring regions. The influence of the spillover effect will be greater the larger the externalities generated by the physical capital accumulation, ϕ, and the coefficient γ that measures the strength of technological interdependence.

In order to determine the equation describing the real income per worker of region i at steady-state, rewrite the production function in matrix form: $y = A + \alpha k$, and substitute A by its expression in equation (4) to obtain:

$$y = (I - \gamma W)^{-1} \Omega + \alpha k + \phi (I - \gamma W)^{-1} k \tag{13}$$

premultiplying both sides by $(I - \gamma W)$, we have:

$$y = \Omega + (\alpha + \phi) k - \alpha \gamma W k + \gamma W y \tag{14}$$

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[4]See Section 2.3 for the proof.

Rewrite this equation for economy i:

$$\ln y_i^* (t) = \ln \Omega (t) + (\alpha + \phi) \ln k_i^* (t) - \alpha\gamma \sum_{j\neq i}^{N} w_{ij} \ln k_j^* (t) + \gamma \sum_{j\neq i}^{N} w_{ij} \ln y_j^* (t) \qquad (15)$$

Finally, introducing equation (12) in logarithms for $i = 1, ..., N$ in equation (15), we obtain the real income per worker of region i at steady-state:[5]

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\begin{aligned}
\ln y_i^* (t) \quad = \quad & \frac{1}{1 - \alpha - \phi} \ln \Omega (t) + \frac{\alpha + \phi}{1 - \alpha - \phi} \ln s_i - \frac{\alpha + \phi}{1 - \alpha - \phi} \ln (n_i + g + \delta) \\
& - \frac{\alpha\gamma}{1 - \alpha - \phi} \sum_{j\neq i}^{N} w_{ij} \ln s_j + \frac{\alpha\gamma}{1 - \alpha - \phi} \sum_{j\neq i}^{N} w_{ij} \ln (n_j + g + \delta) \qquad (16) \\
& + \frac{\gamma (1 - \alpha)}{1 - \alpha - \phi} \sum_{j\neq i}^{N} w_{ij} \ln y_j^* (t)
\end{aligned}
$$

This spatially augmented Solow model has the same qualitative predictions as the textbook Solow model about the influence of the own saving rate and the own population growth rate on the real income per worker of a region i at steady-state. First, the real income per worker at steady state for a region i depends positively on its own saving rate and negatively on its own population growth rate. Second, it can also be shown that the real income per worker for a region i depends positively on saving rates of neighboring regions and negatively on their population growth rates. In fact, although the sign of the coefficient of the saving rates of neighboring regions is negative, each of those saving rates $(\ln s_j)$ positively influences its own real income per worker at steady state $(\ln y_j^* (t))$ which in turn positively influences the real income per worker at steady state for region i through spatial externalities and global technological interdependence. The net effect is indeed positive as can also be shown by computing the elasticity of income per worker in region i with respect to its own rate of saving ξ_s^i and with respect to the rates of saving of its neighbors ξ_s^j. We then obtain respectively:[6]

$$\xi_s^i = \frac{\alpha + \phi}{1 - \alpha - \phi} + \frac{\phi}{(1 - \alpha)(1 - \alpha - \phi)} \sum_{r=1}^{\infty} w_{ii}^{(r)} \left(\frac{\gamma (1 - \alpha)}{1 - \alpha - \phi} \right)^r \qquad (17)$$

and:

$$\xi_s^j = \frac{\phi}{(1 - \alpha)(1 - \alpha - \phi)} \sum_{r=1}^{\infty} w_{ij}^{(r)} \left(\frac{\gamma (1 - \alpha)}{1 - \alpha - \phi} \right)^r \qquad (18)$$

These elasticities help us to better understand the effects of an increase of the saving rate in a region i or in one of its neighbors j on its income per worker at steady state. First, we note that an increase of the saving rate in a region i leads to a higher impact on the real income per worker at steady state than in the textbook Solow model because of technological interdependence modelled as a spatial multiplier effect which represents the knowledge

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[5]Note that when $\gamma = 0$, we have the model elaborated by Romer (1986) with $\alpha + \phi < 1$ and when $\gamma = 0$ and $\phi = 0$, we have the Solow model.

[6]See appendix for details

diffusion. Furthermore, an increase of the saving rate of a neighboring region j positively influences the real income per worker at steady state in the region i.

We can also compute the elasticity of income per worker with respect to the depreciation rate for region i denoted by ξ_n^i, and for neighboring regions j, denoted ξ_n^j:

$$\xi_n^i = -\frac{\alpha + \phi}{1 - \alpha - \phi} - \frac{\phi}{(1 - \alpha)(1 - \alpha - \phi)} \sum_{r=1}^{\infty} w_{ii}^{(r)} \left(\frac{\gamma(1 - \alpha)}{1 - \alpha - \phi} \right)^r \tag{19}$$

and

$$\xi_n^j = -\frac{\phi}{(1 - \alpha)(1 - \alpha - \phi)} \sum_{r=1}^{\infty} w_{ij}^{(r)} \left(\frac{\gamma(1 - \alpha)}{1 - \alpha - \phi} \right)^r \tag{20}$$

In section (4.1), we will test these qualitative and quantitative predictions of the spatially augmented Solow model.

2.3 Transitional dynamic and local convergence

As the textbook Solow model, our model predicts that income per worker in a given region converges to that region's steady state value. Rewritting the fundamental dynamic equation of Solow (10) including the production function (7), we obtain:

$$\frac{\dot{k}_i(t)}{k_i(t)} = s_i \Omega^{\frac{1}{1-\gamma}}(t) k_i^{-(1-u_{ii})}(t) \prod_{j \neq i}^{N} k_j^{u_{ij}}(t) - (n_i + \delta) \tag{21}$$

The main element behind the convergence result in this model is also diminishing returns to reproducible capital. In fact, $\partial \left(\dot{k}_i(t)/k_i(t) \right)/\partial k_i(t) < 0$ since $u_{ii} < 1$ because of the hypothesis $(\alpha + \frac{\phi}{1-\gamma} < 1)$. When a region increases its physical capital per worker, the rate of growth decreases and converges to its own steady state. However, an increase of physical capital per worker in a neighboring region j increases the firm's productivity of the region i because of the technological interdependence. We have: $\partial \left(\dot{k}_i(t)/k_i(t) \right)/\partial k_j(t) > 0$ since $u_{ij} > 0$. Physical capital externalities and technological interdependence only slow down the decrease of marginal productivity of physical capital, therefore the convergence result is still valid under the hypothesis $\alpha + \frac{\phi}{1-\gamma} < 1$, in contrast with endogenous growth models where marginal productivity of physical capital is constant.

In addition, our model makes quantitative predictions about the speed of convergence to steady state. As in the litterature, the transitionnal dynamics can be quantified by using a log linearisation of equation (21) around the steady state, for $i = 1, ..., N$:

$$\begin{aligned} \frac{d \ln k_i(t)}{dt} = {} & -(1 - u_{ii})(n_i + g + \delta) \left[\ln k_i(t) - \ln k_i^* \right] \\ & + \sum_{j \neq i}^{N} u_{ij}(n_i + g + \delta) \left[\ln k_j(t) - \ln k_j^* \right] \end{aligned} \tag{22}$$

We obtain a system of differential linear equations. Let us note $\chi_i(t) = \left[\ln k_i(t) - \ln k_i^* \right]$ and $\dot{\chi}_i(t) = \frac{d \ln k_i(t)}{dt}$, for $i = 1, ..., N$, we obtain in matrix form:

$$\dot{\chi}(t) = A\chi(t) \tag{23}$$

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where:

$$A = -(1-\alpha)\,diag\,(n+g+\delta) + \phi diag\,(n+g+\delta)\,(I-\gamma W)^{-1} \qquad (24)$$

is the matrix of the system, with $diag\,(n+g+\delta)$ the diagonal matrix with the terms $(n_i+g+\delta)$. We will show that the hypothesis $\alpha + \frac{\phi}{1-\gamma} < 1$ implies the following relation for all lines j of the Jacobian matrix A:

$$|a_{ii}| > \sum_{j=i}^{N} |a_{ij}| \qquad \text{for all } i = 1, ..., N. \qquad (25)$$

Proof:

$$\alpha + \frac{\phi}{1-\gamma} < 1$$

$$\Leftrightarrow u_{ii} + \sum_{j\neq i}^{N} u_{ij} < 1$$

$$\Leftrightarrow \alpha + \phi + \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} + \phi \sum_{j\neq i}^{N} \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} < 1$$

$$\Leftrightarrow \phi \sum_{j\neq i}^{N} \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} < (1-\alpha-\phi) - \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)}$$

$$\Leftrightarrow \sum_{j\neq i}^{N} \left| \phi \sum_{i=1}^{\infty} \gamma^i w_{ij}^{(i)} \right| < \left| -(1-\alpha-\phi) + \phi \sum_{i=1}^{\infty} \gamma^i w_{ii}^{(i)} \right| \qquad \blacksquare$$

Therefore, with the dominant diagonal theorem, the matrix A is d-stable and then the system is locally stable. The general solution of the system can be write in the following matrix form: $\chi(t) = VDb$, where D is the diagonal matrix with the terms $e^{\lambda_A t}$ with λ_A the eigenvalues of the matrix A, V the matrix of characteristic vectors associated with the eigenvalues of A and b a vector of constant which we can evaluate with the initial condition. Indeed, since the matrix A is d-stable, its eigenvalues are negatives and so: $\chi(0) = Vb$, then: $b = V^{-1}\chi(0)$. Finally the general solution can be written in the following form: $\chi(t) = VDV^{-1}\chi(0)$, or:

$$\ln k(t) - \ln k^* = VDV^{-1}\left[\ln k(0) - \ln k^*\right] \qquad (26)$$

and substracting both sides by $\ln k(0)$ and rearranging terms:

$$\ln k(t) - \ln k(0) = -\left(I - VDV^{-1}\right)\ln k(0) + \left(I - VDV^{-1}\right)\ln k^* \qquad (27)$$

Replacing $\ln k^*$ by its expression (12) in matricial form:

$$\ln k^* = \left[(1-\alpha)I - \phi(I-\gamma W)^{-1}\right]\left[(I-\gamma W)^{-1}\Omega + S\right] \qquad (28)$$

where S is the $(N\times 1)$ vector of logarithms of saving rate divided by the effective rate of depreciation, we obtain after rearranging terms:

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$$
\begin{aligned}
\ln k\left(t\right) - \ln k\left(0\right) \quad = \quad & -\left(I - VDV^{-1}\right)\ln k\left(0\right) + \frac{\phi}{1-\alpha}\left(I - VDV^{-1}\right)\left(I - \gamma W\right)^{-1}\ln k\left(0\right) \\
& + \frac{1}{1-\alpha}\left(I - VDV^{-1}\right)\left(I - \gamma W\right)^{-1}\Omega + \frac{1}{1-\alpha}\left(I - VDV^{-1}\right)S \quad (29) \\
& + \frac{\phi}{1-\alpha}\left(I - VDV^{-1}\right)\left(I - \gamma W\right)^{-1}\left(I - VDV^{-1}\right)^{-1}\left[\ln k\left(t\right) - \ln k\left(0\right)\right]
\end{aligned}
$$

This equation shows that the convergence process is more complicated than the usual equation in the litterature. However, we can note that if there is no physical capital externalities, that is $\phi = 0$, we can reduce this equation to those the traditional conditional convergence equation except for the constant term with exogenous technical progress. Another case in on interest: when we consider the case of unconditional convergence process, we have $n_i = n \quad \forall i = 1, ..., N$, and then the eigenvalues of the matrix A can be rewrite in function of those of W matrix denoted by λ_W. Indeed, we have:

$$
\lambda_A = -\left(1 - \alpha - \frac{\phi}{1 - \gamma\lambda_W}\right)\left(n + g + \delta\right) \tag{30}
$$

3   Data and spatial weight matrix

All data are extracted from the Cambridge database. More precisely, I consider 204 European regions belonging to 17 countries over the 1977-2000 period at NUTS2 level for Belgium (11), Denmark (1), Germany (31), Greece (13), Spain (16), France (22), Ireland (2), Italy (20), Luxembourg (1), the Netherlands (12), Austria (9), Portugal (1), Finland (6), Sweden (8), United Kingdom (37), Norway (7), Switzerland (7). I measure n as the average growth rate of the working-age population (ages 15 to 64), real income per worker is mesured by the GVA (Gross Value Added) divided by the number of worker, and finally the saving rate s is mesured as the average share of gross investment in GVA.

The Markov-matrix W defined in equation (3) corresponds to the so called spatial weight matrix commonly used in spatial econometrics to model spatial interdependence between regions or countries (Anselin 1988). More precisely, each region is connected to a set of neighboring regions by means of a purely spatial pattern introduced exogenously in W. The elements w_{ii} on the diagonal are set to zero whereas the elements w_{ij} indicate the way the region i is spatially connected to the region j. In order to normalize the outside influence upon each region, the weight matrix is standardized such that the elements of a row sum up to one. For the variable x, this transformation means that the expression Wx, called the spatial lag variable, is simply the weighted average of the neighboring observations.

Various matrices are considered in the literature: a simple binary contiguity matrix, a binary spatial weight matrix with a distance-based critical cut-off, above which spatial interactions are assumed negligible, more sophisticated generalized distance-based spatial weight matrices with or without a critical cut-off. The notion of distance is quite general and different functional forms based on distance decay can be used (for example inverse distance, inverse squared distance, negative exponential etc.). The critical cut-off can be the same for all regions or can be defined to be specific to each region leading in the latter case, for example, to k-nearest neighbors weight matrices when the critical cut-off for each region is determined so that each region has the same number of neighbors.

It is important to stress that the connectivity terms w_{ij} should be exogenous to the model to avoid the identification problems raised by Manski (1993) in social sciences. This is the

reason why we consider pure geographical distance, more precisely great circle distance between centroids, which is indeed strictly exogenous; the functional form I consider is simply the k-nearest neighbors weight matrix $W(k)$ with the general term defined as follows in standardized form $\left[w(k)_{ij} \right]$:

$$w(k)_{ij} = w(k)^*_{ij} / \sum w(k)^*_{ij} \qquad \text{with} \qquad w(k)^*_{ij} = \begin{cases} 0 \text{ if } i = j \\ 1 \text{ if } d_{ij} \leqslant d_i(k) \\ 0 \text{ if } d_{ij} > d_i(k) \end{cases} \qquad (31)$$

where d_{ij} is the great circle distance between regional centroid and $d_i(k)$ is a critical cut-off distance defined for each region i. More precisely, $d_i(k)$ is the k-th order smallest distance between regions i and j so that each region i has exactly k neighbors. In this analyse, I consider $k = 10$.

4 Influence of saving rate and population growth on real income per worker distribution and growth

4.1 Empirical model and spatial econometric framework

In this section, we follow Mankiw et al. (1992) in order to evaluate the impact of saving, population growth and location on real income. Taking equation (16), we find that the real income per worker along the balanced growth path, at a given time - $t = 0$ for simplicity - is:

$$\ln\left[\frac{Y_i}{L_i}\right] = \beta_0 + \beta_1 \ln s_i + \beta_2 \ln(n_i + g + \delta)$$
$$+ \theta_1 \sum_{j \neq i}^{N} w_{ij} \ln s_j + \theta_2 \sum_{j \neq i}^{N} w_{ij} \ln(n_j + g + \delta) + \rho \sum_{j \neq i}^{N} w_{ij} \ln\left[\frac{Y_j}{L_j}\right] + \varepsilon_i \quad (32)$$

where $\frac{1}{1-\alpha-\phi} \ln \Omega(0) = \beta_0 + \varepsilon_i$, for $i = 1, ..., N$, with β_0 a constant and ε_i a region-specific shock since the term $\Omega(0)$ reflects not just technology but also resource endowments, climate, institutions, and so on ..., and then it may differ across regions. We suppose also that $g + \delta = 0.05$ as used in the literature since Mankiw et al. (1992) and Romer (1989). We have finally the following theoretical constraints between coefficients: $\beta_1 = -\beta_2 = \frac{\alpha+\phi}{1-\alpha-\phi}$ and $\theta_2 = -\theta_1 = \frac{\alpha\gamma}{1-\alpha-\phi}$. Equation (32) is our basic econometric specification in this section.

In the spatial econometrics literature, this kind of specification, including the spatial lags of both endogenous and exogenous variables, is referred to as the spatial Durbin model (see Anselin, 1988), we have in matrix form:

$$y = X\beta + WX\theta + \rho Wy + \varepsilon \qquad (33)$$

here y is the $(N \times 1)$ vector of logs of real income per worker, X the $(N \times 3)$ matrix with the sum vector, the vectors of logs of investment rate and the logs of physical capital effective rates of depreciation, W the $(N \times N)$ spatial weight matrix, $\beta' = [\beta_0 \ \beta_1 \ \beta_2]$, $\theta' = [\theta_1 \ \theta_2]$ and $\rho = \frac{\gamma(1-\alpha)}{1-\alpha-\phi}$ is the spatial autocorrelation coefficient.[7] ε is the $(N \times 1)$ vector of errors

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[7] In practice, the spatially lagged constant is not included in WX, since there is an identification problem for row-standardized W (the spatial lag of a constant is the same as the original variable).

supposed identically and normally distributed so that $\varepsilon \sim N\left(0, \sigma^2\right)$. We can easily see that model (32) reduces to the textbook Solow model when $\phi = \gamma = 0$.

Noting that β and θ can be expressed as:

$$\beta = (\widetilde{X}'\widetilde{X})^{-1}\widetilde{X}'y \tag{34}$$

$$\theta = (\widetilde{X}'\widetilde{X})^{-1}\widetilde{X}'Wy \tag{35}$$

we can write the concentrated log-likelihood function for this model as shown in (36) where C denotes an inessential constant:

$$\ln(L) = C + \ln|I - \rho W| - \frac{n}{2}\ln(e_1'e_1 - 2\rho e_2'e_1 + \rho^2 e_2'e_2) \tag{36}$$

with $e_1 = y - \widetilde{X}\beta$, $e_2 = Wy - \widetilde{X}\theta$ and $\widetilde{X} = \begin{bmatrix} X & WX \end{bmatrix}$. Given a value of ρ that maximizes the concentrated likelihood function (say $\widehat{\rho}$), we compute estimates for β and θ using:

$$\widehat{\zeta} = (\beta - \widehat{\rho}\theta) = \begin{bmatrix} \widehat{\beta} \\ \widehat{\theta} \end{bmatrix} \tag{37}$$

Finally, an estimate of σ_ε^2 is calculated using:

$$\widehat{\sigma}^2 = (y - \widehat{\rho}Wy - X\widehat{\zeta})'(y - \widehat{\rho}Wy - X\widehat{\zeta})/n \tag{38}$$

In the first column of table 1, we estimate the textbook Solow model. Our results about its qualitative predictions are essentially identical to those of Mankiw et al. (Table 1, p. 414 of their article) since the coefficients of saving and population growth have the predicted signs. However, the coefficients are weakly significant and the effect of saving rate is lower than as expected. The overidentifying restriction is not rejected in contrast to the recent results in the literature (Bernanke et al. 2003) and the estimated capital share is close to 0.2 the lower bound of this value generally found. The Solow model is misspecified since it omits variables due to technological interdendence and physical capital externalities. Indeed, we can write the spatially augmented Solow model in the following matrix form:

$$y = \frac{\alpha}{1-\alpha}S + \frac{\phi}{1-\alpha}(I - \gamma W)^{-1}\ln k^* + (I - \gamma W)^{-1}\varepsilon \tag{39}$$

with S the $(N \times 1)$ vector of logarithms of investment rate divided by the effective rate of depreciation. Therefore the error term in the Solow model contains omitted information since we can rewrite it:

$$\varepsilon_{Solow} = \frac{\phi}{1-\alpha}(I - \gamma W)^{-1}k^* + (I - \gamma W)^{-1}\varepsilon \tag{40}$$

We also note the presence of spatial autocorrelation in the error term even if there is no physical capital externalities, and then the presence of technological interactions between all countries through the inverse spatial transformation $(I - \gamma W)^{-1}$.

In the second column of table 1, I estimate the spatially augmented Solow model.[8] Many aspects of the results support the model. First, all the coefficients have the predicted signs and the spatial autocorrelation coefficient, ρ, is highly positively significant. Second, the coefficients of saving rates of the region i and its neighboring regions j are significant at 5% and 6% respectively. Third, the joint theoretical restriction $\beta_1 = -\beta_2$ and $\theta_2 = -\theta_1$ is not

---

[8]James LeSage provides a function to estimate this model in his Econometric Toolbox for Matlab (http://www.spatial-econometrics.com)

rejected since the p-value of the LR-test is 0.572. Finally the α implied by the coefficients in the constrained regression is significantly close to one-third as expected. However, many aspects of the results seem do not support the model. Indeed, the coefficient γ, representing the strenght of spatial externalities, is very strong since it is higher than 1. This result shows the importance of spatial externalities in the distribution of income in Europe. The implied value of $\alpha + \frac{\phi}{1-\gamma}$ is too high since its value is higher than 3 but it is not significant. Moreover, the ϕ estimated is negative with a p-value of 0.116 which is indicate there is not physical capital externalities in the european regions. This result is convergent with the evidence against the importance of permanent within-industry knowledge spillovers for growth at the regional and urban level (see Glaeser et al. 1992). More specifically, we can test the absence of physical capital externalities represented by ϕ since $\phi = 0$ implies in the specification (32) the following expression:

$$
\begin{aligned}
\ln\left[\frac{Y_i}{L_i}\right] =\ & \beta_0' + \beta_1' \ln s_i + \beta_2' \ln\left(n_i + g + \delta\right) \\
& + \theta_1' \sum_{j\neq i}^{N} w_{ij} \ln s_j + \theta_2' \sum_{j\neq i}^{N} w_{ij} \ln\left(n_j + g + \delta\right) + \gamma \sum_{j\neq i}^{N} w_{ij} \ln\left[\frac{Y_j}{L_j}\right] + \varepsilon_i \quad (41)
\end{aligned}
$$

with $\beta_1' = -\beta_2' = \frac{\alpha}{1-\alpha}$, $\theta_2' = -\theta_1' = \frac{\alpha\gamma}{1-\alpha}$ hence $\theta_1' + \beta_1'\gamma = 0$ and $\theta_2' + \beta_2'\gamma = 0$. Specification () is the so-called constrained spatial Durbin model which is formally equivalent to a spatial error model written in matrix form:

$$
y = X\beta' + \varepsilon_{Solow} \qquad \text{and} \qquad \varepsilon_{Solow} = \gamma W \varepsilon_{Solow} + \varepsilon \qquad (42)
$$

where $\beta' = \begin{bmatrix} \beta_0' & \beta_1' & \beta_2' \end{bmatrix}$ and ε_{Solow} is the same as before with $\phi = 0$. Hence, we have the textbook Solow model with spatial autocorrelation in the errors terms. We estimate the Spatial Error Model in the third column of the table 1. We note that the coefficients have the predicted signs and the spatial autocorrelation coefficient in error term, γ, is also highly positively significant. We can test the non-linear restrictions with the common factor test (Burridge, 1981). The LR value of the test is 1.883 and its p-value is close to 0.19, so we can't reject the non-linear restrictions, but we can't conclued about the hypthesis $\phi = 0$.

Finally, we should note that these regressions based on the methodology proposed by Mankiw et al. (1992), are valid only if the regions are their steady states or if deviations from steady state are random. So, as already shown by Jones (1997) with international data, most of the regions in Europe have probably not reached their stady-state level. Then, in order to study more precisely the distribution of real income per worker in Europe, we must take into account out-of-steady-state dynamics with a spatial conditional convergence.

4.2 A spatial conditional convergence model

The spatial convergence model can't be estimate directly with equation (29). In this section, we suppose, with the results of the section (4.1), that there is no physical capital externalities ($\phi = 0$), which implies that the matrix A reduces to a diagonal matrix with the terms $-(1-\alpha)(n+g+\delta)$ on its diagonal. As a result, the resolution is now identical to the traditional problem in the litterature. Indeed, for each region $i = 1, ..., N$, the equation (22) can be rewrite for the income per worker[9]:

---

[9]I suppose also that the speed of convergence is identical for all regions as in the traditional literature about conditional convergence (Barro and Sala-i-Martin 1991, 1992, 1995, Mankiw et al. 1992).

$$\frac{d\ln y_i\,(t)}{dt} = \frac{g}{1-\gamma} - (1-\alpha)\,(n+g+\delta)\,[\ln y_i\,(t) - \ln y_i^*] \qquad (43)$$

The solution for $\ln y_i\,(t)$, substracting $\ln y_i\,(0)$, the real income per worker at some initial date, from both sides, is:

$$
\begin{aligned}
\ln y_i\,(t) - \ln y_i\,(0) \;=\;& \left(1 - e^{-\lambda t}\right)\frac{g}{1-\gamma}\frac{1}{\lambda} - \left(1 - e^{-\lambda t}\right)\ln y_i\,(0) \\
& + \left(1 - e^{-\lambda t}\right)\ln y_i^*
\end{aligned}
\qquad (44)
$$

The model predicts convergence since the growth of real income per worker is a negative function of the initial level of income per worker, but only after controlling for the determinants of the steady-state. Rewrite equation (44) in matrix form: $\ln y\,(t) - \ln y\,(0) = \left(1 - e^{-\lambda t}\right)[C - \ln y\,(0) + \ln y^*]$ where $\ln y\,(0)$ is the $(N \times 1)$ vector of the logarithms of initial level of real income per worker, $\ln y^*$ is the $(N \times 1)$ vector of the logarithms of real income per worker at steady-state, C is the $(N \times 1)$ vector of constant. Introducing equation (16) in matrix form: $\ln y^* = (I - \gamma W)^{-1}\left[\frac{1}{1-\alpha}\Omega + \frac{\alpha}{1-\alpha}S - \frac{\alpha\gamma}{1-\alpha}WS\right]$, where S is the $(N \times 1)$ vector of logarithms of saving rate divided by the effective rate of depreciation, premultiplying both sides by the inverse of $(I - \rho W)^{-1}$ and rearranging terms we obtain:

$$
\begin{aligned}
\ln y\,(t) - \ln y\,(0) \;=\;& \left(1 - e^{-\lambda t}\right)\left(C + \frac{1}{1-\alpha}\Omega\right) - \left(1 - e^{-\lambda t}\right)y\,(0) \\
& + \gamma\left(1 - e^{-\lambda t}\right)Wy\,(0) + \frac{\alpha}{1-\alpha}\left(1 - e^{-\lambda t}\right)S \\
& - \frac{\alpha\gamma}{1-\alpha}\left(1 - e^{-\lambda t}\right)WS + \gamma W\,[\ln y\,(t) - \ln y\,(0)]
\end{aligned}
\qquad (45)
$$

Finally, dividing by T in both sides, we can rewrite this equation for a region i:

$$
\begin{aligned}
\frac{\ln y_i\,(t) - \ln y_i\,(0)}{T} \;=\;& \beta_0 + \beta_1 \ln y_i\,(0) + \beta_2 \ln s_i + \beta_3 \ln\,(n_i + g + \delta) \\
& + \theta_2 \sum_{j\neq i}^{N} w_{ij}\ln s_j + \theta_3 \sum_{j\neq i}^{N} w_{ij}\ln\,(n_j + g + \delta) \\
& + \theta_1 \sum_{j\neq i}^{N} w_{ij}\ln y_j\,(0) + \gamma \sum_{j\neq i}^{N} w_{ij}\frac{\ln y_j\,(t) - \ln y_j\,(0)}{T} + \varepsilon_i
\end{aligned}
\qquad (46)
$$

where $\beta_0 = \left(1 - e^{-\lambda T}\right)\left(\frac{g}{1-\gamma}\frac{1}{\lambda} + \frac{1}{1-\alpha}\Omega\,(T)\right)$ is a constant, $\beta_1 = -\frac{\left(1-e^{-\lambda T}\right)}{T}$, $\beta_2 = -\beta_3 = \frac{\left(1-e^{-\lambda T}\right)}{T}\frac{\alpha}{1-\alpha}$, $\theta_1 = \frac{\left(1-e^{-\lambda T}\right)}{T}\gamma$, $\theta_3 = -\theta_2 = \frac{\left(1-e^{-\lambda T}\right)}{T}\frac{\alpha\gamma}{1-\alpha}$. In matrix form, we have also the non-constrained spatial Durbin model which is estimated in the same way as the model in the section (4.1).

In the first column of table 2, I estimate a model of unconditional convergence. This result shows that there is convergence between european regions since the coefficient on the initial level of income per worker is negative and strongly significative. Therefore, there is tendency for poor regions to grow faster on average than rich regions in Europe. Note that this result is different to the traditional result in the literature about the failure of income

convergence in international cross-countries (De Long 1988, Romer 1987 and Mankiw et al. 1992). I test the convergence predictions of the textbook Solow model in the second column of table 2. I report regressions of growth rate over the period 1977 to 2000 on the logarithm of income per worker in 1977, controlling for investment rate and growth of working-age population. The coefficient on the initial level of income is also significantly negative; that is, there is strong evidence of convergence. The results support also the predicted signs of investment rate and working-age population growth rate. However, the speed of convergence associated with both estimations is close to 0.7% far below 2% usually found in the convergence literature (Barro and Sala-i-Martin 1995 for instance). The half-life is about 96 years which indicates that the process of convergence is indeed very weak.

The textbook Solow model is misspecified since it omits variables due to regional technological interdependence and physical capital externalities. Therefore, as in Section (4.1), the error terms of the Solow model contains omitted information and are spatially auto-correlated. In table 3, I estimate the spatially augmented Solow model. Many aspects of the results support this model. First, all the coefficients are significants and have the predicted signs. The spatial autocorrelation coefficient ρ is highly positively significant which shows the importance of the role played by regional technological interdependence on the convergence process. Second, the coefficient on the initial level of income is significantly negative, so there is strong evidence of convergence after controlling for those variables that the spatially augmented Solow model says determine the steady state. Third, the λ implied by the coefficient on the initial level of income is about 1.4% which is more closer to the value usually found about the speed of convergence in the literature. However, the common factor test is strongly reject since the LR value is 18.664 with a p-value of 0.000. The theoretical non-linear constrains are then reject by the data, so we don't conclued precisely about the hypothesis of the absence of physical capital externalities ($\phi = 0$). The Spatial Error Model implied by this hypothesis fits good the data since all the coefficients are significants and have the predicted signs and the implied λ is about 1.2%, a value less by those implied by the Spatial Durbin Model above.

5 Income distribution in Europe

5.1 Conterfactual income density estimates

In this section, following Di Nardo, Fortin and Lamorieux (1996) and Desdoigts (2002), I look at the implied distribution in order to analyse the distribution of income per worker would have if look like if regions had been characterized by technological interdependence as well as by different initial levels of income per worker after having controlled for differences in steady state. This allows us to focus on contrefactual dynamics of the european income distribution implied by the spatially augmented Solow model.

In the figure 1, univariate densities observed of the european real output per worker in 1977 and 2000 are displayed. The final density is represented by the thick line and the initial density by the solide line. Notice that the so-called phenomenon of twin peaks distribution generally observed across countries (Quah, 1996) is not significantly at work for the european regions. The middle-income class does'nt vanishes really but we note a group of very rich regions at the upper tail of the distribution. These regions are essentially the most urbanized regions of the sample as Bruxelles, Hamburg, Luxembourg, Oslo for instance and the swiss regions. However, the distribution in 2000 seems show a disturbance in the vinicity of 18000 euros per worker.

In order to compare the observed distribution and the implied distribution by the models,

I estimate counterfactuel income distribution issued by growth regressions over the period 1977-2000 on the Solow model and the spatially augmented Solow model. Such counterfactual income density estimate are plotted in Figure 2 where I superimpose both counterfactual income density estimates that would have been observed at the end of the period 1977-2000 if the growth model was either the textbook Solow model (solid line) or the spatially augmented Solow model (dotted line) and the true income density estimate in 2000 (thick line). Following Di Nardo, Fortin and Lemieux (1996) and Desdoigts (1999), I also plot in the figure 3 the difference between the density estimate of the european income distribution in 2000 and each couterfactual density implied by either the textbook Solow model (solid line) or the spatially augmented Solow model (dotted line). The closer to the zero line and the flatter is the estimated line, the better the counterfactual density estimate fits the shape of the observed income distribution at the end of the period. The local impact of each model on the evolution of the european income distribution can now be clearly seen.

Globally, both model fit good the true distribution. The spatially augmented Solow model fits better the upper part of the income distribution whereas the textbook Solow model seems fit better the lower part of the income distribution. Both models don't take into account the dynamics on the most upper tail of the distribution and in the vinicity of the main modal. However, this method is not adapted to an analyse of spatial distribution of income per worker in Europe. This is the reason for which I use an other method to visualize this distribution.

5.2   An exploratory spatial data analysis of the income distribution

In order to study the local geographic distribution of real income per worker, I use the Moran scatterplot and suggest an application of the transition probability matrices. Local spatial autocorrelation and instability can be studied by means of the Moran scatterplot (Anselin, 1996), which plots the spatial lag Wx, that is in this paper the mean of income per worker in the neighborhood of a region, against the original value x. The four different quadrants of the scatterplot correspond to the four types of local spatial association between a region and its neighbors: HH a region with a high[10] value surrounded by regions with high values, LH a region with a low value surrounded by regions with high values, etc. Quadrants HH and LL (resp. HL and LH) refer to positive (resp. negative) local spatial autocorrelation indicating spatial clustering of similar (resp. dissimilar) values. Moreover, I study the local geographic distribution of real income per worker comparing the true local distribution displays by the Moran scatterplot of the real income per worker in 2000 and the implied distributions by the models display by the Moran scatterplot of the real income would have if look like if regions had been characterized by technological interdependence as well as by different initial levels of income per worker after having controlled for differences in steady state. The results are locally convergent if a region that is in a particular state (i.e. in a quadrant HH, HL, LH, LL) with the observed distribution remains in this state for the textbook Solow model or the spatially augmented Solow model. More the model fit good the geographic local distribution and more the transition probability on the diagonal is close to 1.

The Moran scatterplot of the true distribution in 1977 and 2000 are display in the figures 4 and 5. We note a strong positive global spatial autocorrelation between the european regions in the sample.[11] The dynamic of the distribution can be seen in the table 4 displaying the transition probability matrices between the distribution in 1977 and 2000. Most of the change are the regions in the quadrant LH or HL in the begining of the period which are

---

[10] High (resp. low) means above (resp. below) the mean.

[11] A complete study of exploratory spatial data analysis of EU15 and EU27 can be found in Ertur and Koch (2004).

in the quadrant HH at the end. We note that the textbook Solow model and the spatially augmented Solow model fit good the spatial distribution since the transition probability on the diagonal are very close to 1 for the quadrant HH and BB. Only local spatial instability represented by quadrant LH and HL are weakly fitted by both model. Note also that both model predict a global spatial autocorrelation[12] higher than the true value of the distribution of real income per worker.

6   Conclusion

In this paper, I developed a neoclassical growth model which explicitly takes into account technological interdependence between regions under the form of spatial externalities. The qualitative predictions of this spatially augmented Solow model provided us with a better understanding of the important role played by geographical location and neighborhood effects in the growth and convergence processes. In addition, the econometric model leads to estimates of structural parameters close to predicted values. The estimated capital share parameter is close to one third, but the physical capital externalities are not significant and we can conclued to absence of Marshallian externalities in European Regions. This result is close to those found in the literature as Glaeser et al. (1992) for instance. The strong value of technological parameter is convergent with the high spatial autocorrelation usually found in the literature and shows also the important role played in the economic growth and income distribution processes.

Our results are then important to better understand the phenomena of spatial autocorrelation generally found in the spatial distribution of income and in the regional economic growth and convergence. Moreover the empirics consequences show that the traditional econometrics results are misspecified, since they omit spatially autocorrelated errors and spatially atoregressive variable.

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[12]The Moran's I statistic, representing a measure of global spatial autocorrelation, is indeed formally equivalent to the slope coefficient of the linear regression of Wx on x using a row-standardized weight matrix.

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Appendix

Take equation (16) in matrix form:

$$y = \frac{1}{1-\alpha-\phi}\Omega + \frac{\alpha+\phi}{1-\alpha-\phi}S - \frac{\alpha\gamma}{1-\alpha-\phi}WS + \frac{(1-\alpha)\gamma}{1-\alpha-\phi}Wy \tag{47}$$

where S is the $(N \times 1)$ vector of logarithms of saving rate divided by the effective rate of depreciation. Substracting $\frac{(1-\alpha)\gamma}{1-\alpha-\phi}Wy$ in both sides, and pre-multiplying both sides by $\left(I - \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W\right)^{-1}$, we obtain:

$$y = \frac{1}{1-\alpha-\phi}\left(I - \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W\right)^{-1}\Omega + \left(I - \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W\right)^{-1}\left(\frac{\alpha+\phi}{1-\alpha-\phi}I - \frac{\alpha\gamma}{1-\alpha-\phi}W\right)S \tag{48}$$

Deriving this expression in respect to the vector S, we obtain the expression of elasticities in matrix form:

$$\Xi = \left(I - \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W\right)^{-1}\left(\frac{\alpha+\phi}{1-\alpha-\phi}I - \frac{\alpha\gamma}{1-\alpha-\phi}W\right) \tag{49}$$

$$= \left(I + \frac{(1-\alpha)\gamma}{1-\alpha-\phi}W + \left(\frac{(1-\alpha)\gamma}{1-\alpha-\phi}\right)^2 W^2 + ...\right)\left(\frac{\alpha+\phi}{1-\alpha-\phi}I - \frac{\alpha\gamma}{1-\alpha-\phi}W\right) \tag{50}$$

$$= \frac{\alpha+\phi}{1-\alpha-\phi}I + \left(\frac{\phi}{(1-\alpha)(1-\alpha-\phi)}\right)\sum_{r=1}^{\infty}W^r\left(\frac{(1-\alpha)\gamma}{1-\alpha-\phi}\right)^r \tag{51}$$

Finally, we can rewritte these expressions for each region i and we obtain the expressions in the text.

Table 1: Estimation results: Textbook Solow and spatially augmented Solow models

Model	TextBook Solow	Spatial aug. Solow (SDM)	Spatial aug. Solow (SEM)
Dep. var.	$\ln y_i$ (2000)	$\ln y_i$ (2000)	$\ln y_i$ (2000)
Obs.	204	204	204
$constant$	10.256	1.628	10.239
	(0.000)	(0.198)	(0.000)
$\ln s_i$	0.292	0.303	0.262
	(0.074)	(0.038)	(0.067)
$\ln(n_i + 0.05)$	-0.135	-0.102	-0.077
	(0.566)	(0.569)	(0.666)
$W \ln s_j$	$-$	-0.504	$-$
		(0.059)	
$W \ln(n_j + 0.05)$	$-$	0.330	$-$
		(0.409)	
$W \ln y_j$ (SDM) / γ (SEM)	$-$	0.872	0.866
		(0.000)	(0.000)
Restricted regression			
$constant$	9.862	1.597	9.794
	(0.000)	(0.001)	(0.000)
$\ln s_i - \ln(n_i + 0.05)$	0.245	0.233	0.199
	(0.101)	(0.074)	(0.121)
$W[\ln s_j - \ln(n_j + 0.05)]$	$-$	-0.431	$-$
		(0.057)	
$W \ln y_j$	$-$	0.867	0.864
		(0.000)	(0.000)
Test of restriction	0.237 (Wald)	1.119 (LR)	0.953 (LR)
p-value	(0.627)	(0.572)	(0.329)
Implied α	0.197	0.332	0.166
	(0.040)	(0.005)	(0.063)
Implied ϕ	$-$	-0.143	$-$
		(0.116)	
Implied γ	$-$	1.052	0.866
		(0.000)	(0.000)
$\alpha + \frac{\phi}{1-\gamma}$	$-$	3.071	$-$
		(0.923)	

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Table 2: Estimation results: Textbook Solow and spatially augmented Solow models

Model	Unconditional conv.	TextBook Solow
Dep. var.	$\frac{\ln y_i(2000)-\ln y_i(1977)}{23}$	$\frac{\ln y_i(2000)-\ln y_i(1977)}{23}$
Obs.	204	204
$constant$	0.085	0.073
	(0.000)	(0.045)
$\ln y_i(1960)$	-0.007	-0.007
	(0.000)	(0.000)
$\ln s_i$	–	0.019
		(0.000)
$\ln(n_i+0.05)$	–	-0.013
		(0.001)
Implied λ	0.014	0.012
	()	()
Half-life	51.34	57.28

Table 3: Estimation results: Textbook Solow and spatially augmented Solow models

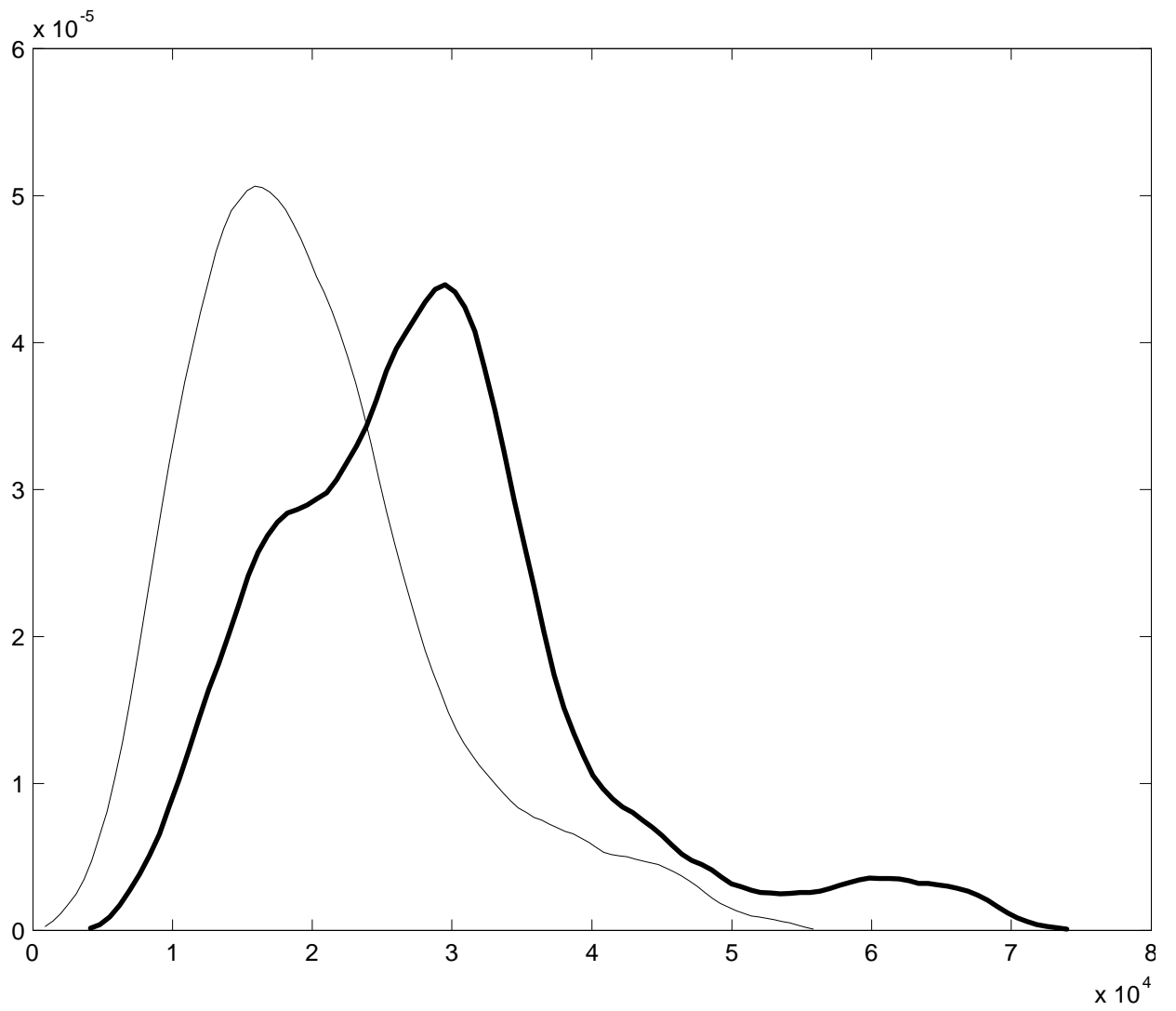| Model	Spatial aug. Solow (SDM)	Spatial aug. Solow (SEM)
Dep. var.	$\frac{\ln y_i(2000)-\ln y_i(1977)}{23}$	$\frac{\ln y_i(2000)-\ln y_i(1977)}{23}$
Obs.	204	204
$constant$	-0.001	0.114
	(0.979)	(0.000)
$\ln y_i(1960)$	-0.012	-0.011
	(0.000)	(0.000)
$\ln s_i$	0.031	0.028
	(0.000)	(0.000)
$\ln(n_i+0.05)$	-0.019	-0.017
	(0.000)	(0.001)
$W\ln y_j(1960)$	0.010	–
	(0.000)	
$W\ln s_j$	-0.041	–
	(0.000)	
$W\ln(n_j+0.05)$	0.015	–
	(0.165)	
$W\frac{\ln y_j(1995)-\ln y_j(1960)}{35}$ (SDM) / γ (SEM)	0.447	0.664
	(0.000)	(0.000)
Implied λ	0.014	0.012
	()	()
Half-life	51.34	57.28

Figure 1: Observed distribution in 1977 and 2000
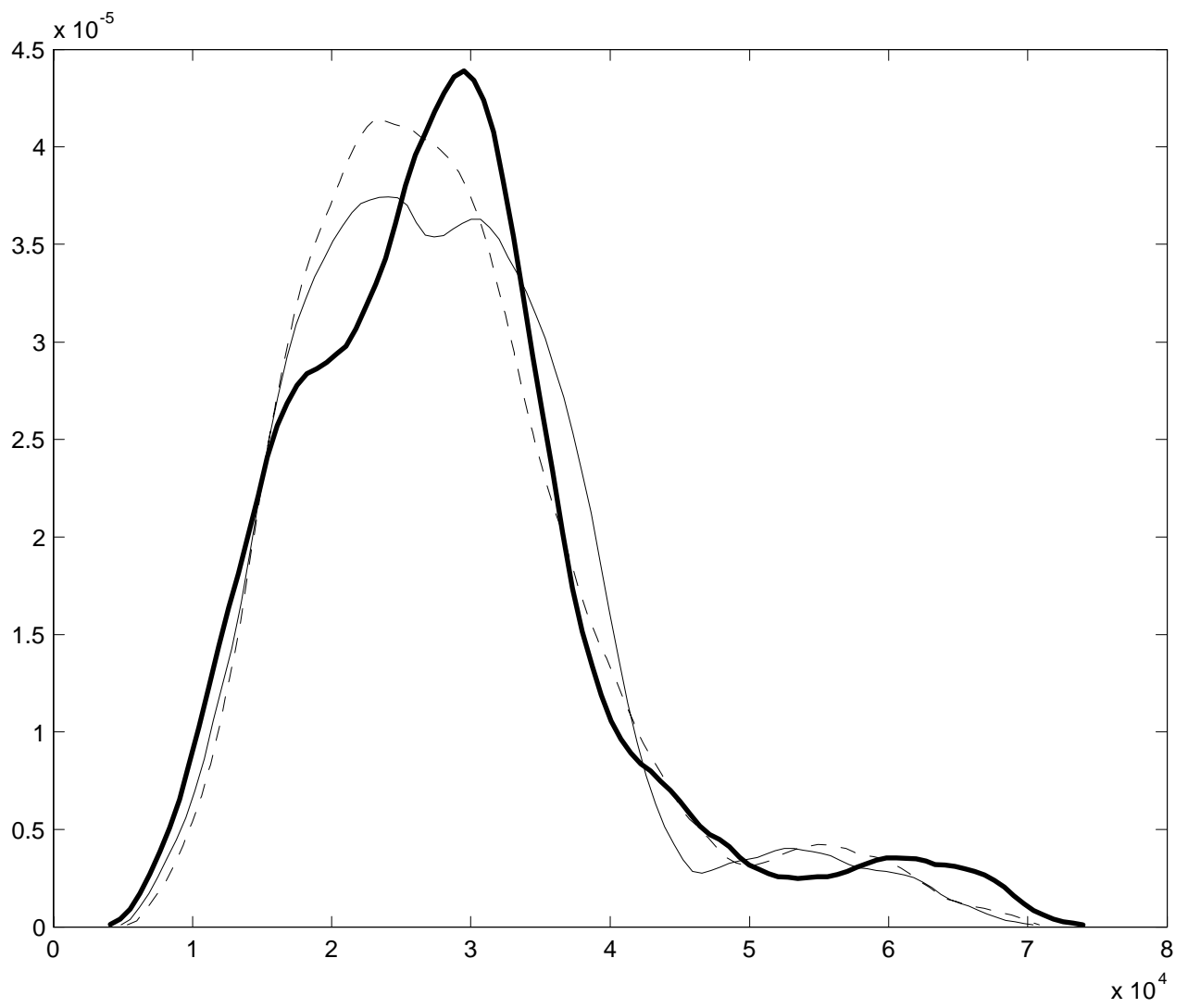
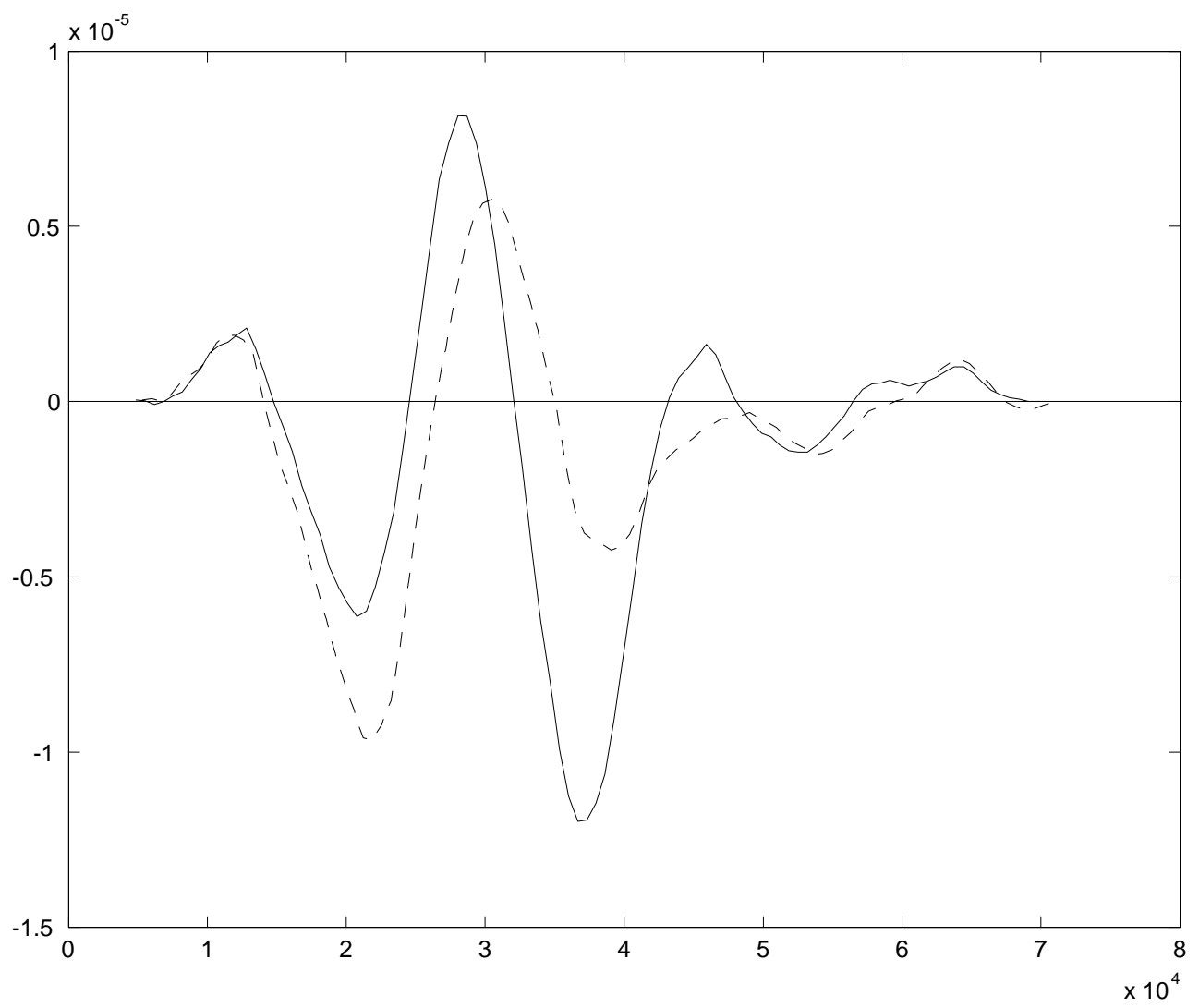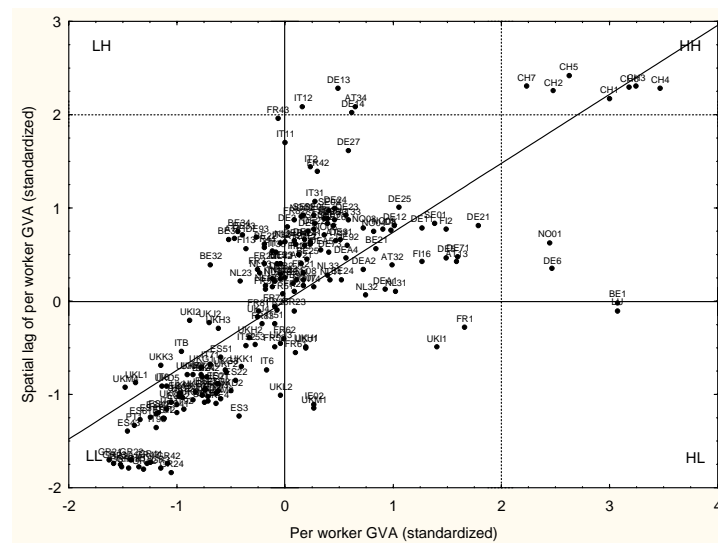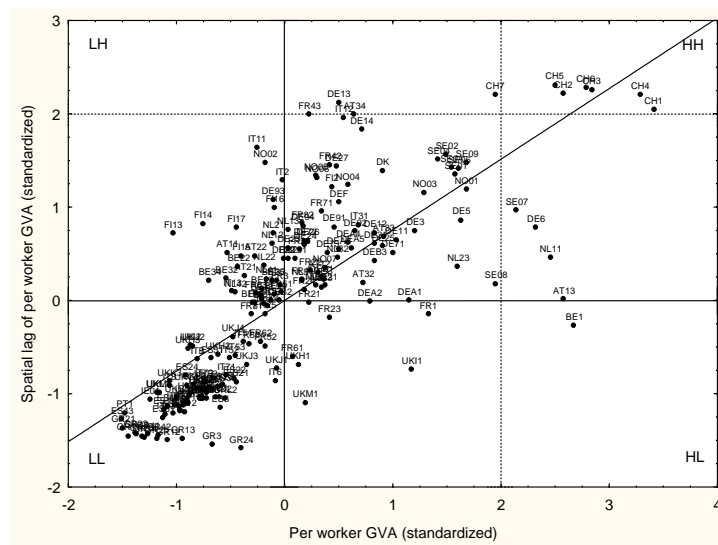Figure 2: Contrefactual density estimates

Figure 3: Differences between the true and the contrafactual densities

Figure 4: Moran scatterplot for GVA per worker in 1977

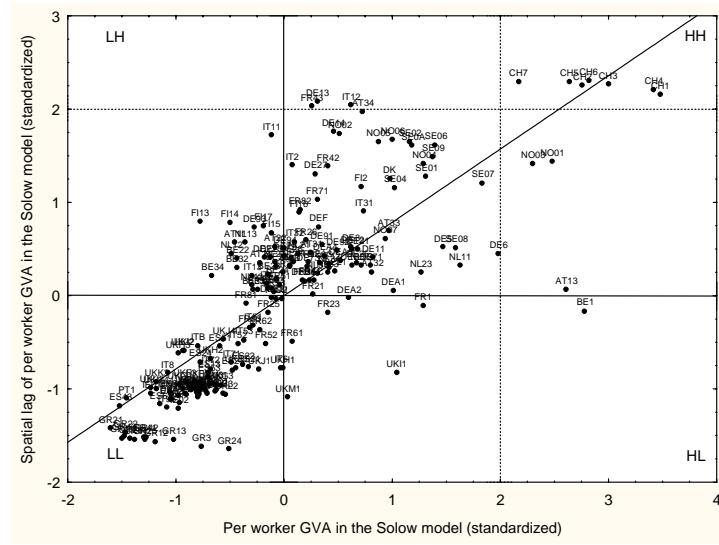Figure 5: Moran scatterplot for GVA per worker in 2000

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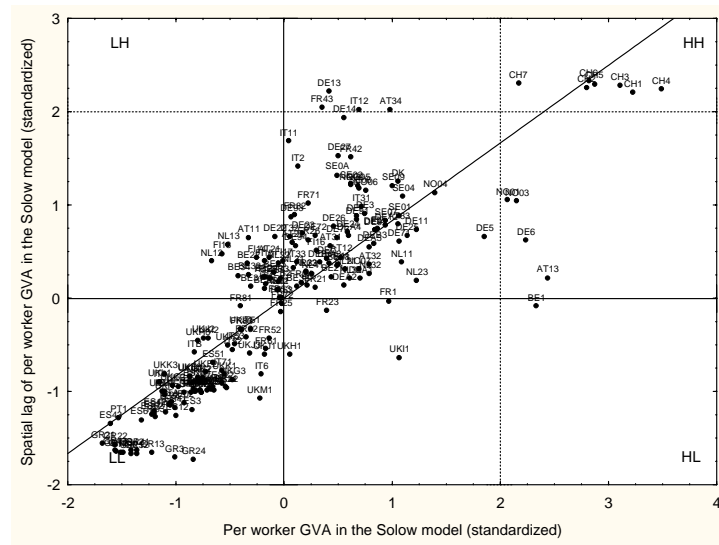Figure 6: Moran scatterplot for GVA per worker implied by the Solow model

Figure 7: Moran scatterplot for GVA per worker implied by the spatial Solow model

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Table 4: Transition probability matrix between the distributions in 1977 and 2000

		Quadrant in 1977			
		HH	HL	LH	LL
Quadrant in 2000	HH	0.907	0.222	0.333	0.023
	HL	0.000	0.778	0.030	0.023
	LH	0.093	0.000	0.636	0.011
	LL	0.000	0.000	0.000	0.943

Table 5: Transition probability matrix between the distributions in 2000 and implied by the Solow model

		Quadrant in 2000			
		HH	HL	LH	LL
Quadrant in Solow	HH	0.867	0.000	0.276	0.000
	HL	0.012	0.600	0.000	0.000
	LH	0.120	0.000	0.690	0.000
	LL	0.000	0.400	0.034	1.000

Table 6: Transition probability matrix between the distributions in 2000 and implied by the spatially augmented Solow model

		Quadrant in 2000			
		HH	HL	LH	LL
Quadrant in Spatial Solow	HH	0.916	0.100	0.345	0.000
	HL	0.000	0.500	0.000	0.000
	LH	0.084	0.000	0.655	0.000
	LL	0.000	0.400	0.000	1.000