Welfare Economic Aspects of Land Use Planning

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Paper prepared for the 45th Congress of the

European Regional Science Association,

Vrije Universiteit Amsterdam,

23-27 August 2005.

Abstract

This paper develops a unifying framework for spatial and environmental economics, based on equilibrium considerations for population games. The main contribution of this paper consists of introducing a consistent concept for *spatial welfare*. Following the introduction of estimable locational sorting models for valuation methods in environmental economics, the relationship between the theoretical underpinnings of the hedonic pricing model and the bid rent concept in urban economics is re-examined. This is done along the definition of the ideal general equilibrium willingness to pay (GE-WTP) that is at the heart of most applications of locational sorting models in environmental economics. A GE-WTP should be able to account for the value

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of non-marginal changes in a spatially explicit distribution of local public goods. Commonly, such a GE-WTP is derived as a Hicksian WTP adjusted for endogenous prices. Endogenous prices are typically enforced by a market clearing condition, often a fixed supply, constraining the relocation of a population in response to the changes in local quality. This paper offers an alternative interpretation of a GE-WTP. It demonstrates how for a discrete choice formulation, a fixed supply generically results in a Nash equilibrium in a population game. Furthermore, it is shown that this Nash equilibrium corresponds exactly to a spatial equilibrium in urban economics. This observation allows for a novel spatially explicit approach to the evaluation of land policy options, combining current valuation practice with the optimization of land use. Finally it is shown, how the GE-WTP can be adjusted for developers' decisions, based on the analogy with urban economic models. It allows this spatial welfare measure to be extended with endogenous, instead of fixed, supply. This makes the concept also suitable for comparing the social welfare implications of entirely different land use patterns.

Keywords: Welfare Economics, Locational Sorting, Capitalization, Willingness to Pay

JEL Classification: Q58, R52, D61

1 Introduction

In this paper aims at clarification of the position of a government involved in land use planning along the line of welfare economics applied to the land market. The task envisioned for the government in this paper might be captured with the term *spatial social planning*. Like in neoclassical welfare economics the role for a social planner will be contrasted with allocation by markets. A market that conforms to the conditions stated as assumptions the Arrow-Debreu framework, is known to support an efficient allocation. Put in other words, decentralization supports the maximization of social welfare. The task of a benevolent social planner is an abstraction for an allocation that reflects central decision-making. Centrally planned allocation might yield a higher social welfare, or well-being, if the market would be distorted by external effects, or the case allocation would involve public goods.

When the allocation of land is concerned, a brief consideration already reveals that the distinction between the allocation mechanisms of market vs. planning is more complicated than for normal goods in an Arrow-Debreu market. To summarize a few aspects:

1. a parcel of land usually is unique, in terms of distinguishing characteristics,
2. in line with the *New Economic Geography* (Fujita et al., 1999; Fujita and Thisse, 2002), agglomerations are now commonly considered the result of the presence of positive external effects,
3. some characteristics of parcels of land are part of the set of production possibilities of the developer (e.g., the type of house build on it), other are public goods (e.g., the local air quality),
4. land is a scarce good as the total amount of space is limited,

5. land as a production factor could be considered a non-renewable resource, for which—in general—the impact on social welfare is part of an ongoing societal and academic debate (*sustainable development*).

These aspects show that from a public policy perspective spatial and environmental economic elements of land markets are difficult to separate. A selection is needed with enough information for a practical interpretation. The main problem addressed in this paper is limited to the allocation of land to developers by a spatial social planner. This can be interpreted as assigning locations that are allowed to be used for housing, where the housing is rented by tenants at rental prices established in a market. The task of a spatial social planner therefore consists of maximizing social welfare directly by means of *land use planning*. The planner is assumed to simultaneously optimize two impacts on social welfare:

1. the optimality of the consumption of space, when allocated by markets, as in the spatial economics tradition of von Thünen (1826),
2. the distribution of public goods, as local amenities, in the environmental economics tradition following Mäler (1974).

It will be shown that both impacts can be brought together under the one heading of the *bid rent*, allowing for a consistent but simple expression for social welfare. The main inspiration for the approach developed in this paper is supplied by a relatively new valuation method, which is based on so-called *locational sorting models* (Epple and Sieg, 1999; Bayer et al., 2002) and the concept of a *general equilibrium to pay* (GE-WTP) (Smith et al., 2004). The intention behind these models lies in the derivation of a measure for property valuation with endogenous prices, where changes in prices can be assumed to result from changes in demand. Changes in demand in turn reflect the reconsideration of location choices by individual consumers. And models of location choices, with prices based on a bid rent have a well established position as land use models in urban economics, following Alonso (1964). Therefore, locational sorting models by their construction combine property valuation and land use.

In urban economics, for a population of identical agents a spatial equilibrium is characterized by the same level of utility for each agent at every location. It is argued in this paper that the a market clearing condition adopted in the currently existing locational sorting models—assuming the supply of housing to be fixed[1]—, is essentially equivalent to the assumption of equalizing utility in a spatial equilibrium. The equivalence is shown to be valid indirectly for a model with discrete locations, by identifying a spatial equilibrium with a *mixed-strategy Nash equilibrium in bid rents* for a population game. With three ways of deriving the same overall level of utility, finally, this level allows for a comparison of entirely different land use patterns on the basis of their implication for the level of *social welfare*.

This paper is organized as follows. First, the relation between land use and social welfare will be addressed, as well as the possible role of locational sorting models in the assessment of this relation. In the third section, the use of the bid rent in spatial and environmental economics will be compared. In the third part of this section, it is shown how for a formulation with discrete locations, a spatial equilibrium might be interpreted as a Nash equilibrium in bid rents. The fourth section is devoted to valuation and capitalization. It demonstrates how the valuation concept of a *general equilibrium willingness to pay* (GE-WTP) in locational sorting models can not only be interpreted as an extension of hedonic pricing, but also as a measure for spatial welfare. Spatial welfare is addressed separately in section 5, focusing on the public policy aspects of land use planning. Finally, conclusions are stated in section 6.

[1]This amounts to the assumption that the population will *resort* itself over the existing housing stock.

2 Land Use and Social Welfare

In this paper the focus is on land, or rather *space*, as a consumption good. The individual consumer, or economic agent, can be thought of as a single household choosing a location. A utilitarian social welfare function is adopted, which is defined as the sum of individual utilities from consumption of space and a consumption bundle that will serve as numéraire. Following the urban economics tradition, it will further be assumed that all economic agents are identical. Therefore, the social welfare function might be interpreted as similar to the utility of a representative consumer, although agents differ in their location, so that it is more accurate to consider the social welfare function representing an *average* utility level. Issues concerning the relation between average and representative utility be addressed in more detail in section 3 and 4.

Adopting a utilitarian approach facilitates the integration of the contribution of market and non-market goods to social well-being, in line with common practice in environmental economics (Mäler, 1974). The non-market good in the framework developed in the following sections concerns one composite local public good, representing the amenity level or environmental quality in a broad sense. Unlike the consumption of space and and the composite good, the consumption of quality is not part of the optimization problem for the agent at a *given* location. Instead, the agent optimizes the benefits of quality by choosing the location that yields the highest level of utility from the combination of (local) market good, *space*, and the local public good, or *quality*. With hedonic pricing as a reference, it is further assumed that the value of the local quality is capitalized in the price of land[2]. In effect, the land market itself is simplified as a real (money) market for capitalized local public goods. The maximization of social welfare involves the selection of

[2]Taking this interpretation one step further would lead to the definition of *quality-adjusted prices* with the local price-quality combination as a single characteristic (see e.g. Feenstra (1995)).

the optimal set of parcels, while capitalization is left to the market. Although this perspective on land use planning is a highly simplified model of reality, it is believed that it illustrates the main features of the trade-off between central vs. decentral allocation of land.

3 Bid rent

One binding element for land prices in both environmental and spatial economics is the concept of a *bid rent*. In spatial economics this concept plays a central role in land use models. In environmental economics, where land use would be at first sight an obvious research topic, the literature until recently showed little coverage of the spatial dimension (Bockstael, 1996; van der Veen and Otter, 2001). Housing prices, on the other hand, have been used extensively already for a long time in environmental economics for valuation studies. In this respect, the perspective on land use in environmental economics has been dominated by the theoretic underpinning, also referring to the bid rent concept, by Rosen (1974).

In the urban land use models the bid rents reflect a spatial equilibrium. The spatial equilibrium itself is in urban economics defined as an equilibrium distribution of agents. It is Pareto optimal in the sense that no agent is able to improve her utility by moving to another location, without reducing the utility of other agents. For a population of identical agents, this definition of a spatial equilibrium implies that every agent—regardless her location—enjoys the same level of utility. And because bid rents often implicitly assume a mechanism in which agents establish their bids in strategic interaction on the basis of equalizing differences in utility, this mechanism suggests an analogy with interpersonal comparisons of utility similar to *game theory* and strategically price setting producers in oligopoly. In urban economics in the Alonso tradition consumers would seem to be involved in strategic price setting by means of bid rents. This analogy will be elaborated upon in section 3.3. First, the concept of the bid rent in urban economics and environmental economics will be summarized in section 3.1 and 3.2, respectively.

3.1 Land Prices in Spatial Equilibrium

The best-documented spatial economic tradition in land use models employing a bid rent dates back to von Thünen (1826) for agricultural land use. Von Thünen's method was extended by Alonso (1964), Muth (1968) and Mills (1972) for location choices of consumers and producers. These prototypes of spatial economic models have always been interpreted as part of the neoclassical economic tradition, because they conform to the conditions of competitive markets. The market equilibrium price for land is assumed to be identical to the maximum bid rent. It represents the price a consumer or producer is willing to pay as a rent, after travel or transport costs are subtracted from her income. Travel or transport costs are the only connotation with geography, as they are calculated based on the distance to an exogenously given Central Business District (CBD), or market place. Fujita and Thisse (2002, p.79) derive the Alonso model starting with the following maximization problem:

$$\max_{z,s} u(z,s) \qquad s.t. \qquad z + Rs = y - T(r). \tag{1}$$

Here, y is income and R is the rental price per quantity, s, of *space*. The transportation costs T are a function of the distance, r, to the CBD. In a more general interpretation, r can be thought of as merely *location*. The bid rent is defined as:

$$R(r,u^*) = \max_{z,s} \left\{ \frac{y - T(r) - z}{s} \qquad s.t \qquad u(z,s) = u^* \right\}. \tag{2}$$

This definition shows the relation between a bid rent function and a expenditure function which would be the dual of (1). It expresses the *willingness to pay* per quantity of space in order to attain a utility level of u^*. Because u is strictly increasing in z, z can defined as a function (by inversion) of s and u^*. The bid rent is then only a function of s:

$$R(r, u^*) = \max_s \frac{y - T(r) - z(s; u^*)}{s}. \tag{3}$$

By definition of the bid rent the following equality holds (Fujita and Thisse, 2002, p. 80)

$$u^* \equiv v[R(r, u^*), y - T(r)]. \tag{4}$$

This is due to the maximization of space, s, and the definition of the indirect utility. Assuming a homogeneous population, supported by the same problem (1) for every individual agent, a spatial equilibrium is *defined* by the same level of utility for every agent, u^*, regardless her location, r. This is consistent with the notion of Pareto efficiency, stated above. Market equilibrium prices in spatial equilibrium are therefore only a function of the location

$$R^*(r) = R(Y - T(r), u^*). \tag{5}$$

For a given population size N, social welfare—which could here be identified with $U^* \equiv Nu^*$—can be interpreted as a function of the *city size*. The city size is measured in the maximum distance to the CBD, $r_{\max}$, where land is still occupied. This can be expressed as (Fujita and Thisse (2002, p. 82))

$$R^*(r_{\max}) = R_A. \tag{6}$$

This results from the general condition $R^*(r) \geq R_A$, meaning that for the land owner the revenues from residential use need at least to be equal to rent earned from alternative (e.g. agricultural) use. In urban economics it is often assumed that the land owners are different from the members of the population ('absentee landlords').

3.2 Bid Rent and Value

In the hedonic pricing literature following Rosen (1974), equilibrium prices are commonly interpreted as bid rents. Analogous to a willingness to pay, a bid rent can be defined on the basis of the expenditure needed to attain (or maintain) a given level of utility. Rosen proposed a perfectly competitive market for the characteristics of sites and houses, making use of the bid rent concept, thereby referring to Alonso (Rosen, 1974, p. 38). He defines a bid rent as a willingness to pay, according to:

$$u(y - Rs; q) = \widetilde{u}. \tag{7}$$

The parameter q is an exogenous quality index or public good[3] (amenity). In spatial economics issues concerning public goods are seldom identified as such. Occasionally the existence of a CBD is justified by the assumption that a composite public good is supplied there. As suggested in Scotchmer (1986, p. 68, footnote 8), in this paper the distance to the CBD might be considered an amenity in itself. If a fully consistent notion were to adopted, quality then would depend on the location, r, because of which it could be expressed as $q(r)$. However, because q has a local impact on utility, it always represents the *local quality*. Therefore, in

[3]Throughout this paper the quality index, q, is assumed to be a scalar. This done mainly to stress the similarity with the travel cost, T, in spatial economic models.

the following equations q takes a place similar to r in the urban economic model. This is more in line with Rosen (1974, p. 34) referring to 'locational decisions in characteristics space'.

This section follows Scotchmer (1986), rather than Rosen (1974). Scotchmer (1986) uses a reduced expenditure function, consisting of all expenditures except those for housing, in the derivation of the bid rent. Her derivation can thereby easily be compared with Fujita and Thisse (2002) as referred to in eq. (2). The maximization problem then reads

$$\max_{z,s} u\left(z, s; q\right) \qquad s.t. \qquad z + Rs = u. \tag{8}$$

And the bid rent is defined as

$$R\left(q, u\right) = \max_{z,s} \left\{ \frac{y - z}{s} \qquad s.t \qquad u\left(z, s; q\right) = \widetilde{u} \right\}. \tag{9}$$

Again, with z strictly increasing in z, substitution by $z(s; q, \widetilde{u})$ allows the bid rent to be written as a function of s only:

$$R\left(q, \widetilde{u}\right) = \max_{s} \frac{y - z\left(s; q, \widetilde{u}\right)}{s}. \tag{10}$$

For reference purposes, the corresponding market equilibrium will be defined by

$$\widetilde{u} \equiv v\left[\widetilde{R}\left(q\right), y; q\right]. \tag{11}$$

It is to be noted that the qualitative relation between travel cost and quality is inverse. A short distance to the CBD would have a similar impact on utility as a high quality level. The similarities between (11) and (4) will be exploited in the next subsection.

3.3 Sorting Equilibrium as Nash Equilibrium

In section 3.1 it was argued that in urban economics, a spatial equilibrium is characterized by the assumption that identical agents enjoy the same level of utility. In this section it is shown how the equilibrium level of utility can be derived for *discrete locations choices*, drawing a parallel with *population games.* Discrete locations facilitate drawing a parallel between choosing strategies and choosing locations. They also conform to the characteristics of a discrete choice framework applied in the econometric implementation of so-called *locational sorting*, to be addressed in the section 4.2. More specifically, a game theoretic analysis of a discrete choice model corresponds to the framework Bayer et al. (2002) and especially Timmins (2003), based on *discrete choice with social interactions* (Brock and Durlauf, 2003). And, finally, a setting with discrete locations facilitates the translation from travel costs to quality, which allows for the direct derivation of a *spatial* equilibrium on the bases of differences in amenity level.

3.3.1 Demand and Supply

Assuming a population of N agents, sorting over M locations, with j as the index for a location, the equilibrium price in spatial equilibrium at location j is defined in

$$u_j^* = v\left[R\left(q_j, u_j^*\right), y; q_j\right]. \tag{12}$$

On the other hand, individual demand[4] at equilibrium prices, in a spatial equilibrium is given by

$$s_j^* = s\left(q_j, u_j^*\right). \tag{13}$$

The number of agents at location j is given by n_j. Total demand for location j in spatial equilibrium therefore equals $S_j^* = n_j^* s_j^*$. Continuing the establishment of a relation with the urban economics model in the Alonso tradition, a *fixed supply* would close the model. Fixed supply would in this case correspond to *scarcity* in the sense of a limited total amount of space per location, plus the condition that all space is used (no vacancy[5]). For simplicity it is assumed that the supply of land at every location j equals the same amount[6]:

$$n_j^* s_j^* = A. \tag{14}$$

Given the population size N, equation (14) can also be written in terms of fractions ($x_j \equiv n_j/N$):

$$N x_j^* s_j^* = A. \tag{15}$$

Referring to a population game, these fractions correspond to probabilities in a mixed strategy equilibrium. The supply constraint (14) effectively relates individual local demand s_j^*, through inversion of the demand function to price R_j^*. It also

[4]In order to avoid confusion, demand here is denoted by the variable s for the individually consumed amount *space*, as before.

[5]Fujita and Thisse (2002, p. 82) define a similar condition as $ns = 1$, or $ns = 2\pi r$, depending on whether the city is depicted on a line or a circular plane.

[6]This is not necessary, as A_j could also be defined as location dependent.

resolves the issue of scarcity, because the individual (Marshallian) demand s_j^* acts like for a normal good, in the sense that

$$s_j \sim \frac{1}{R_j}, \tag{16}$$

but for a 'local total demand' in terms of the number of people per location, normalized to a fraction, eq. (15) implies $x_j^* \sim R_j^*$. This accounts for the notion of relative *scarcity*, concerning space. Finally, in terms of evolutionary game theory, the equilibrium distribution could be derived from—a *ad hoc* use of —the *replicator dynamics* (Weibull, 1995), similar to its use in the core-periphery model of the *New Economic Geography* (Fujita et al., 1999):

$$\frac{dx_j}{dt} = x_j\left(v_j - \overline{u}\right) = 0. \tag{17}$$

Here, $\overline{u}$, would be the average (expected) fitness or payoff. If $x_j^* > 0$, it follows that $u_j^* = \overline{u}^*$, which is consistent with the assumption that of the level of utility being equal across all locations. Following Alonso's notion of a spatial equilibrium, the level of utility is assumed to be equal across all locations: $u_j^* = \overline{u}^* = u^* \quad \forall j$. The level of utility in a Nash equilibrium in a discrete locational sorting model is therefore shown to be consistent with the equilibrium utility level in urban economics. The main difference with urban economics is that the same equilibrium level of utility in the framework developed here has become an endogenous (*emergent*) characteristic of the model.

3.3.2 Cobb-Douglas Case

By means of illustration and for future use in section 5, the individual will be specified as a Cobb-Douglas function:

$$u_j = s_j^{\beta} z_j^{1-\beta} q_j^{\gamma}. \tag{18}$$

As quality level q_j is beyond the local choice opportunities for the individual[7], individual demand is given by

$$s_j = \frac{\beta y}{R_j}. \tag{19}$$

Quality only reappears in the indirect utility:

$$v_j = \beta^{\beta}(1-\beta)^{(1-\beta)} \frac{\beta Y}{R_j^{\beta}} q_j^{\gamma}. \tag{20}$$

Equilibrium utility—as an *average* cf. the *replicator dynamics* (17)—reads as

$$v_j = \overline{u} \equiv \sum_k x_k v_k. \tag{21}$$

The relation between the price and population density can be established on the basis of fixed demand per location

$$A = N x_j \frac{\beta y}{R_j}. \tag{22}$$

As is readily seen, this relation constitutes the basis of the indicated *scarcity rent* by means of $R_j \sim x_j$. Even more specifically, in this example it is a linear relation:

$$R_j = \frac{N \beta y}{A} x_j = \alpha x_j, \tag{23}$$

[7] The individual chooses a location in the *second stage* of her optimization problem.

which allows for a direct substitution of the rent R_j in the replicator dynamics, reading

$$\frac{dR_j}{dt} = R_j (v_j - \overline{u}) = 0. \tag{24}$$

Although the replicator dynamics is used here primarily as a heuristic tool for finding an equilibrium condition, the intuitive interpretation is appealing. It would allow for a spatial equilibrium to be regarded the equilibrium of some tatônnement process, where bid rents eliminate differences in local utility levels. This corresponds to the more narrative justification of hedonic pricing, where the bidding process continues up to the point where consumers become indifferent between choices.

4 Capitalization and Valuation

In 3.3 is was shown that, under the condition of a capacity constraint for each location, a sorting model allows for drawing parallels between a spatial equilibrium and a market equilibrium when valuing amenities, based on a average utility level, $\overline{u}$, that determines the same level of utility at every location

$$u^* = \widetilde{u} = \overline{u}. \tag{25}$$

However, up to this point, there is no actual reason for the identification of a market equilibrium in a valuation method with a spatial equilibrium. The observation important for the analysis in this paper is that if only a hedonic price function for *marginal changes* is derived, the level of utility, $\widetilde{u}$ is implicitly assumed as given (see also Scotchmer (1986, p. 64)). In 4.1 it will be shown that this type of valuing marginal changes imply that the market equilibrium remains fixed. It is exactly for this reason that sorting models have been developed with applications in valuation issues. The underlying locational sorting *mechanism* is primarily used for deriving *endogenous prices*.

4.1 Hedonic Pricing

In a market equilibrium resulting from utility maximization, Marshallian demand will equal Hicksian demand:

$$\widetilde{s}(q_j, \widetilde{u}) \equiv s^M [R(q_j, \widetilde{u}), y] \equiv s^H [R(q_j, \widetilde{u}), \widetilde{u}]. \tag{26}$$

Resulting from the first order conditions, the bid rent in equilibrium is essentially equal to the *price*, or the marginal rate of substitution:

$$\widetilde{R}_j = -\frac{\partial \widetilde{u}/\partial s}{\partial \widetilde{u}/\partial z} = -\frac{\partial \widetilde{z}_j}{\partial s}. \tag{27}$$

This observation leads to the following justification of using hedonic prices for deriving a willingness to pay (WTP) for a marginal change in amenity q_j, embodied in $WTP \approx \Delta \widetilde{z}_j$, with $\widetilde{s}_j$ (equilibrium amount of space) kept fixed:

$$\frac{\partial \widetilde{R}_j}{\partial q_j} = -\frac{1}{\widetilde{s}_j} \frac{\partial \widetilde{z}_j}{\partial q_j}. \tag{28}$$

The interpretation of (28) is facilitated by going back to (7) with equilibrium space and price. The willingness to pay (WTP) for a amenity improvement $\Delta q = \hat{q} - q$ for tenants, assuming *no resorting* (i.e., sufficiently high moving costs), is defined by (Haab and McConnell, 2002, p. 250)

$$u\left(y - \widetilde{R}_j \widetilde{s}_j - WTP; \hat{q}_j\right) = u\left(y - \widetilde{R}_j \widetilde{s}_j; q_j\right) = \widetilde{u}_j. \tag{29}$$

Given (11), (29) might also be expressed in terms of indirect utility, thereby conforming to the definition of a regular, Hicksian WTP:

$$v\left(y - WTP; \hat{q}\right) = y\left(Y; q\right) = \widetilde{u}. \tag{30}$$

For a marginal change, (28) is always valid. For a non-marginal change however, (29) assumes that (local) equilibrium rent $\widetilde{R}_j$ and equilibrium demand for space $\widetilde{s}_j$ remain unchanged.

In terms of a rental market for space, this assumption translates to the *condition* that the individuals in the population will not move to other locations in response to changes in q. Hence the interpretation that moving costs are sufficiently high to prevent the population from *resorting*. This has inspired the search for a WTP based on a endogenous market equilibrium, or *general equilibrium willingness to pay* (GE-WTP). The following subsection is based on relating $\widetilde{u}$ to u^* in light of a GE-WTP.

4.2 General Equilibrium Willingness To Pay

Hedonic pricing is probably the most popular revealed preference method for valuing non-market goods. As was argued before, the impact of changes in quality on welfare is usually confined to measuring marginal price effects. Therefore, the changes have to be assumed to be marginal too, effectively excluding the market response to relocation. In order to circumvent this limitation, recently frameworks based on a locational equilibrium have been introduced. A locational equilibrium can be thought of as a mapping of a spatial equilibrium on price equilibrium in the land (or housing) market, where demand is derived from individual location choices (Epple and Sieg, 1999; Bayer et al., 2002). Locational sorting models contribute to the valuation literature by defining a general equilibrium willingness to pay (GE-WTP) per individual by (Smith et al. (2004)):

$$v\left(\widetilde{R}_l, y - WTP_{GE}; \hat{q}_l\right) = v\left(\widetilde{R}_j, y; q_j\right). \tag{31}$$

Here, $\widetilde{R}_l$ denotes the equilibrium price corresponding to a change from vector (of all locations) $\mathbf{q}$ to $\hat{\mathbf{q}}$, where the index l signals that in the new equilibrium the location choice for the individual might have changed. This GE-WTP is to be contrasted with the general definition of a Hicksian (partial equilibrium, or short-run) WTP for changes in the quality level of a public good only (keeping prices, R^*, fixed)

$$v\left(\widetilde{R}_j, y - WTP_{PE}; \hat{q}_j\right) = v\left(\widetilde{R}_j, y; q_j\right). \tag{32}$$

While (31) intuitively makes sense, the problem is to find the new location choices and the new equilibrium prices. The value of WTP_{GE} critically depends on the definition of the new market equilibrium. In short, a mechanism needs to be designed that derives consistent values for $\widetilde{R}_l$. This mechanism is then applied to calculate the equilibrium values of a counterfactual equilibrium (i.e., the equilibrium with hypothetical changes in the values of the quality levels). In the literature on locational sorting, the total supply of housing is often taken to be fixed, assuming that the population would resort over the existing stock of houses. The specification of both demand and supply introduces endogenous prices, and thereby the definition of the 'general equilibrium,' in the model.

The assumption of a fixed supply resembles the supply constraint (14) in the discrete location choice model of section 3.3. If consumers are assumed identical, the level of utility in a new market equilibrium, after changing the state from $\mathbf{q}$ to

$\hat{\mathbf{q}}$, is given by the following indirect utility:

$$v_j \left[\widetilde{R}_j \left(\hat{\mathbf{q}} \right), y; \hat{\mathbf{q}} \right] = \hat{\widetilde{u}}_j. \tag{33}$$

In section 3.3 it was shown that the supply constraint introduced the Nash equilibrium, while the Nash equilibrium implies the same level of utility at every location for identical agents. Hence, the market equilibrium bares all features of spatial equilibrium $\widetilde{u} = u^*$.

When the market equilibrium of a locational sorting model is identified with a spatial equilibrium, it can be conjectured that a general equilibrium willingness to pay is likely to value the difference in utility of two spatial equilibria. In general, it is to be expected that

$$\hat{u}^* \neq u^*. \tag{34}$$

Stated otherwise, the counterfactual equilibrium is probably characterized by a different level of equilibrium utility. Against this background, the GE-WTP is mainly restoring the old utility level:

$$v \left[R^* \left(\hat{\mathbf{q}} \right), y - WTP_{GE}; \hat{\mathbf{q}} \right] = u^*. \tag{35}$$

Both connotations of the market equilibrium shed different lights on a GE-WTP. The relation with a hedonic bid rent facilitates an interpretation in terms of adjusting, or compensating, a pure Hicksian willingness to pay for the capitalization of quality changes in the rent. From the perspective of a spatial equilibrium, the GE-WTP would be monetary measure for comparing the welfare level of two different simultaneous distributions of agents and amenities. Therefore, a GE-WTP could also be read directly as a monetary measure for a change in *spatial welfare*.

5 Welfare

Now the market level of utility and the utility level in spatial equilibrium have been identified as being the same under the condition of a fixed supply of space per location, the welfare economic analysis can be completed. In the case of a Cobb-Douglas specification for the individual utility function, substitution of (23) in (20) and (20) in (21) results, after some manipulation, in an analytical solution for the *replicator dynamics* (17):

$$x_j^* = \frac{q_j^{\frac{\gamma}{\beta}}}{\sum_k q_k^{\frac{\gamma}{\beta}}}. \tag{36}$$

Population density is a increasing function of quality and because of $x_j \sim \frac{1}{s_j}$, due to (15), space per person is a decreasing function in quality.

Elimination of the equilibrium rent, R_j^*, which can now be derived of the substitution of (36) in (22), in the social welfare function, $U^* \equiv Nu^*$, results in

$$U^* = A^\beta \left[(1-\beta)Ny\right]^{1-\beta} \left(\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta}} \right)^\beta. \tag{37}$$

From (37), the following implications can be derived

1. The effect of capitalization on social welfare is captured still in (37), while the rent price for space is eliminated,

2. As a result, the level of social welfare depends on the distribution of quality levels, q_j, over the locations.

3. The level of social welfare also depends on the *number*, M,—or more specifically—the *set* of locations.

If the set of locations would be selected by the market, it can be assumed that the land owners, or *developers*, will to to maximize their profit. Because of the the supply constraint for each location, the developer will receive a positive profit.

$$\pi_j = A\left(R_j^* - R_A\right). \tag{38}$$

Here, R_A can—more generally—be thought of the costs per quantity of space. The rent R_j^* can be considered a *pure economic rent*. Due to the elimination of the rent in the social welfare function, this mark-up is already accounted for. The only impact on social welfare developers could have concerns the set of locations to be developed. Assuming *free entry* for the developers, every location which yields a positive profit would be developed. Profit maximization on the side of the developers, therefore would imply a maximization of the number of locations until the quality of the last location to be developed is just enough for a non-negative rent

$$R_j^* \geq R_A. \tag{39}$$

This is the spatial equivalent of the welfare maximization in a free land market. The result is similar to that for regular consumption goods in the traditional Arrow-Debreu framework (see Fujita and Thisse (2002, p. 84–85)). Therefore the traditional policy directive concerning liberalization and free markets based on neoclassical Welfare Economics could be extended to land markets for these models.

6 Conclusions

Under the conditions stated in this paper, a market allocation of land, while accounting for the capitalization of the value of amenties in the rent, will yield the highest level of social welfare. Market allocation of land is characterized by the optimal set of locations—differentiated by local quality—where only the last developed location will yield a profit close, or equal to zero, for the land owner. This concept thereby reunites the Thunian with the Ricardian differential rent concept.

A spatial social planner involved in land use planning, in principle could assign the set of locations that yields the highest level of social well-being under market conditions, to the developer for residential land use. This, however, implies that the (absentee) land owners are allowed to maximize positive profits based on relative scarcity. The implication might seem counterintuitive at first sight, since the relation between welfare maximization and marginal cost pricing is lost in a spatial context (see also Fujita and Thisse (2002, p. 159) for traditional urban economics models).

Any land use pattern other than the decentrally optimized set would result in a lower level of social well-being. However, in a broader perspective, undeveloped land could also be assigned an asset-like quality for society in terms of a *resource*. The amount—or stock—of undeveloped land might be interpreted as a nearly non-renewable and exhaustible resource, especially in densily populated regions. The framework developed here could easily be extended for the *depletion* of this stock, facilitating the perspective on the impact of protected areas, based on non-use value, social well-being. This combination of resource management and land use, would suit the use of a similar social welfare function as the basis for the definition of *sustainable development* (Dasgupta and Mäler, 2000). Such an approach is considered a task for future research.

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