Dynamic road pricing optimization with heterogeneous users

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Abstract

In transport networks, travelers individually make route and departure time choice decisions that may not be optimal for the whole network. By introducing (time-dependent) tolls the network performance may be optimized. In the paper, the effects of time-dependent tolls on the network performance will be analyzed using a *dynamic traffic model*. The network design problem is formulated as a *bi-level optimization problem* in which the upper level describes the network performance with chosen toll levels while the lower level describes the dynamic network flows including user-specific route and departure time choice and the dynamic network loading. In case studies on a simple hypothetical network it is shown that network improvements can be obtained by introducing tolls. It is also shown that finding a global solution to the network design problem is complex as it is non-linear and non-convex.

Index Terms— Bi-level optimization, dynamic road pricing, dynamic traffic assignment, heterogeneous users

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1. Introduction

In this paper we consider a network design problem (NDP) in which the aim is to determine a set of *design parameters* that will lead to an optimal network state. In our case the design parameters are (time-varying) road-pricing levels on road segments on a transportation network. Road pricing presents one of the market-based policy instruments having influence on the travel behavior of users. Many researchers have been working on road pricing problems trying to solve problems in transportation caused by increasing congestion levels (see e.g. [1], [2]). Congestion pricing is a type of *responsive pricing* that can change consumption (demand) patterns by influencing users' travel choices at various levels. The problem is to determine the prices that should be charged to travelers in order to improve the overall level of service of the system.

Nevertheless, congestion still remains one of the unresolved problems having impact not only on the transportation side but also on the economic and social life of people. How then optimize network performance using dynamic road pricing measures? In this paper we try to gain *more insight* into the dynamic toll design problem and this rather complex question.

The nature of pricing (static and especially time-varying and dynamic pricing) in combination with *dynamic traffic assignment* (DTA) model, leads to a very complicated problem formulation and complex solution procedures. The computational effort required to solve such a problem by well-known numerical methods grows prohibitively fast with the dimensions of the problem, restricting researchers to apply their algorithms only on simple hypothetical networks.

In this paper we mathematically formulate a network design problem in which time-varying road prices need to be determined that will minimize the total travel time in the system, taking route and departure time changes of travelers as a response to the prices into account. The contributions of the paper can be listed as follows: a) *dynamic* instead of static traffic flows and road pricing strategies are considered, b) the *Mathematical Programming with Equilibrium Constraints* (MPEC) method is used (up to now, MPEC formulation is applied to static cases only); c) not only route choice but also *departure time choice* is modeled; d) analysis of a very simple network shows that the network design problem is in general a complex *non-convex* optimization problem.

2. Literature study

Network design problems have been proposed in various studies, e.g. [3], [4]. The problem of congestion pricing has been studied in the literature from different modeling perspectives and under various assumptions. A classification can be made depending on different criteria: *pricing theory* (marginal cost pricing vs. second best pricing), *objectives* to be reached (minimizing total travel time or maximizing net social welfare or maximizing revenues), *type of analysis* (static or dynamic pricing), *pricing strategy* (link-based, path-based, or zone-based pricing) and *user classes* (users can be classified based on travel cost perceptions, information access, value of time, or vehicle category).

Dynamic congestion pricing models in which network conditions and link tolls are time varying, have been addressed in [5] in which the effectiveness of various pricing policies (time-varying, uniform, and step tolls) are compared. Limitations of those models are that they consider either a single bottleneck or a single destination-network. In [6] and [7] dynamic marginal (first-best) cost pricing models for general transportation networks have been developed. As indicated by the authors, the application of their model is limited to destination-specific (rather than route or link-based) tolling strategies, which might not be easy to implement in practice. Moreover, since tolls are based on marginal cost pricing, it is implicitly assumed that all links can be priced.

Modeling the joint choice of route and departure time depending on the behavioral assumptions and the significance on unobserved effects is given in [8]. In [9] a dynamic congestion-pricing model is formulated as a bi-level program, and the prices are allowed to affect the (sequentially modeled) route and departure time choice of travelers.

Most previous studies assume that all road users have identical characteristics. The toll design problem that can accommodate *multiple user groups* applied for the static case is presented in the work of [10]. Our aim is to include multiple user classes in *dynamic* traffic network.

In the framework proposed in this paper, departure time and route choices are modeled simultaneously according to the results of a survey showing that the most important behavioral changes to road pricing are the route and departure time choices [11]. The dynamic road pricing design problem is formulated using the mathematical program with equilibrium constraints (MPEC) formulation. Our main contribution to the state-of-the-art is the MPEC formulation of the dynamic optimal toll design problem in dynamic traffic networks (up to now, only static optimal toll design is considered). The main purpose of studying the dynamic optimal toll problem is to address difficulties in modeling and complexity in optimizing the objective function even if it is only for very small examples. Moreover, class-specific second-best pricing, which is more appropriate for practice, is modeled and analyzed. Although complex, the feasibility of the model is illustrated by solving a simple case study.

3. Problem definition and assumptions

Considering the optimal toll design problem, the aim of the road authority is to optimize system performance (e.g. to minimize the total travel time) by choosing the optimal tolls for a subset of links, within realistic constraints and subject to the dynamic traffic assignment. The optimal toll design problem can be seen as a *bi-level problem* where on the upper level the road authority determines tolls on the links while the travelers on the lower level react to these tolls and adapt their travel behavior accordingly (see Fig.1).

Considering the framework of the *classical DTA model* (see e.g. [12]), the research aim is to determine which *extensions and modifications* of classical DTA model are needed for capturing impacts of road pricing. If road pricing is imposed on only a subset of links and not to all links, travelers could decide, on the one hand to change *routes* and travel on non-tolled routes or less tolled routes. On the other hand, if a toll level is dependent on a certain period of the day, travelers could be better off by changing their *departure time* and travel earlier or later.

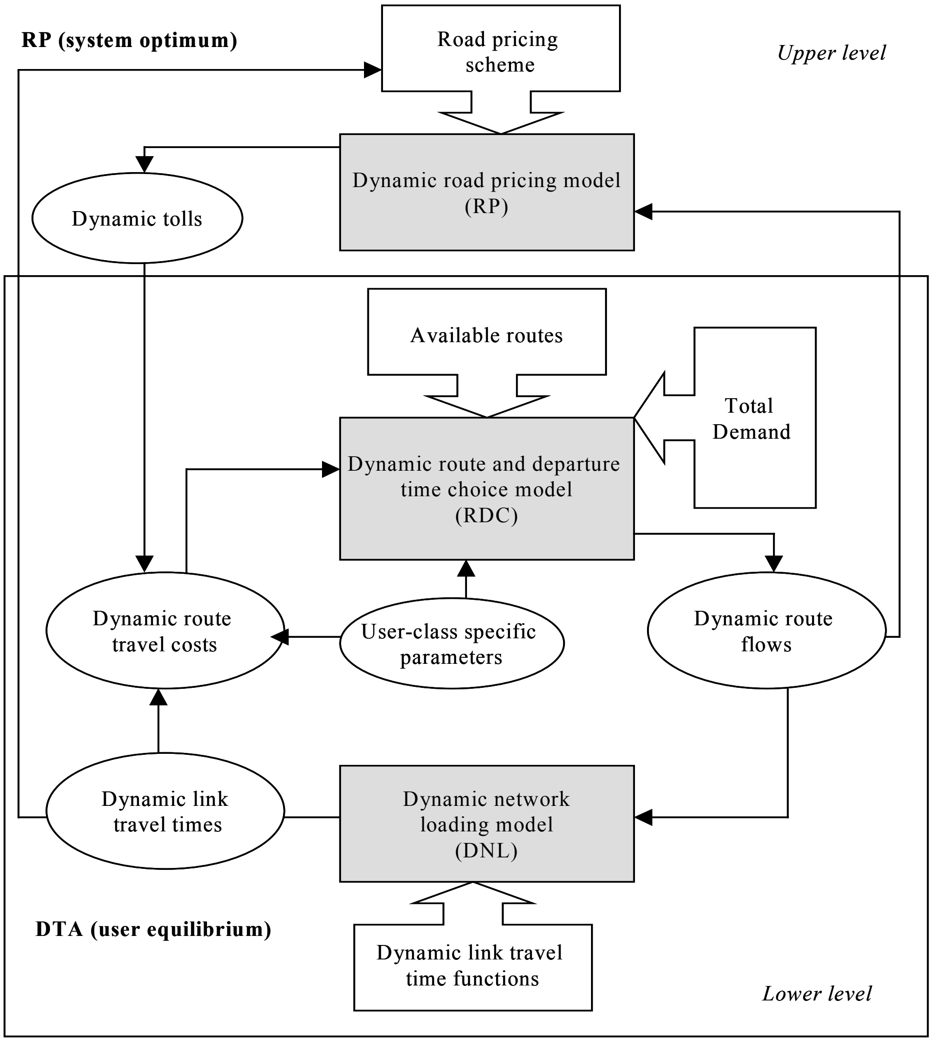

Fig.1. Conceptual framework for dynamic optimal toll design problem in dynamic traffic networks with heterogeneous users

From Fig.1 we can distinguish three different submodels: The dynamic network loading (DNL) model, the route and departure time choice (RDC) model, and the road-pricing (RP) model. The DNL and route and departure time choice models combined constitute the DTA model at the lower level. DNL simulates propagation of traffic along available routes in the network giving as a result dynamic link travel costs, travel times, and flows. The route and departure time model takes individual travel behavior into account in which travelers aim to individually minimize their travel costs, yielding dynamic user equilibrium. While the DTA model captures traveler's behavior, the road-pricing model captures different influences on that behavior. Given (time-varying) tolls as an element of the route travel cost function, one can consider the different impacts of variable tolls on the traveler's behavior. The aim of solving the road-pricing problem is to find an optimal space-time pattern of tolls over the whole network (e.g. minimize the total travel time of all network users). More about the problem definition and the theoretical framework can be found in [13].

4. Mathematical formulation of the optimal toll design problem in dynamic networks

Let $\Omega = \{N, A\}$ denote a transportation network consisting of a set of nodes N and a set of links A. Each origin-destination pair (r,s) is defined by an origin $r \in R \subseteq N$ and a destination $s \in S \subseteq N$. One or more routes $p \in P^{rs}$ may exist between origin-destination (OD) pair (r,s). Every route *p* is comprised of one or more links $a \in A$. We use a discrete time formulation in which the whole studied time period T is divided into a certain number of small time intervals, denoted by *t*. We assume that different user classes *m* are present on the network, each class having its own sensitivity to time and cost expressed by its value of time. Let the set of user classes be defined by *M*. The total travel demand from each origin *r* to each destination *s* per user class *m* is given by D_m^{rs}. The set of departure times is denoted by $K \subseteq T$.

As mentioned before, the proposed modeling framework contains three major components, that is, the DNL and RDC model, together forming the DTA model, and the RP model (see Fig.1). These components are interrelated to find the solution of the dynamic toll design problem.

4.1 DTA Model – Lower level of the problem

The DNL component is formulated as a system of equations expressing link dynamics, flow conservation, flow propagation, and boundary constraints. In this paper, we adopt the DNL problem formulation described in [14]. A discrete choice model describing the combined route and departure time choice as a function of travel time and travel cost will formulate the RDC component. In this section we concentrate on the formulation of the RDC component.

In a *discrete choice model* the travelers aim at maximizing their utility by choosing their subjective optimal route and departure time. We assume that travelers consider the *generalized travel costs* of choice alternatives when making a decision. Let $U_m^{rs}(p,k)$ denote the utility of user *m* when this user selects route *p* and departure time *k* for traveling from origin *r* to destination *s*. Furthermore, let $c_{pm}^{rs}(k)$ denote the corresponding generalized travel costs for this user. Then the utility function can be written as

$$U_m^{rs}(p,k) = -c_{pm}^{rs}(k) + \varepsilon_{pm}^{rs}(k), \tag{1}$$

where $\varepsilon_{pm}^{rs}(k)$ is a *random term* describing the subjective unobserved effects. Note that the generalized costs are a disutility since travelers are assumed to maximize utility, hence minimize travel costs.

In existing models describing route choice behavior, the dynamic generalized travel costs $c_{pm}^{rs}(k)$ usually only consist of the (user-class generic) actually experienced route travel times $\tau_p^{rs}(k)$. We will add several additional elements to the cost function, namely (a) an element for the (dynamic) road-pricing toll, (b) a user-specific value of time (VOT), and (c) penalties for deviating from a preferred departure and arrival time. Adding the toll makes sure that travelers respond to toll strategies, adding the penalties enables departure time choices, and adding the VOT yields that tolls and travel times are additive. Including all these elements yields the following generalized cost function:

$$c_{pm}^{rs}(k) = \underbrace{\alpha_m \tau_p^{rs}(k)}_{\text{travel time}} + \underbrace{\theta_p^{rs}(k)}_{\text{toll}} + \underbrace{\beta \left| k - \zeta^{rs} \right|}_{\substack{\text{penalty deviation} \\ \text{preferred departure}}} + \underbrace{\gamma \left| k + \tau_p^{rs}(k) - \xi^{rs} \right|}_{\substack{\text{penalty devation} \\ \text{preferred arrival}}}, \tag{2}$$

where α_m is the VOT for user *m*, $\theta_p^{rs}(k)$ is the toll to be paid for route *p* when departing at time *k*, ζ^{rs} and ξ^{rs} are the preferred departure and arrival times, respectively, while β and γ are parameters for the penalties for early/late departure and arrival. Note that α_m is the only class-specific element of the cost function. For simplicity sake, we only consider here different classes in terms of travelers having different VOT's. Extensions to more complex class definitions are straightforward. The exact specification and calibration of these cost functions are subject to research in a parallel study (see [10]). The tolls $\theta_p^{rs}(k)$ are input from the RP model component. The route travel times $\tau_p^{rs}(k)$ can be computed from the output of the DNL component that provides dynamic link flows and link travel times $\tau_a(t)$ for each link *a* and each time *t*, which is merely a sum of consecutive dynamic link travel times:

$$\tau_p^{rs}(k) = \sum_{a \in p} \sum_t \delta_{ap}^{rs}(k,t)\tau_a(t), \tag{3}$$

where $\delta_a^p(k,t)$ is a dynamic route-link incidence indicator which equals one if a traveler will at time *t* reach *a* on route *p* from *r* to *s* when departing at time *k*, and is zero otherwise. The route tolls $\theta_p^{rs}(k)$ may have a similar additive structure although this is not necessary. In case the dynamic route tolls are additive (e.g. a road pricing strategy where travelers are independently tolled each time they enter a new road segment), the tolls can be computed by adding the appropriate dynamic link tolls $\theta_a(t)$, i.e.

$$\theta_p^{rs}(k) = \sum_{a \in p} \sum_t \delta_{ap}^{rs}(k,t)\theta_a(t). \tag{4}$$

Various specifications of a discrete choice model can be derived by assuming different joint probability distribution functions for the random terms $\varepsilon_{pm}^{rs}(k)$ in Expression (1). Here we assume these random terms to be identically and independently extreme value type I distributed yielding a multinomal logit (MNL) model for the simultaneous route and departure time choice. The probability $\psi_{pm}^{rs}(k)$ of choosing alternative route *p* and departure time *k* among all other alternatives can then be expressed as follows:

$$\psi_{pm}^{rs}(k) = \frac{\exp\left(-\mu c_{pm}^{rs}(k)\right)}{\sum_{\bar{p} \in P^{rs}} \sum_{\bar{k} \in K} \exp\left(-\mu c_{\bar{p}m}^{rs}(\bar{k})\right)}, \tag{5}$$

where μ is a scale parameter. Equation (5) defines a simple discrete choice model while more sophisticated models may be used to overcome possible problems with e.g. overlapping (route) alternatives. For more details we refer to [8].

Given the proposed route and departure time choice behavior we can define the dynamic stochastic user-equilibrium equilibrium (SUE) state in the DTA model. As an extension of Wardrop's equilibrium law, we define this equilibrium as the state in which no traveler thinks that he/she can decrease his/her generalized travel cost by unilaterally changing routes or departure time. It can be shown that this equilibrium state can be found by solving an equivalent variational inequality (VI) problem, see e.g. [16]. In [17] the VI problem for the dynamic SUE is formulated as following. Find dynamic and class specific route flow rates $\bar{f}_{pm}^{rs}(k)$ such that

$$\sum_{rs\in(RS)}\sum_{p\in P^{rs}}\sum_{k\in K}\sum_{m\in M}\bar{Q}_{pm}^{rs}(k)\cdot\left(f_{pm}^{rs}(k)-\bar{f}_{pm}^{rs}(k)\right)\geq 0,\quad \forall f_{pm}^{rs}(k)\in F, \tag{6}$$

where $\bar{Q}_{pm}^{rs}(k)$ is defined as

$$\bar{Q}_{pm}^{rs}(k)=\left(\bar{f}_{pi}^{rs}\left(k\right)-\psi_{pm}^{rs}(k)D_{m}^{rs}\right)\frac{\partial c_{pm}^{rs}(k)}{\partial f_{pm}^{rs}(k)}. \tag{7}$$

The generalized travel cost $c_{pm}^{rs}(k)$ is a function of the route flow rates $f_{pm}^{rs}(k)$ since the route flows determine the link conditions in the DNL model and in return these determine the travel times and therefore the travel costs. The set F in Expression (6) defines the set of feasible route flow patterns that satisfy the flow conservation constraints (travel demand should be satisfied) and nonnegativity constraints, i.e.

$$F=\left\{f_{mp}^{rs}(k)\mid\sum_{k}\sum_{p}f_{pm}^{rs}(k)=D_{m}^{rs},\, f_{mp}^{rs}(k)\geq 0\right\}. \tag{8}$$

4.2. Road Pricing (RP) Model – Upper level of the problem

Tolls can be introduced as periodical time varying on a specific link, in contrast to uniform tolls or truly dynamic tolls. It should be noted that for the road authority different objective functions can be chosen depending on the aim of the toll strategy (e.g. to minimize the total travel time on the network, to maximize social benefit, or to maximize toll revenues, etc.) subject to some constraints on toll levels. For our experiment, minimization of the total travel time on the network is chosen. Objective function Z describes the total travel time on the network to be minimized, depending on the time-varying link toll values $\Theta\equiv[\theta_a(t)]$:

$$\min_{\Theta\in H}Z(\Theta)=\sum_{(rs)\in(RS)}\sum_{p\in P^{rs}}\sum_{k\in K}\bar{\tau}_{p}^{rs}(k)\sum_{m\in M}\bar{f}_{mp}^{rs}(k), \tag{9}$$

where $\bar{f}_{pm}^{rs}(k)$ are the equilibrium route flow rates and $\bar{\tau}_{p}^{rs}(k)$ are the corresponding equilibrium route travel times, both depending on the link toll levels Θ. The set H

defines the set of feasible toll levels, which in our paper consists of (time-specific) upper and lower bounds $\theta_a^-(t)$ and $\theta_a^+(t)$ on the toll levels, and of a definitional constraint assuming additive toll values:

$$H = \left\{ \Theta \mid \theta_a^-(t) \le \theta_a(t) \le \theta_a^+(t), \theta_p^{rs}(k) = \sum_k \sum_t \delta_{ap}^{rs}(k,t)\theta_a(t) \right\} \tag{10}$$

Since a second-best tolling strategy is followed in this paper, some links may not be tolled. If a link is not tolled, both the upper and lower bounds for these links are set to zero for all time periods.

For each evaluation of the objective function Z a VI problem has to be solved at the lower level. The bi-level optimization problem defined by Expressions (6) and (8) can also be written as a single level optimization problem using a mathematical program with equilibrium constraints (MPEC) problem formulation in which the inequalities in expression (6) are included into the set of constraints of optimization problem (6), see e.g. [18], [19].

The complexity of the problem defined above is NP-hard. In [20] it is already mentioned that bi-level problems are in general NP-hard and in [21] that even single agent problems are NP-hard, already with only a single commodity.

5. Solution approach

Each component of the optimal toll design problem can be solved using various types of algorithms. The solution algorithm for the DNL model we adopt stems from [12]. For the RDC equilibrium model a descent algorithm using the method of successive averages (MSA) is implemented. Because of the complexity of the whole problem we aim to solve it for the simplest case in order to examine the shape and the properties of the objective function to be minimized (non-linearity and non-convexity). For this purpose we use a simple grid-search procedure in order to solve the RP model.

The outline of the complete algorithm is as follows. The algorithm starts with specifying the full grid of considered decision values (tolls) satisfying the constraints. The algorithm starts with an initial feasible price vector satisfying the lower and upper bounds of the tolls. In each iteration the algorithm finds a *user equilibrium solution* based on the current toll levels and sets *new tolls* that can potentially decrease the objective function Z described in Equation (9). Because the algorithm is a grid search method it stops after all feasible toll values have been considered. More sophisticated algorithms may be used, e.g. to compute the gradients to find new tolls (see in [22]). More advanced algorithms will be the focus in the next step of our research.

The two-stage iterative grid-search procedure for the optimal time-varying toll problem with DTA (including joint route and departure time choice) can be outlined as follows:

Begin Outer Loop: PRICING

Step1: Initialization

Specify set of grid points to be calculated; Determine initial feasible solution for time-varying toll values satisfying the lower and upper bounds; Assume an empty network and free flow traffic conditions; Set $n := 0$.

Step 2: Set next toll combination from the grid

Begin Inner loop: DTA

Step 3a: Compute dynamic route costs

Compute travel costs $c_{pm}^{rs}(k)$ using Equation (2).

Step 3b: Compute new intermediate route flows

Determine the new intermediate dynamic route flow pattern $\tilde{f}_{mp}^{rs}(k) = \psi_{mp}^{rs}(k) D_m^{rs}$, using Equation (5).

Step 3c: Flow averaging

Use the MSA method to update the route flow rates:

$$f_{pm}^{rs,(n)}(k) = f_{pm}^{rs,(n)}(k) + \tfrac{1}{n+1}\left(\tilde{f}_{pm}^{rs,(n)}\left(k\right) - f_{pm}^{rs,(n)}(k)\right).$$

Step 3d: Perform DNL

Dynamically load $f_{pm}^{rs,(n)}(k)$ onto the network.

Step 3e: Convergence of the lower level

If the duality gap is sufficiently small, go to Step 4, otherwise return to Step 3a.

End Inner Loop

Step 4: Compute objective function

Compute the objective function in Equation (9).

Step 5: Termination of the algorithm of the upper level

If all toll combinations are explored, then the algorithm is terminated. Otherwise, set $n := n + 1$ and return to Step 2a.

End Outer Loop

Performing this simple iterative procedure, we explore all the possibilities for all toll level combinations and find the minimum of the objective function. Regarding the convergence of this algorithm, the inner loop using the MSA procedure typically converges to an equilibrium solution, although convergence cannot be proven. In the outer loop the whole solution space is investigated using a grid search (yielding a finite number of solutions that are evaluated). For practical studies this procedure will be infeasible, but for the purpose of analyzing the nature of the problem and the shape of the objective function this approach is preferable.

6. Experimental results

The solution procedure proposed in the previous section has been applied to a small network example shown in Figure 2.

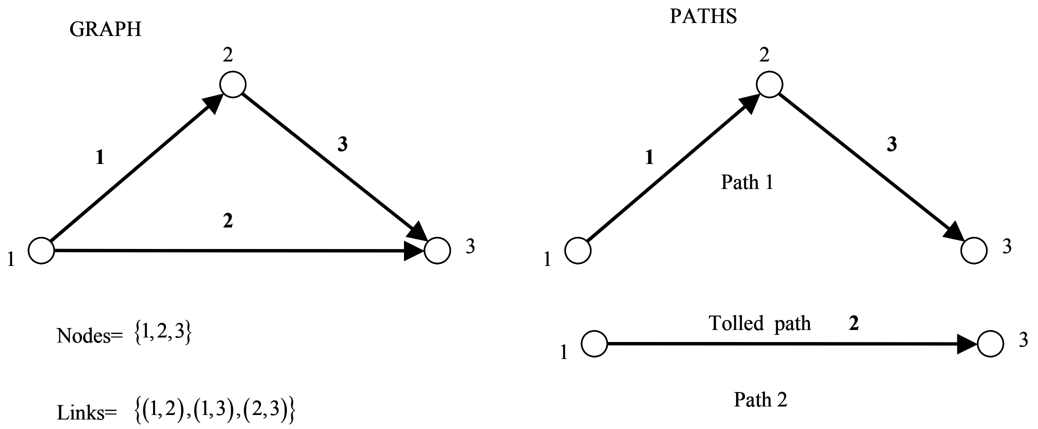

Fig. 2. Example: network description and route composition.

The network consists of one OD-pair connected by two paths. All three links have identical properties with a free flow travel time of 1 time period. Thus without congestion and tolls, route 2 is more attractive for the travelers than route 1. In the experiment we investigate the potential savings in total travel time if tolls are imposed to route 2 only, which means tolling just link 3. Two user classes are distinguished with different VOT's. The total travel demand from node 1 to node 3 is 86 of which 50% high VOT travelers and 50% low VOT travelers. The following parameter values are used: $\alpha_1 = 10$, $\alpha_2 = 15$, $\beta = 0.8$, $\gamma = 2$, $\zeta^{rs} = 5$, $\xi^{rs} = 10$, and $K = \{1, \ldots, 15\}$. Considering the preferred departure time and arrival time it can be expected that most travelers prefer to travel between periods 5 and 10, hence congestion is likely to occur between those periods. Therefore, a tolling strategy is used that link 3 will be tolled (only) on time periods 7, 8, and 9. For simplicity we assume that the toll on periods 7 and 9 are identical, such that two toll levels need to be determined: $\theta_3(7) = \theta_3(9) = \theta_3^1$ and $\theta_3(8) = \theta_3^2$.

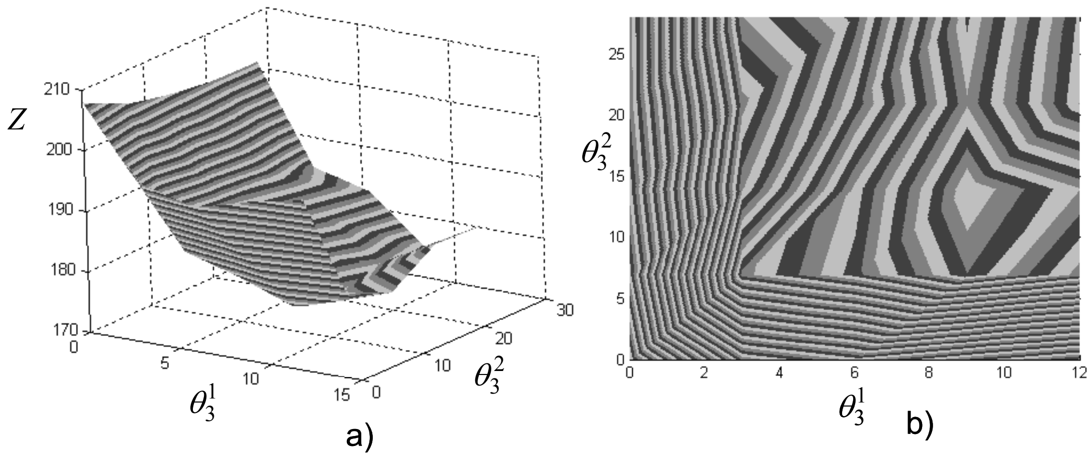

Fig. 3. The resulting objective function values for the optimal toll design problem

Fig. 3(a) shows the resulting shape of the objective function that corresponds to the different toll levels. The toll values have been divided into a grid of possible values and the objective function has been calculated for all the values of this set. For all toll levels the lower bound is zero, while the upper bounds for θ_3^1 and θ_3^2 are 12 and 30, respectively. From Fig. 3(a) we can conclude that the objective function is *non-convex*. Fig. 3(b) indicates the minimal value of the function and corresponding toll levels. We can conclude that the optimal tolls to be applied in time periods 7 and 9 are $\theta_3^2 = 14.8$ while the optimal toll in time period 8 is $\theta_3^1 = 8.5$. The resulting total travel time is *Z*=210 without tolls ($\theta_3^1 = \theta_3^2 = 0$) while being *Z*=170 for the optimal tolls (Fig. 3(a)).

The results corresponding to the situation with optimal toll values are plotted in Fig. 4. DTA convergence based on a dynamic duality gap function is provided in Fig. 4(a). The optimal toll levels are provided in Fig. 4(b), $\theta_3^1 = 8.5$ and the $\theta_3^2 = 14.8$, respectively. We can observe different dynamic route flow patterns for different user classes. We can observe in Fig. 4(e) that route 2 is not used in time periods 7, 8, 9 for travelers with low VOT. This is because of the high tolls (route costs) imposed to these times periods (Fig. 4(c)); hence travelers are shifted to non-tolled time periods. It is also the reason for such high flows in time periods 10, 11, and 12 (Fig. 4(e)). For the travelers with high VOT of tolls, route 2 is not used only in time period 8 (Fig. 4(f)). This is because of high route costs imposed to this time period (Fig. 4(d)). As expected, the travelers with high VOT can afford to pay higher travel cost and on time periods 7 and 9 they don't change their route while the travelers with low VOT are pushed off in these time periods.

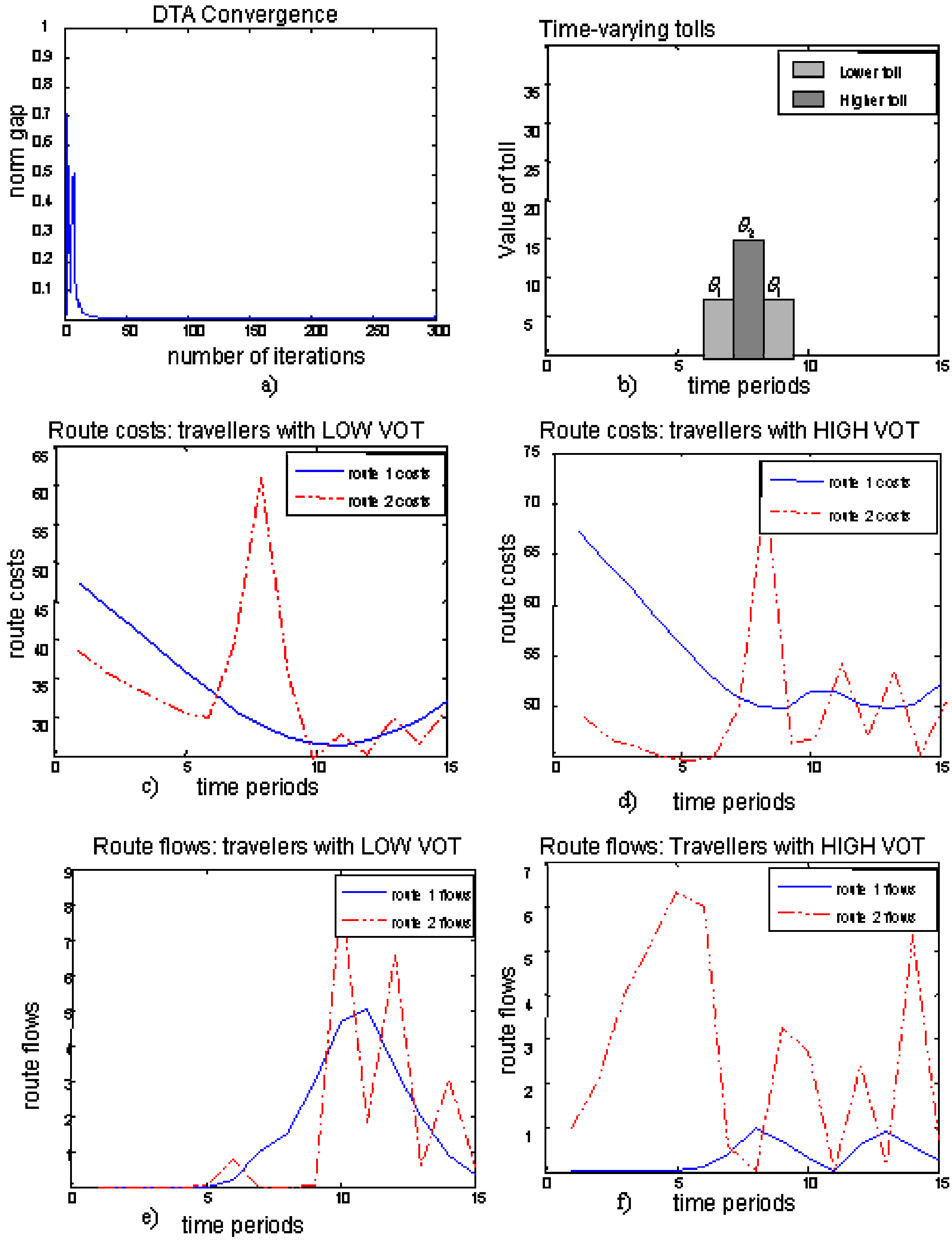

Fig. 4. Dynamic route cost and route flow pattern of both user classes

We can conclude that with increasing the toll some users are pushed from the congested route and time period to the low congested routes and time periods leading to the *decrease* of the total travel time until an optimal value is reached.

7. Conclusions

The dynamic road-pricing problem is formulated as a *bi-level programming* problem, with the road authority (on the upper level) setting the tolls and the travelers (on the lower level) responding by changing their travel decisions. The lower level is formulated and solved using a user-specific *variational inequality problem* (VIP) while the whole problem is formulated using the MPEC method. *Second-best tolling* scenarios are applied imposing tolls only to a subset of links on the network. *Joint route and departure time* choice is modeled because tolls will influence not only route but also departure time choice of travelers. Up to now, a simple procedure (*grid-search method*) is used to indicate optimal tolls that should be applied in order to reach the objective of the road authority. Further, it demonstrates that time-varying pricing may lead to *savings in the total travel time* compared to the no-toll situation.

There does not appear to be any simple solution of the problem time-varying pricing in dynamic traffic networks for real-size networks. The *non-linear* and *non-convex* objective makes it difficult to find an (global) optimal solution. However, it is possible to find the optimal toll values for small hypothetical networks, as we demonstrate in this paper.

We believe that for finding a global solution on larger networks a more appropriate global search algorithm (e.g. genetic algorithm) should be developed. For cases with overlapping routes better choice models can be used.

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