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Influences of the shape of a territory on the optimal locations of human activities? A numerical geography approach.

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Abstract.- This paper aims at showing how far the shape of a studied area influences the results of optimal location-allocation models. Simulations are performed on rectangular toy-networks with an equal number of vertices but with different length/width ratios. The case of merging two such networks into a common market is also considered. We limit our experience to the Simple Plant Location Problem (SPLP) which captures the fundamental trade-off of economic geography between accessibility and economies-of-scales. Results are analysed in terms of locations, allocations and costs. On the average, we confirm that regions that are elongated require a greater number of facilities that those with a compact shape; this effect however depends upon the way a region is merged into a common market (type of border; relative position). The results help at understanding how far an area (country/region) has larger development problems than others just because of its shape and/or of the way this area is linked within a common market (elongation of the country and length of the common border).

Key words: Location-allocation models; S.P.L.P.; regional development; shape; region.

I. INTRODUCTION

For hundreds of years, facility locations have largely been determined qualitatively. Following the advent of Operations Research in the last 40 years, mathematical models have been developed and used in order to solve these problems quantitatively. The purpose of **location-allocation models** is to help find a pattern of locations (firms or services) providing a service to spatially dispersed clients as well as an allocation of the clients to those facilities in order to optimize one (or several) economic criteria often related to accessibility. Optimal location models enable one to find within a studied area the "best" location(s), the design of the service areas, the size of the facilities as well as the costs. The interest in these models is based on the commonly known equity-efficiency dilemma. Demand and supply characteristics are included in these models as part of their inputs. These models are mainly solved by O.R. methods (see e.g. Labbé, Peeters and Thisse 1995; Daskin 1995) and revealed to be quite operational tools in terms of planning (Drezner 1995; Drezner and Hamacher 2004).

However, when applying these methods, a good **definition of their inputs** is crucial for robust solutions (see Peeters and Thomas 2001 for a review). Standard inputs mainly rely on a network: nodes correspond to demand centers, potential supply sites and crossroads; links represent the transportation network. Both can be weighted (for example demand points by population, links by road distance or time, etc.). In order to get an operational decision making tool, numerous sensitivity analyses were previously performed on real-world examples in the 1990s and more recently on toy-network (see e.g. Thomas 2002 for a review as well as some examples). As far as we know, one aspect has never been considered: the shape of the studied area. This should be of prime interest as administrative units suffer from a lack of homogeneity, whatever the scale of analysis: communes, provinces, regions, countries are heterogeneous in size and shape.

The bias of the shape has for long interested geographers (Bunge ***; Boyce and Clark, 1964;). We also know that boundaries (physical, built, administrative also condition geographic relationships, e.g., the geometry of boundaries can strongly influence spatial interaction models (Griffith 1982). Moreover, most administrative units are different in size and shape. This has lead to an interesting field of research called the Modifiable Areal Unit Problem (MAUP) which is a potential source of error that can affect spatial studies using aggregate data sources (see e.g. Unwin 1996; Openshaw and Taylor 1975); this is also the case of location-allocation models (see for instance Daskin et al. 1989, Current and Schilling 1987, Francis et al. 2000). These aggregated basic spatial units are often arbitrary in nature and different areal units can be just as meaningful in displaying the same base level data. The objective of this paper is not to treat the MAUP problem in location-allocation results; we here restrict ourselves to the shape of the studied area, the shape of the envelop. It has indeed already been demonstrated that the boundaries of a state and the shape of the land, which it encompasses, can present problems and/or they can even help to unify the nation/region (de Blij and Muller 2003). Five categories of territorial configurations of countries are distinguished according to their shape: compact, fragmented, protruded, perforated and elongated. The **compact** country is – on the average - the easiest to manage: compactness is supposed to keep the country together. Metropolitan France, Cambodia, Uruguay, and Poland are typical examples. The **fragmented** nation is often an archipelago difficult to govern. When it was created in 1947, Pakistan consisted of two regions separated by more than 2000 km; after a long period of trouble, the country split up and the eastern part became independent in 1971 under the name of Bangladesh. Another example of fragmented nation are the Philippines or Indonesia. The best example of **perforated** nation is South Africa, which completely surrounds Lesotho; the surrounded nation can only be reached by going through one country. In case of hostilities, access to the surrounded country can be difficult. A **protruded** or panhandled country has an extended arm of territory (panhandle), which complicates the shape of the territory. It is a type of territorial shape that exhibits a narrow, elongated land extension leading away from the main body of the territory. Examples : Thailand, Oklahoma, Myanmar. Last but not least, we have the elongated nation. An **elongated** nation such as Chile makes it difficult for the capital Santiago located in the center of country to administrate the peripheral areas in the north and south. Other examples are Italy, Portugal, Vietnam or Laos. Another question could be : does the elongated shape of Czechoslovakia explain why it split in 1992 into two parts (Slovakia and the Czech republic)? Elongated countries will be of interest in this paper: they correspond to states with a territory long and narrow; their length are at least six times greater than their average width.

Let us now take the example of two elongated European countries: **Italy** (57,8 million inhabitants; 294.000 km² of land; north-south crow-fly distance: 1100 km) and **Portugal** (10 millions inhabitants; 92.000 km^2; north-south crow-fly distance: 570 km). They differ in the way they are bound to the mainland: a very long terrestrial border with Spain for Portugal

(1.214 km), a comparatively short border along the Alpine axis for Italy (1.933 km). In Italy, the border is perpendicular to its longest axis; in Portugal it is parallel. Many questions arise when considering the shape of a state: everything else being equal, how far does the shape of the country influence its governance? Does it result in inefficiencies for the provision of goods and services? Do respective shapes have an influence when countries merge in a common market? May shape explain (at least partially) why some countries separated in the past?

When studying optimal locations, we know from former analyses (Peeters, Thisse and Thomas 1998; Thomas, 2002) that the permeability of the **border** has a limited effect on the results. We also know that **border effects** often appear when finding optimal solutions: by ignoring the surrounding world, sub-optimal solutions might be found. We however don't know how far the shape of the studied area affects optimal locations and hence development. This paper aims at testing the effect of changes in the shape of the studied area on location-allocation results – every other sources of variation being held constant. We here aim at conducting some tests on toy-networks of fixed size in order to see how far elongation can affect optimal results. If we can easily imagine optimal solutions in the case of autarky, we have no idea of the influence of one elongated entity within a common market.

The paper is organised as follows. Section II summarises the design of the experiment; Section III presents the results. Conclusions and discussion are reported in Section IV.

II. DESIGN OF THE EXPERIMENT

2.1 The location model

We here limit ourselves to one location-allocation model: the Simple Plant Location Problem (SPLP). Given some requirements for a composite good distributed over space, the purpose of this model is to determine the number and locations of facilities in order to minimize the sum of the production and transportation costs (Mirchandani and Francis 1990, Drezner 1996 or Drezner and Hamacher 2004). Given preceding sensitivity analyses (see e.g. Thomas, 2002) , this choice should not affect the conclusions of this paper.

For the sake of clarity, let us here simply remind that, for the SPLP model, on the demand side, social needs are expressed by some fixed requirement for a composite good. Requirements are distributed over a finite number of points *j* and the requirement for the composite good in site j is denoted by δ_j. On the supply side, facilities can be placed at a finite number of potential locations denoted i while production involves scale economies. The set-up cost and the marginal cost at location i are denoted respectively by F_i and c_i; hence the production cost of a facility at i with output q_i is given by $F_i + c_i\, q_i$. Fixed costs may account for differences in fixed factors endowments, while the marginal costs may reflect particularities in local competition for variable production factors. Finally, the cost of shipping one unit of the composite good from site i to site j is denoted t_{ij}. Clearly, the matrix (t_{ij}) of transportation costs is general enough to allow for different shapes of the transportation network and various access conditions to local markets. Note that a rise in fixed production costs is formally equivalent to a fall in transportation costs; hence studying the impact of F_i on the locational pattern amounts to studying the impact of the t_{ij}.

The model can be formalized as follows

$$\text{Min} \quad \sum_i \sum_j \left(c_i + t_{ij}\right) \delta_j\, x_{ij} + \sum_i F_i\, y_i$$

subject to

$$0 \le x_{ij} \le y_i, \quad \forall i, j$$

$$\sum_i x_{ij} = 1, \quad \forall j$$

$$y_i \in \{0,1\}, \quad \forall i$$

where x_{ij} stands for the (nonnegative) fraction of the demand at j supplied by a facility at i, and y_j is a 0-1 variable which equals 1 when a facility is located in j and 0 otherwise. The first set of constraints implies that no demand can be supplied from a site where no facility has been built. The second set of constraints means that the total requirement in each i must be met.

This model is based on a discrete representation of the geographical space: i and j nodes summarize the information of the space which is by definition continuous. Each node summarizes information about a basic spatial unit. The shape of the studied area is determined by the location (coordinates) of all these points.

The central feature of the SPLP, and the reason why we singled it out, is that it captures the fundamental trade-off of economic geography: the existence of economies-of-scales expressed by the magnitude of the fixed costs which tend to reduce the number of facilities (centripetal forces) and the accessibility to the clients or markets which, on the contrary, leads to multiply them (centrifugal forces).

2.2 The settlement system

Computational experiments are here conducted on idealized networks (toy-networks). Choosing a theoretical lattice rather than a real-world layout enables us to better isolate the tested problems from many other sources of variation and to control as much as possible for spatial layout. This technique has been shown to be quite relevant in former publications (Peeters, Thisse and Thomas 1998 and 2000; Thomas 2002). The set of points chosen as **benchmark** among all possible spatial configurations is a 15×15 squared lattice (225 points) where each point *j* has the same spatial environment as every other, at the exception of the border points. Each point *j* of the lattice is simultaneously a demand point and a potential location for a facility. Each *j* is characterized by its coordinates (x_j, y_j) and is linked by an edge to its closest neighbors. The number of edges and their spatial organization define the transportation network. In our previous papers, we let this latter vary. In this paper, it is fixed: horizontal and vertical edges (length = 100) are considered as transportation links, forming a grid network. We know that this type of network favors dispersion of facilities, whereas overall accessibility is stronger in radial or circumradial networks, which amplifies the role of the centripetal forces generated by scale economies. This choice enables to better isolate the effect of shape. In our simulations, transportation costs are set equal to distances measured on the network. Finally, the model (Section 2.1) requires that a quantity δ_j is to be associated to each point *j* of the lattice. It stands for the quantity of composite good demanded in that point. In this paper, it is supposed to be invariant with *j*, that is, the distribution of demand is uniform with $\delta_j = 1$ for all *j*.

In a second step, we let the **shape of studied area** vary. That is to say, starting with a 15×15 squared lattice, several shapes of rectangle parallelepipeds are designed; they all have the same size (225 points). Let us here define *b* as the length of the basis, and *h* the height (Figure 1). For 225 points, five shapes (*b*×*h*) are designed: 15×15, 25×9, 45×5, 75×3 and 225×1. We are aware that several researchers have developed shape measures (e.g., Boyce and Clark 1964; Massam and Goodchild 1971; Moellering and Rayner 1981; Tobler 1978) for describing and comparing geographic objects. Fractals have also been used for describing morphologies (see MacLennan et al. 1991). We here limit ourselves to basic shapes measures.

In a third and last step, we merge two rectangular regions into a **common market**. *LHS* refers to the left hand side region and *RHS* to the right hand side. The two regions are first linked together in such a way that there is a symmetry axis and they are placed side by side. In a second step, the condition of the axis of symmetry is alleviated. In our common market, the *LHS* region is always a square (15×15); the *RHS* varies in shape as previously mentioned. Horizontal as well as vertical elongations of the *RHS* region are considered. Further more, we let the position of the *RHS* vary : *RHS* can indeed be differently bound to the *LHS* region. In this way, nine different common markets are designed. This enables one to see how far the shape of the *RHS* region and the way it is bound to the common market influence the optimal locations, and further handicap or favor the regional development.

For the sake of clarity, let us call $\varepsilon = b/h$, the elongation that characterizes the shape of the studied area. When b is larger than h we have a horizontal elongation, while when b is smaller that h we've a vertical elongation. Both regions have the same size and are fully linked: border edges are created between all contiguous points belonging to the regions. We know from former experiments that the number of edges does not influence the results (see Peeters, Thisse and Thomas 2002, and Thomas 2002, pp.107-119). Simulations are here limited to rectangles; more complex geometrical shapes would blur the results. Simulations are performed on individual rectangular regions (autarky) and compared to solutions obtained on the same regions integrated in a common market. In a common market, the median axis of the *RHS* region can be horizontal or vertical, and its position relative to the *LHS* can be different. In this paper, after looking at the solution in the case of autarky (section 3.1), we first comment the horizontal solution (Section 3.2), then shortly the vertical solution (Section 3.3) as well as the effect of a north-south gliding *RHS* region (Section 3.4). Most analyses and comparisons are done in terms of locations and allocations (market areas, hierarchy of centers) and interpreted in a New Economic Geography context (relation between trade and the location of production inside countries – see Henderson 1996 or Crozet and Soubeyran 2004).

We are aware that these modelling assumptions are rather heroic. Cities are not regularly distributed on a grid, fixed costs and transportation costs are not everywhere the same, etc. History, physical geography, economic development, etc. will make the problem much more intricate.

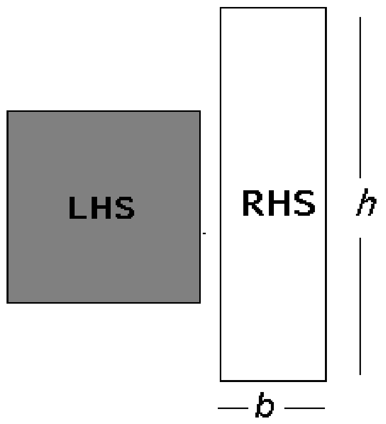

Figure 1: Studied toy-networks

III. COMPUTATIONAL EXPERIENCE

3.1 The case of autarky

Let us consider the case of a rectangular lattice in an isolated state. Fixed costs (F_i) and variable costs (c_i) are equal across locations: $c_i = c$ which is here equal to 1; $F_i = F$ and varies from 5,000 to 2,000,000. Two outputs are of interest in this paper: the number of facilities and their location. Figure 2 compares the number of facilities (p) for different shapes ($b \times h$) and for different values of F. Note that we here compare isolated effects; in reality, they should combine!

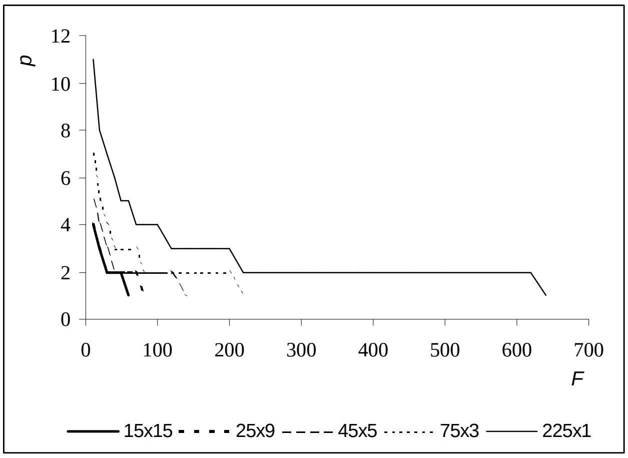

Figure 2: The effect of the shape of the region ($b \times h$) on the number of facilities (p) for different fixed costs ($F \times 1{,}000$), in the case of autarky.

The results are those expected. They illustrate the compromise between equity and efficiency: p (number of facilities) decreases when F (fixed costs) increases and the elongation of the rectangle influences the rate of the decrease. We notice that the square (b = h) is more efficient, whatever the value of F: it gives the smallest number of facilities for a given value of fixed cost. This also means that in the case of a compact studied area (15×15), the solution with one facility (p = 1) is reached for small fixed costs (F= 60): one facility is then located in the centre. When the difference between b and h increases, it costs more to the authority to cover the entire area, whatever F. Small differences in ε lead to comparable solutions; strong elongation leads to very expensive spatial solutions: whatever F, the number of facilities necessary to cover the area is much larger. Figure 2 also shows that in our toy-networks, we need fixed costs ten times larger for a 225×1 rectangle than for a squared (15×15) and that 4 sites are necessary in a squared lattice when F=5,000, whereas 11 are necessary for the 225×1 rectangle!

Locations are here briefly discussed, but not illustrated. As expected, in the squared and compact studied area, we start with a Christaller type distribution when F is small and rapidly end up with a central location when F increases. However, as soon as $b \neq h$, we observe an alignment of the locations on the principal axe of the rectangle. These locations are regularly spread on that main axis. More surprisingly, the centre of the rectangle is not often selected as an optimal location. When looking at the allocations, we see that the size of the market areas is quite homogeneous in a squared compact studied area, while discrepancies between the largest and the smallest service area become larger when elongating the studied area.

We can here conclude that in the context of an isolated state, there are **elongation costs**; these extra costs depend upon the extend of the elongation (here noted ε). In other words, the number of facilities decreases when fixed costs increase and the elongation of the rectangle influences the rate of the decrease. The result is that, when fixed costs are small, the relative difference in the number of facilities between differently-shaped regions is big; when fixed costs are larger, the difference in the number of facilities is smaller. This is however counter-intuitive, since our real-world experience tells us that when transportation cost decreases we can better overcome the friction of distance, i.e. an elongated shape should matter less if transportation cost is small.

3.2 The case of a common market

Let us now consider a common market where *LHS* is always a 15×15 square and *RHS* is a rectangle where *b* and *h* vary. Market integration is here simply considered as the physical attachment of two regions; we are aware that in reality, it is understood as increased trade and interactions between regions each having their comparative advantages. In this section, we only consider the case where *b* is always larger than *h* (horizontal rectangle). Once again (Figure 3), *p* decreases when *F* increases and the elongation of the *RHS* region has a strong effect on the efficiency of the entire spatial system. A small elongation (25×9) has little effect on the optimal organisation; when ε increases, differences become very large in terms of costs of covering the region. In other words, **elongation leads to insularity**!

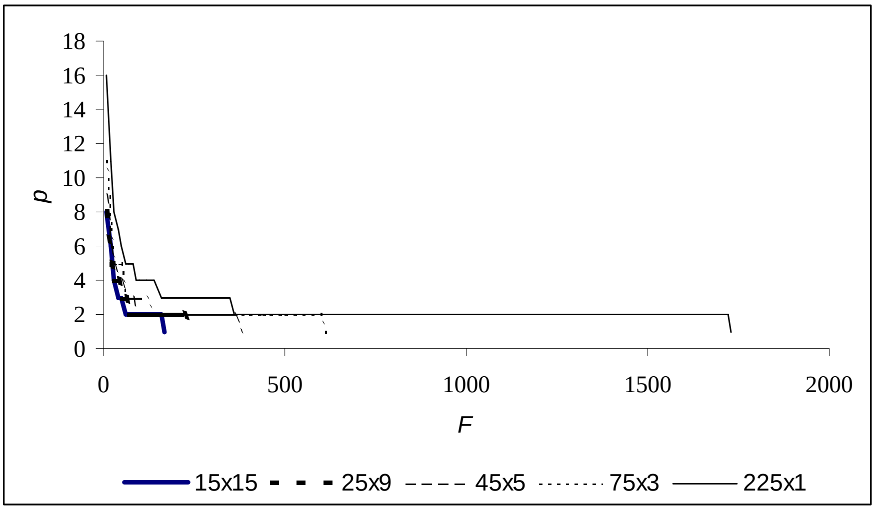

Figure 3: The effect of the elongation of the *RHS* region on the variation of the number of facilities (*p*) with fixed costs (*F*), in the case of a common market.

In Figure 4, we give the optimal number *p* of facilities in an integrated economy as well as in the sum of the two economies that are separated (autarkies). Results are similar. In other words, the opening of the economies is not likely to have a strong impact on the locational pattern (assuming of course that the spatial distribution of demand remains the same). When there is a difference, however, the integrated economy typically involves a number of facilities smaller than the total number of facilities operating in the separate economies. In this case, market integration seems to yield more but small geographic agglomerations. This confirms former results obtained in a common market where the *RHS* = *LHS* (Peeters, Thisse and Thomas 1998; Thomas 2002).

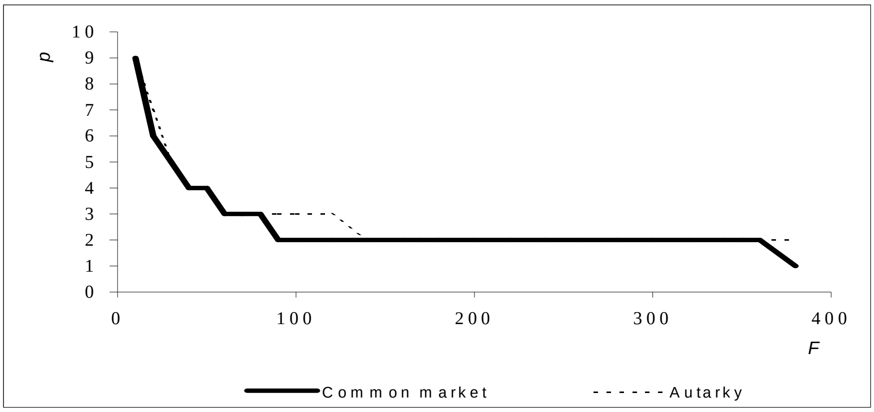

Figure 4: Variation of the number of facilities (*p*) with fixed costs (*F* × 1,000) in case of autarky and common market for *RHS* = 45×5

Optimal locations (Figure 5 in appendix) are affected by the elongation ε of the *RHS* region in several terms. First, the compact area (*LHS*) attracts more sites when the elongation of the *RHS* is small. When ε increases, the rectangular area is characterised by more sites than the *LHS* square: more locations are necessary for covering the *RHS*. This means – among other things – that compact regions will tend to higher centralisation than others and that total transportation costs depend upon the shape of the region.

Second, when *F* increases and hence *p* decreases, the optimal locations are situated on the spinal line of the common market: the properties of the rectangle expand to the square. Geometry matters within a common market.

Third, when only one site is located ($p = 1$) there is no predatory effect. The optimal site is the geometrical centre, which is not the centre of gravity. This location corresponds to a gate point (Labbé, Peeters and Thisse 1995); as the sum of the weights in each region is equal, this corresponds to a median location. Thus distances do not matter anymore: shape prevails !

Demand is homogeneous (no weight variation) ; this explains why the allocation process partitions the two regions quite regularly; there is a very slight predatory effect of the *LHS* region on the elongated *RHS* region for average values of *F* and for average elongations (not illustrated)

Figure 6 gives the variation of the total transportation costs associated with each location pattern. It corroborates the results discussed in the former paragraphs : transportation costs increase with *F* and with elongation : compactness is more sustainable.

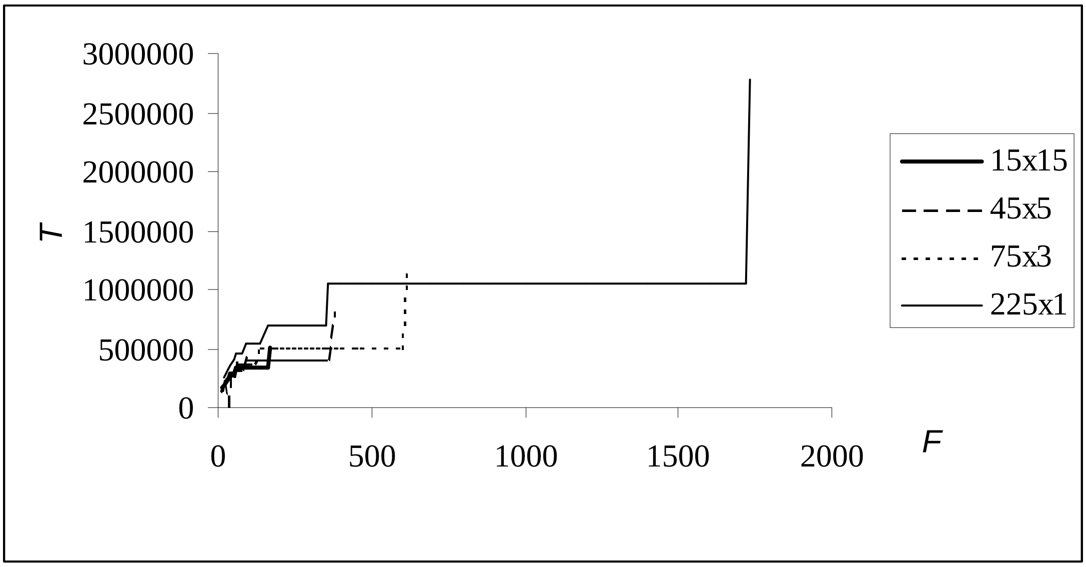

Figure 6: Variation of the total transportation costs (*T*) with fixed costs ($F \times 1{,}000$) and the elongation ($b \times h$) of the *RHS*.

3.3 Vertical versus horizontal elongation

Let us now compare two common market structures: one in which the *RHS* region is vertical ($\varepsilon < 1.0$) and the other in which the *RHS* region has the same shape but is horizontal ($\varepsilon > 1.0$). Results are reported in Figure 7. The total number of facilities suggested by the model decreases similarly with *F* for small values of *F* ; however, the vertical organisation is always "covered" by one site at a much lower value of *F* than in the horizontal organisation. The greater the elongation of the *RHS*, the more important is this difference. This is mainly due to transportation costs. Hence, a vertical organisation costs less than a horizontal configuration.

Verticality partitions the *RHS* in three areas: two appendices (one north, one south), and a central area. In terms of locations, the two appendices react as 2 peninsulas. The isthmuses linking them to mainland do not play a great role anymore as those are not dominant regions (in terms of weights). These sub-regions are hence little affected by the common market in terms of locations; the central area, close to the border with *LHS*, is attached to the compact *LHS*. Once again: geometry matters.

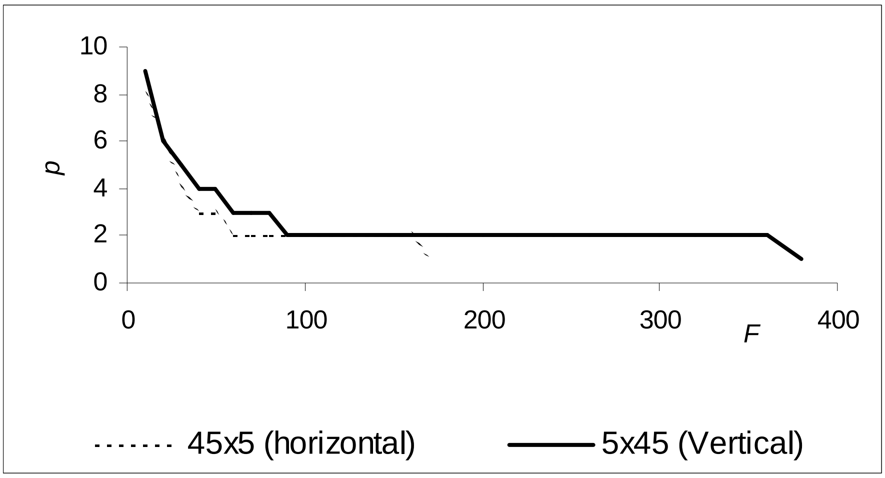

Figure 7: Variation of the number of facilities (*p*) with fixed costs (*F* × *1,000*) when comparing a vertical (5×45) versus a horizontal (45×5) rectangle of equal size and shape on the *RHS*.

When looking at the optimal locations (not illustrated here), we observe that the spinal locations remain on the northern and southern appendices of the vertical region, while they totally disappear in the central-eastern region. This is of particular interest when comparing the way Portugal or Italy is attached to the EEC. Moreover, when the vertical elongation is not too high that is to say when the *RHS* region still has some compactness, we observe that a move of the optimal locations in *LHS* towards the common border.

In the vertical organisation we also see that the bottleneck city is never selected as optimal location. The isthmus disappears for another kind of geometry. Border cities do not have any strategic role in the settlement system. Finally, verticality generates a larger predatory effect in terms of allocation: the *LHS* locations capture the *RHS* demand. Thus ε is here an important parameter in analysing spatial structures.

3.4 Position of the *RHS*

In this last section, we consider the effect of position of the *RHS* compared to that of the *LHS*; the *RHS* region translates (glides) from north to south. We once again consider a common market with two equal sized economies; the *LHS* is a square (15×15), the *RHS* is a vertical rectangle. The relative position the *RHS* here changes from top (north) to bottom (south): from only one link north, to 15 central links or to one link south (Figure 8). We considered several values of *b* and *h* but here only refer to *RHS* = 9×25. Results are independent of this choice.

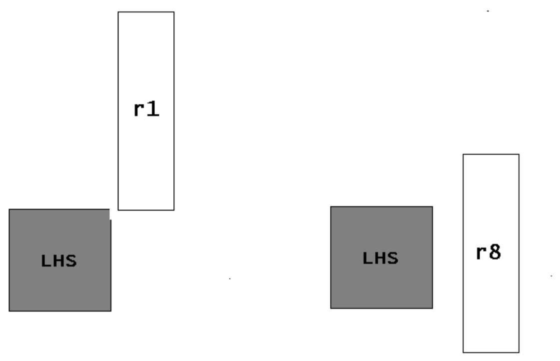

Figure 8: Simulations performed in Section 3.4: in *r1* there is only one possible link between *LHS* and *RHS*, in *r8* there are 15 possible links.

Figure 9 gives the results in terms of *p* and *F*. We see that (1) the more compact the common market that is to say the more central the *RHS* compared to the *LHS (r8)*, the cheaper are the costs for covering the entire area with one facility. Geometry matters, once again. We can also state that (2) the relationship between *p* and *F* does not (or so little) vary with the position of the *RHS* for small values of *F* (*F* < 130). More over differences in terms of locations (not illustrated) are more to be explained by local optima: the effect of elongation is more important than that of relative position !

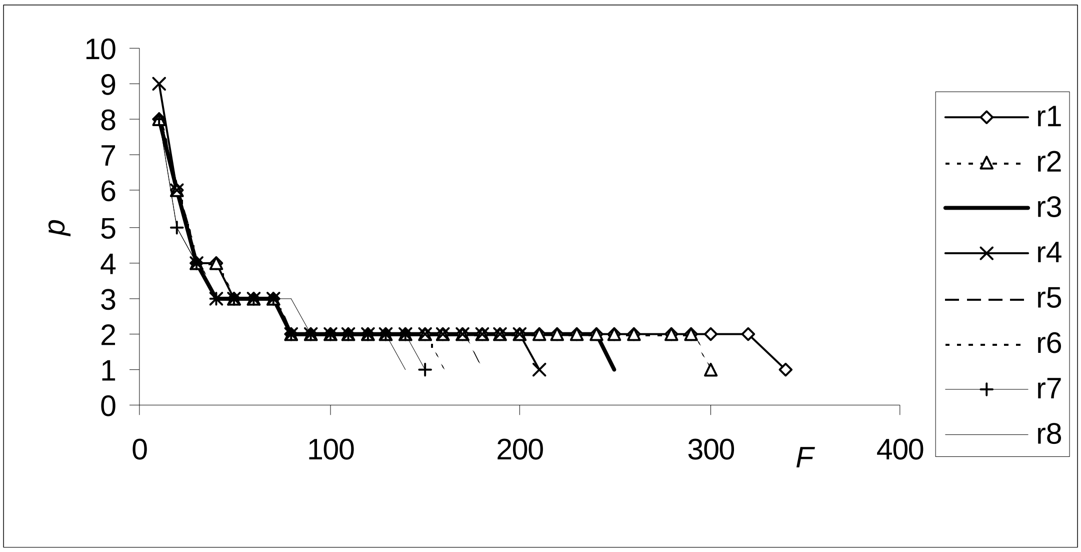

Figure 9:Variation of *F* (× 1,000) and *p* with the position (*r*) of the *RHS*.

IV. CONCLUDING REMARKS AND POLICY IMPLICATIONS

We know that the location of people and activities determines the nature and the quality of our lives but also regional growth. Geographers, spatial economists and regional scientists share this central conviction about the importance of space. In this paper we simply analysed the effect of the shape of a region/country on the optimal location of human activities. By means of an optimal location-allocation model applied on toy-networks, we here managed to isolate the effect, and showed how far geometry matters in optimal locations. Elongation of a country such as Italy implies important sources of extra costs for developing some parts of the country just because they are more remote than others (Mezzogiorno). We also showed that the way the elongated region is linked to the common market is also of prime importance: Portugal and Italy are two different and good examples for that. This should also be true at other scales of interpretation (regions).

Until the 1990's, most studies on economic growth have tended to ignore or underplay the role of physical geography (Clark, Feldman and Gertler 2000). Since the development of NEG, economists have re-discovered geography and include climate, access to the sea, soil quality,… as explanatory variables in their equations for explaining economic development and cross-country differences in the level and growth of per capital GDP. Our paper considers an additional and neglected aspect (the shape of the nation) and confirms that "physical geography" also matters: spatial differences in economic performance may arise even when the economies are initially similar in structure. The administrative boundaries can generate differences in locations and hence economic development.

We here considered the shape of the studied area as an exogenous source of variation (instrumental variable), all other source of variation being held constant. We are aware that this is artificial: all explanatory variables are exogenous and quite potentially correlated with other variables that may truly be driving forces. Moreover we isolated the effects; in reality they interact.

The model we used, the SPLP, is built on the fundamental trade-off of economic geography between minimizing transportation costs by creating facilities (centrifugal force) and minimizing the number of facilities due to the presence of economies of scales (centripetal force). The paper shows that the magnitude of the trade-off strongly depends on the shape of the region: the more compact the region, the more sensitive the interplay of the two forces. Merging regions raises several interesting questions related not only to their shapes but also to their relative positions. This can be alternatively viewed as a way to tackle more complicated shapes than the rectangular ones we have discussed in this paper. Our investigation leads to a not too surprising result: all types of appendages are costly.

One of the most obvious limitations of our simulations is that we have assumed a uniformly distributed population. Taking into account geographical heterogeneity in the demand would be an interesting extension, although we can infer some results. Consider for instance the USA and Canada in the NAFTA-framework. As most of the Canadian population is located close to the southern border, we may reasonably argue that the relative position of the two countries is quite similar to the Spain/Portugal case.

Another limitation is the static framework in which we remained. One can expect a relocation of the populations in the long run to adjust to the provision of public and private goods by the facilities and to the job market. Different evolution paths can appear and one may suspect that they will emphasize the trends we observed in the static analysis. One path would lead towards the centralization of the country, with a “core” population. An alternative path could lead to the emergence of local population centres. One may suspect that compact shapes will more likely generate the first path, while elongated countries or “legged” countries more likely the second.

A final comment concerns the agglomeration economies that are not captured by the SPLP while they are at the core of the discussion of the New Economic Geography (Fujita et al. 1999, Fujita and Thisse 2002). It is well proved that these types of externalities influences the patterns of location of firms. The fact is their influence is limited in space, and one of the questions raised in several papers is to measure their geographical extent (see, e.g., Hanson 1998, Amiti and Cameron 2004, Mion 2004, Rosenthal and Strange 2004). For instance, Mion (2004) found in the case of Italy that access to consumers’ demand was significant up to 200 km. Obviously, when comparing two countries such as France and Italy, the impact of a shock in demand or in intermediate product at one location is likely to exhibit very different patterns according to the shape of the territory. Interestingly, the morphology of the regions is almost totally absent from the various economic geography models proposed in the literature.

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Figure 5: Optimal locations in the case of autarky (+) and in the case of the common market (•). *RHS* = 25×9 (for increasing values of *F*(×1000) : 10, 20, 30, 50, 70 and 240)