The general interregional quantity model

Bjarne Madsen
akf, institute of local government studies – Denmark
Nyropsgade 37,
DK-Copenhagen K
Tel.: +45 3311 0300
Fax: +45 3315 2875
E-mail: bm@akf.dk

Chris Jensen-Butler
Department of Economics,
University of St. Andrews,
St. Andrews, Fife KY 16 9 AL,
Scotland, UK
Tel.: +44 1334 462442
Fax: +44 1334 462444
E-mail: cnj@st-and.ac.uk

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Abstract

Taking the Leontief and Miyazawa formulations of the interregional economic quantity model as the point of departure the general interregional quantity model is developed. This model incorporates a number of conceptual and theoretical changes, which have become necessary as economies become more diverse and differentiated. In particular, there is a need to integrate essentially sub-regional and local/urban activities covering such areas as commuting, shopping, tourism and trade into a general interregional modelling framework. The theoretical changes examined include a set of new geographical concepts and in the context of an interregional SAM the development of the two-by-two-by-two approach, involving two sets of actors (production units and institutional units), two types of markets (commodities and factors) and two locations (origin and destination). The equations of the general interregional quantity model are presented together with the solution of the model. Comparisons are made with the Danish interregional CGE-model LINE and a typology of regions is proposed using the general model as a conceptual foundation.

Keywords: Interregional quantity models, Interregional SAM, Sub-regional models, Urban models, Regional typology

1. Introduktion

As society has become more differentiated and diverse it has become necessary to reflect this differentiation and diversity in regional and urban economic models designed to explain and forecast economic and social development. This greater diversity has meant that our understanding of regional growth and development increasingly is based on sub-regional spatial units and their interactions, involving such concepts as urban, semi-rural and rural areas as well as labour market catchment areas and shopping hinterlands. This has created new challenges for regional and interregional modelling, where traditional approaches are less adequate for dealing with the new diversity. These new conditions have, for example, highlighted the necessity of incorporating such essentially sub-regional phenomena as commuting and shopping as well as trade in commodities at sub-regional and local levels into a more general regional and interregional modelling framework.

In this paper, the most important and necessary changes to established regional and interregional modelling theory and practice arising out of the new diversity are examined. Taking as a point of departure the institutional industry by industry regional and interregional input-output model a number of developments of this framework are first examined. We conclude that a further development of theoretical framework is necessary in order to build models capable of the new realities. The extension of the theoretical framework involves a number of components. First, four geographical concepts are introduced to regional and interregional models. Second, these concepts are integrated with established social accounting matrix concepts. This leads to development of a two–by–two–by–two theoretical framework involving actors (production units and institutional units), two markets (commodities and factors) and two locations (origin and destination)

2. Geographical concepts in local models – conceptual lock-ins

Sub-regional components in sub-regional macro-economic model necessitate the introduction of four geographical concepts: place of production, place of residence, place of commodity market and place of factor market. Mainstream single regional macroeconomic models are focused on place of production. There is limited or no treatment of place of residence or the market places being focussed on place of production. They also build on a division of productive activities by industry with limited differentiation between types of production factor, institutions and commodities. The traditional input-output approach has these features and the conceptual inertia of this approach constitutes a lock-in in terms of model developments. The approach has also had practical consequences for the way in which data has been constructed and collected, contributing significantly to inertia.

In the conventional interregional quantity based input-output model, place of production and place of commodity market are included. In this sense this type of model can be regarded as a limited spatial model, as transfers between place of production and place of residence (commuting) and from place of residence to place of commodity market (shopping, tourism etc.) are not included.

Demo-economic models (Batey & Madden (1981), Madden & Batey (1983), Madden & Trigg (1990), Oosterhaven & Folmer (1985), Stelder & Oosterhaven (1995)) represent a step forward, as they normally include a distinction between place of production and place of residence.

Land-use and transportation models employ some relevant spatial concepts, but they only reflect selected elements in the local economy and as such are partial models. For example commuting models (Wang (2001), Renkow&Hoover (2001), Casado-Diaz (2000), Gitelsen & Thorsen (2002) Artis et al (2000)) data and models use the concept of place of production (work) and place of residence, but these models do not include production and interregional trade. In shopping models (Lakshmanan & Hansen 1965), Guy 1996, Munroe 2001, Baker (2000), Cadwallader (1995)) the concepts of place of residence and place of commodity markets are used but they do not reflect the interregional structure in the determination of production and demand.

In single regional or non spatial CGE models no spatial dimensions are included (Shoven & Whalley (1992),). In the sub-model for cost and prices no corrections for the transformation of producer prices from place of production to place of commodity market is normally not included. In interregional CGE models (van den Bergh et al, 1996, Brocker, 1995, 1998, 2002, Harrigan et al 1991, McGregor et al 1998, Haddad et al 2002 spatial interaction is usually restricted to trade flows at a very aggregate level (typically two commodities). Commuting, shopping and tourist interaction are usually excluded.

3. Local and urban models based upon interregional input-output and SAM approaches

As a response to the issues described above, a new generation of local economic models is emerging, based upon further development of the traditional input-output approach and introducing new spatial concepts together with a disaggregated SAM (Round, 1995, Kilkenny & Rose 1995, Hewings & Madden 1995a, 1995b.

Jun(1999) formulated an integrated metropolitan model, which captures intersectoral and interspatial relations as well as impacts on transport networks within a metropolitan area, including three components: multizonal input-output linkages, land use forecast models and transport demand forecast models.

Hewings et.al. (2001) set up a model for Chicago with 4 regions. Following Miyazawa's (1975) modelling approach economic interaction is first divided into indirect effects, driven by intermediate consumption and induced effects driven by private consumption. However in the basic version, the model only includes a conventional interregional input-output model. There is no explicitly modelling of the effect of commuting which necessitates the introduction of a spatial division between place of production and place of residence. Likewise the introduction of shopping would necessitate economic link between place of residence and place of commodity market.

In the interregional CGE/SAM tradition Madsen & Jensen-Butler (2004) have constructed a local economic model for Denmark. The model consists of an interregional input-output model, includes and integrates models of trade, commuting, shopping and tourism. At the core of the model there is an interregional model for costs and prices and a link between the interregional quantity model and the interregional price model.

3.1 Extensions of the interregional quantity model

In the following a general interregional quantity model based upon the original interregional Leontief quantity model and developed using the two-by-two-by-two principle, described above, is presented.

3.1.1 The Leontief single region quantity model

The point of departure for our model construction is the Leontief quantity model, where gross output is determined by demand:

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{f} \quad \text{.................................(1)}$$

where
$\mathbf{x}$: gross output by sector
$\mathbf{A}$: intermediate consumption by sector of origin as share of gross output, by purchasing sector
$\mathbf{f}$: final demand, by sector

In this model it is assumed that gross output in a sector (x) is determined by intermediate consumption ($\mathbf{Ax}$) and final demand ($\mathbf{f}$) by sector. Using the equilibrium condition for the commodity market the analytical solution to the Leontief model is:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} \quad \text{..................................(2}\mathbf{a}\text{)}$$
$$= (\mathbf{I} + \mathbf{A}^1 + \mathbf{A}^2 + \mathbf{A}^3 \text{......})\mathbf{f} \quad \text{............(2}\mathbf{b}\text{)}$$

Equation 2a shows that the solution of the Leonteif model can also be found using gross output by sector is equal to the product of the Leontief inverse and the vector of final demand. Equation 2b shows that the solution of the Leontief quantity model can be found sequentially using the power series approximation of the spill over and feedback effects between and within sectors. Each term expresses the effects of an extra round of intermediate consumption.
This model is single region as economic activities take place in one region without interaction with other regions. The model is based upon sectors (industries) and does not include explicit transformations from sectors to commodities (output) or transformations from commodities to sectors (input). The Leontief quantity model is therefore a reduced form model with underlying transformations, which appear when the model is applied to local and urban economies.

3.1.2 The interregional Leontief quantity model

Setting up an interregional quantity model involves extensions of the reduced form Leontief quantity model. The interregional quantity model includes intra- and interregional trade, which in spatial terms leads to a distinction between place of production and place of commodity market. The interregional quantity model establishes a link between place of commodity market, where intermediate consumption or final demand originates and place of production, where production takes place.

The Isard model (Isard 1951) is often described as the ideal interregional quantity model, which establishes a direct link between the intermediate consumption by purchasing sector in region S and gross output in the producing sector in region P. In the Isard model the A-matrix is simply extended, so that the same sector in two regions is defined as two different sectors. This in turn gives the same solution as in equation 2.

The Chenery-Moses model (Chenery 1953, Moses 1955) uses a pool approach, where intermediate consumption and final demand by region and sector are added together. Aggregate demand by sector enters into a demand pool. In simple models demand is met by production from other regions, which supply the pool. Both supply and demand are by sector. In more complex models a trade model establishes a link between economic activity at place of commodity market and at place of production. In some of the models it is simply assumed that supply is distributed amongst the supplying regions in proportion to the region's share of supply to the pool. In other approaches it is assumed that transport cost is an impediment to trade. Interregional trade can be modelled using a gravity model or an entropy maximising model.

However, both types of model involve the problem as they establish intra- and interregional trade in sectors using a methodology which relies on the assumption that the make matrix for the region of demand can be used as the make matrix for the region of production. It seems more straight forward to use a commodity approach in trade assuming that demand by place of commodity market is transformed into commodity demand, then being transformed from place of commodity market to place of production still in commodity form and finally at the place of production being transformed from production in commodities to production in sectors (Greenstreet 1987).

Establishing a model with a spatial market for commodities also leads to inclusion of shopping for commodities for intermediate consumption. Assuming that intermediate consumption is determined at the place of production and that commodities for intermediate consumption are purchased at the place of the commodity market, the interregional quantity model can instead be written:

$$\mathbf{x} = \mathbf{D}\mathbf{T}\mathbf{S}_{\mathbf{IC}}\mathbf{B}_{\mathbf{IC}}\mathbf{x} + \mathbf{D}\mathbf{T}\mathbf{f} \quad \text{..............................(3}\mathbf{a}\text{)}$$

$$= \mathbf{D}\mathbf{T}(\mathbf{S}_{\mathbf{IC}}\mathbf{B}_{\mathbf{IC}}\mathbf{x} + \mathbf{f}) \quad \text{................................(3}\mathbf{b}\text{)}$$

where

$\mathbf{x}$: gross output by sector and by place of production

$\mathbf{D}$: the Make matrix or gross output by commodity as share of gross output, by sector by place of production

$\mathbf{T}$: the intra- and interregional trade matrix or sales originating from place of production as share of total sales, by place of commodity market by commodity

$\mathbf{S}_{\mathbf{IC}}$: the shopping matrix for intermediate consumption or intermediate consumption at the place of the commodity market as share of total intermediate consumption, by place of production by commodity

$\mathbf{B}_{\mathbf{IC}}$: the Use matrix or intermediate consumption by commodity as share of gross output by sector and by place of production

$\mathbf{f}$: final demand by commodity and by the place of the commodity market

The model now follows a real circle, which corresponds to reading from right to left in equation (3a). Starting with production at place of production ($\mathbf{x}$) in the first element of equation (3a) intermediate consumption by commodity is calculated employing a Use matrix ($\mathbf{B}_{\mathbf{IC}}$). Moving again to the left, commodities for intermediate consumption are purchased at the place of the commodity market, which involves transport from place of production to place of commodity market ($\mathbf{S}_{\mathbf{IC}}$, in effect a shopping model for intermediate consumption commodities). Moving again to the left the demand for commodities for intermediate consumption is transformed back to place of production using an intra- and interregional trade model ($\mathbf{T}$). Finally, gross output by sector and by place of production is calculated using a Make matrix ($\mathbf{D}$). The second element in equation (3a) transforms final demand from place of commodity market to place of production using an intra- and interregional trade model ($\mathbf{T}$) and further from production in commodities to production by sector using a make matrix ($\mathbf{D}$). Using the principle that supply equals demand the following analytical solution to the interregional Leontief quantity model can be derived:

$$\mathbf{x} = (\mathbf{I} - \mathbf{DTS_{IC}B_{IC}})^{-1}\mathbf{DTf} \ldots\ldots(4\mathbf{a})$$
$$= (\mathbf{I} + (\mathbf{DTS_{IC}B_{IC}})^{1} + (\mathbf{DTS_{IC}B_{IC}})^{2} + (\mathbf{DTS_{IC}B_{IC}})^{3} + \ldots\ldots)\mathbf{DTf} \ldots\ldots(4\mathbf{b})$$

The power series expansion of the model (equation 4b) shows that the interregional quantity model can be solved numerically in a sequential procedure starting with exogenous final demand and then continuing with the first round effects, the second round effects etc.

This circle represents economic flows in the real economy, and solves a number of conceptual problems in the conventional interregional input-output model. First, the spatial division into place of production and place of commodity market is followed by a division of the SAM-axis including both sectors and commodities, represented by the Use and the Make matrices. Second, the Make matrix now is defined for the region where production takes place. Finally, the introduction of a shopping matrix for intermediate consumption commodities reflects the fact that these commodities often are purchased at the location of the wholesaler, this being the place of the commodity market.

3.2 The Miyazawa single region quantity model

The next step in derivation of the general local/urban model is inclusion of interaction between production and institutional demand. This involves application of the Miyazawa quantity model (Miyazawa (1975)), which includes a transformation of income by sectors into income by institutions. In matrix notation the model is

$$\mathbf{x} = \mathbf{Ax} + \mathbf{CVx} + \mathbf{f} \ldots\ldots(5)$$

where
$\mathbf{C}$: Private consumption by sector as share of Gross Value Added (GVA), by type of institution
$\mathbf{V}$: GVA by type of institution as share of gross output, by sector

Following Miyazawa (1975) this quantity model including the indirect effects (the A-matrix) and the induced effects (the V- and the C-matrix) can be solved in 3 ways:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A} - \mathbf{CV})^{-1}\mathbf{f} \ldots\ldots(6\mathbf{a})$$
$$= \mathbf{B}(\mathbf{I} - \mathbf{CVB})^{-1}\mathbf{f} \ldots\ldots(6\mathbf{b})$$
$$= \mathbf{B}(\mathbf{I} + \mathbf{CKVB})\mathbf{f} \ldots\ldots(6\mathbf{c})$$

where

$$\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$$
$$\mathbf{K} = (\mathbf{I} - \mathbf{L})^{-1}$$

where

$$\mathbf{L} = \mathbf{VBC}$$

$$\mathbf{x} = (\mathbf{I} + (\mathbf{A} + \mathbf{CV})^{1} + (\mathbf{A} + \mathbf{CV})^{2} + (\mathbf{A} + \mathbf{CV})^{3} + ..)\mathbf{f} \ldots\ldots(6\mathbf{d})$$

The first solution (equation 6a) is simply the direct solution, where the multiplier includes both the indirect effects (the A-matrix) and the induced effects (the product of the C and V matrices).

The second solution (equation 6b) separates the indirect effect multiplier matrix (the Leontief inverse), which in turn is multiplied by the induced effect multiplier matrix (the second bracket $(\mathbf{I}-\mathbf{CVB})^{-1}$).

The third solution (equation 6c) transforms the induced effect (the second bracket $(\mathbf{I}-\mathbf{CVB})^{-1}$) into a multiplicative expression starting with the traditional Leontief indirect effects multiplier matrix (B) multiplied by the GVA coefficient matrix (V), the interrelational income multiplier (K) and the consumption coefficient matrix (C). The interrelational income multiplier matrix shows the multiplier effect of the consumption of one institutional group on the income of another institutional group.

Finally, in equation (6d) the Miyazawa quantity model can be formulated as a sequential model using the power series expansion to expand the steps in the real circle with inclusion of the induced effects.

The Miyazawa extension of the Leontief quantity model includes the transformations from sectors to institutions and from institutions back to sectors. Miyazawa thereby changes the Leontief model from a one-dimensional model (sector by sector) into a model with transactions from sectors to institutions (V) and from institutions back to sectors (C).

3.2.1 The Miyazawa interregional quantity model

When used in a local or urban context, the spatial dimension must also be adjusted in such a way that production is located at the place of production whereas the institution is located at the place of residence of the institution. Private consumption is derived from demand originating at the place of residence and is purchased at the place of production. Introducing the real circle into the Miyazawa extended quantity model involves a transformation in 2 steps: i) commuting, which transforms GVA from place of production to place of residence (and from sectors to types of institutions); ii) combined shopping and trade, which transforms private consumption from place of residence to place of production (and from type of institution back to sector). Following these definitions the Miyazawa model can be reformulated:

$\mathbf{x}=\mathbf{Ax}+\mathbf{S_{CP}CJVx}+\mathbf{f}$................................(7)

where

$\mathbf{S_{CP,j}}$: shopping/trade at place of production as share of total demand, by sector and by place of residence

$\mathbf{J}$: GVA by place of residence as share of total GVA, by type of institution and by place of production

Accordingly, the analytical solution now can be reformulated:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A} - \mathbf{S_{CP}CJV})^{-1}\mathbf{f}........(8\mathbf{a})$$

$$= \mathbf{B}(\mathbf{I} - \mathbf{S_{CP}CJVB})^{-1}\mathbf{f}.......(8\mathbf{b})$$

$$= \mathbf{B}(\mathbf{I} + \mathbf{S_{CP}CKJVB})\mathbf{f}.......(8\mathbf{c})$$

where

$$\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$$

$$\mathbf{K} = (\mathbf{I} - \mathbf{L})^{-1}$$

where

$$\mathbf{L} = \mathbf{JVBS_{CP}C}$$

$$\mathbf{x} = (\mathbf{I} + (\mathbf{A} + \mathbf{S_{CP}CJV})^{1} + (\mathbf{A} + \mathbf{S_{CP}CJV})^{2} + (\mathbf{A} + \mathbf{S_{CP}CJV})^{3} + ..)\mathbf{f}......(8\mathbf{d})$$

The induced effects refer in quantity modelling terms to transformation of economic activity from sectors to type of institution and back again from type of institution to sectors. Treating the quantity model system as a power series expansion, the Leontief - Miyazawa quantity model now consists of two sub-circles. The first includes the indirect effects from sectors back to sectors and the second the induced effects from sectors to type of institution and from type of institution back to sectors.

3.2.2 The urban or local economy extended input-output system

Transforming the interregional Leontief and the interregional Miyazawa quantity model into a local or an urban input-output model involves introduction of three spatial dimensions and three SAM-dimensions into the modelling framework. In this section a local or urban model including these extensions is presented. In the following section the full extended four dimensional quantity model (using the two-by-two-by-two principle) is presented.

In this 3 dimensional version the quantity model includes the following spatial transformations:

- from place of production to place of residence corresponding to a transformation from sectors to type of institution
- from place of residence to place of commodity market corresponding to a transformation from type of institution to commodities
- from place of commodity market back to place of production corresponding to a transformation from commodities to sectors

The model assumes that intermediate consumption and final demand are added before entering into the intra- and interregional trade system[1]. Gross output can now be expressed as follows:

$$\mathbf{x} = \mathbf{DT}(\mathbf{S_{IC}B_{IC}x} + \mathbf{S_{CP}B_{CP}J} \cdot \mathbf{J}_{\mathbf{j,h}}(\mathbf{x} - \mathbf{i_i}\mathbf{B_{IC}} \circ \mathbf{x}) + \mathbf{f})........(9\mathbf{a})$$

$$\begin{aligned} &= \mathbf{DTS_{IC}B_{IC}x} \\ &\quad + \mathbf{DTS_{CP}B_{CP}J} \cdot \mathbf{J_{j,h}i_i}(\mathbf{I} - \mathbf{B_{IC}}) \circ \mathbf{x} \\ &\quad + \mathbf{DTf}...(9\mathbf{b}) \end{aligned}$$

where

$\mathbf{J}_{\mathbf{j,h}}$: transformation of GVA from sectors (j) to type of institution (h), by place of production

$\mathbf{i_i}$: aggregation vector by commodity (i)

$\mathbf{S_{CP}}$: shopping for private consumption at the place of commodity market place as share of total demand, by commodity and by place of residence

$\mathbf{B_{CP}}$: the Use matrix for private consumption or private consumption by commodity as share of GVA, by place of residence.

From equation 9b it can be seen that the quantity model has two sub-circles. The first represents the indirect effects, whereas the second includes the private consumption (induced) effects. The solution of the model is now straightforward:

$$\mathbf{x} = (\mathbf{I} - \mathbf{DTS_{IC}B_{IC}} - \mathbf{DTS_{CP}B_{CP}J} \cdot \mathbf{J_{e,h}i^h}(\mathbf{I} - \mathbf{B_{IC}}))^{-1}\mathbf{DTf} \text{...................}(10\mathbf{a})$$

$$= (\mathbf{I} - \mathbf{DTS_{IC}B_{IC}})^{-1}(\mathbf{I} - \mathbf{DTS_{CP}B_{CP}J} \cdot \mathbf{J_{e,h}i^h}(\mathbf{I} - \mathbf{B_{IC}}))^{-1}\mathbf{DTf})$$

$$= \mathbf{B}(\mathbf{I} - \mathbf{DTS_{CP}B_{CP}J} \cdot \mathbf{J_{e,h}i^h}(\mathbf{I} - \mathbf{B_{IC}}))^{-1}\mathbf{DTf}) \text{..................................}(10\mathbf{b})$$

$$= \mathbf{B}(\mathbf{I} + \mathbf{DTS_{CP}B_{CP}KJ} \cdot \mathbf{J_{e,h}i^h}(\mathbf{I} - \mathbf{B_{IC}}))\mathbf{DTf}) \text{..................................}(10\mathbf{c})$$

where

$$\mathbf{B} = (\mathbf{I} - \mathbf{DTS_{IC}B_{IC}})^{-1}$$

$$\mathbf{K} = (\mathbf{I} - \mathbf{L})^{-1}$$

$$\mathbf{L} = \mathbf{J} \cdot \mathbf{J_{e,h}i^h}(\mathbf{I} - \mathbf{B_{IC}})\mathbf{BTDS_{CP}B_{CP}}$$

$$\begin{aligned}\mathbf{x} = (\mathbf{I} &+ (\mathbf{DTS_{IC}B_{IC}} + \mathbf{DTS_{CP}B_{CP}J} \cdot \mathbf{J_{e,h}i^h}(\mathbf{I} - \mathbf{B_{IC}}))^1 \\ &+ (\mathbf{DTS_{IC}B_{IC}} + \mathbf{DTS_{CP}B_{CP}J} \cdot \mathbf{J_{e,h}i^h}(\mathbf{I} - \mathbf{B_{IC}}))^2 \\ &+ (\mathbf{DTS_{IC}B_{IC}} + \mathbf{DTS_{CP}B_{CP}J} \cdot \mathbf{J_{e,h}i^h}(\mathbf{I} - \mathbf{B_{IC}}))^3 \\ &+ \text{...})\mathbf{f} \text{..........................}(10\mathbf{d})\end{aligned}$$

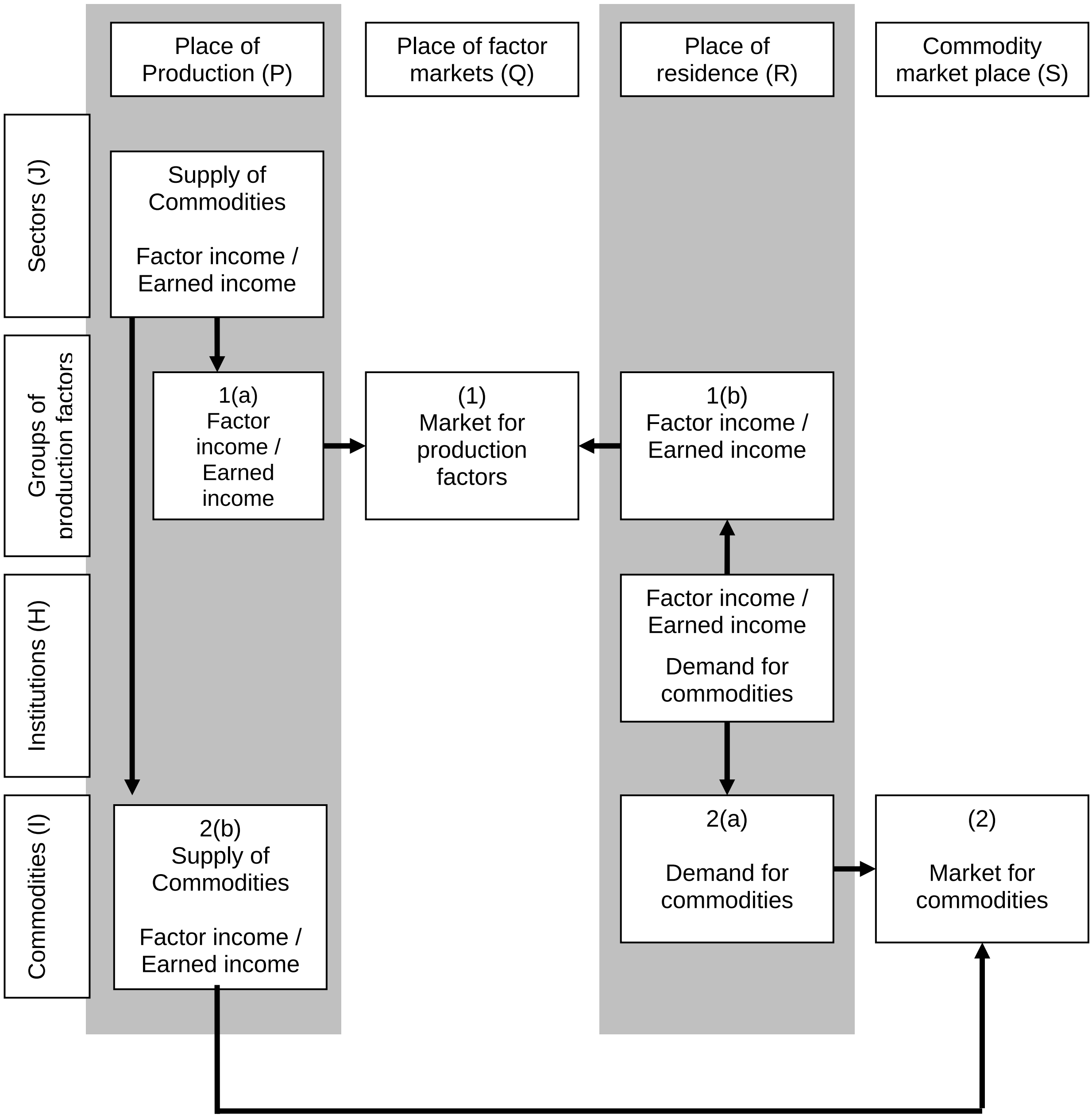
4. The general interregional quantity model

In section 3 the Leontief and Miyazawa interregional quantity models are presented and the two models are integrated and extended to derive a single model incorporating the fundamental spatial concepts identified above. To formulate the general interregional quantity model the two-by-two-by-two principle is used as the basic structure of the model of the local economy.

4.1 Basic concepts and dimensions in the general interregional quantity model - A graphical presentation[2]

There are three fundamental dimensions in the general quantity model, following the two-by-two-by-two principle. First, both producers and households are represented in the general quantity model. Second, two markets – the commodity market and the factor market – are included in the general model. Third, interaction between markets and actors include information on origins and destinations. For both actors and markets basic geographical concepts have been used as well as social accounting concepts for activities. The model structure is presented in figure 1.

Figure 1
The conceptual basis of the general interregional quantity model



In comparison with the 3 dimensional model above, the factor market has been added, which involves a SAM-dimension (groups of production factors) and a spatial concept (place of factor market). In factor markets supply and demand of production factors are to be found. Demand for production factors (g) is determined by production by sector (j) at the place of production (p). In figure 1 factor demand by sector is transformed into factor demand by type of production factor (g). On the supply side, supply of production factors by type of institution (h) is transformed into supply by type of production factor (g). Supply of a production factor is related to the place of residence of the institution (r). The factor market is geographically assigned to the market place for factors (q).

Completing the presentation of the general model based on the two-by-two-by-two principle, in figure 1 in the commodity market there is a distinction between place of residence (r), the market place for commodities (s) and place of production (p). The market place for commodities links the demand for the commodity (from place of residence to the market place for commodities) to the supply of the commodity (from place of production to the market place for commodities). Before the transformation to the market place for commodities, the demand for commodities is transformed from institutional group (h) to commodity (i). On the supply side, production by sector (j) is transformed into production by commodity (i) and then supply is related geographically to the market place for commodities (s).

Second, in the general model both domestic and foreign sectors are represented in all markets. This involves not only international trade in commodities, but also other types of international interaction, such as cross border commuting (income flows to and from abroad through commuting), border shopping, which includes one-day tourist expenditure in both directions, and tourism, again in both directions. This extension is included to make the general model more applicable as most regional systems do not encompass the world, but are surrounded by »the rest of the world«.

4.2 The model

In appendix 1 the equations of the general local/urban quantity model are presented. The model can be treated as a national model (the integrated Leontief and Miyazawa model), where a spatial dimension and a social accounting dimension have been included. The equations in the real circle are presented in structural form together with their partial solutions.

The equations follow the real circle as illustrated in figure 1. Starting in the upper left hand corner at place of production by sector (cell Pj) in equation 1 intermediate consumption $\mathbf{u}_{\mathbf{j,IC}}^{\mathbf{P,f}}$ is determined. **u** is demand. The subscript IC indicates intermediate consumption by sector j. The superscript shows, that intermediate consumption is determined at the place of production P and in fixed prices f. Intermediate consumption is a function of gross output $\mathbf{X}_{\mathbf{j}}^{\mathbf{P,f}}$ (by sector j, by place of production P in fixed prices f) and intermediate consumption's share of production $\mathbf{B}_{\mathbf{j,IC}}^{\mathbf{P}}$ (by sector j by place of production R in fixed prices f). In equations 2-6 intermediate consumption is determined in the following sequence

i) transformation from sectors to commodities (equation 2),
ii) commodities for intermediate consumption purchased abroad are derived and subtracted (equation 3 and 4),
iii) transformation from place of production to place of commodity market (equation 5) and
iv) commodities for foreign intermediate consumption purchased at the place of the commodity market are added (equation 6)

The sequential structure of the equations of the real circle shown in appendix 1 is clear and follows the graphical presentation in figure 5.1. The real circle corresponds to a straightforward, but extended version of the Leontief and Miyazawa interregional quantity model and moves clockwise in figure 1.

Continuing in the upper left corner (cell Pj), production generates factor income in basic prices and is transformed from fixed prices to current prices (equation 7). This factor income is transformed from sectors j to factor groups g and includes factor payments received abroad (equations 8 to 10). Then factor income is transformed from place of production P to place of factor market Q and further to the place of residence R through a commuting model (from cell Pg to cell Rg, going through the factor market, cell Qg, equations 11-12) and including factor payments received from abroad (equation 13). GVA is the basis for determination of private consumption in market prices, by place of residence (cell Rg). First, GVA is transferred to groups of households (cell Rh), transformed from current prices to fixed prices and used in the determination of private consumption (equations 14-15).

The remaining equations reflect the following overall path: Private consumption is divided into tourism (domestic and international) and local private consumption (cell Ri) and is assigned to the place of the commodity market (cell Si) using a shopping model for local private consumption. Private consumption, together with intermediate consumption, public consumption and investment constitute the total local demand for commodities (cell Si). Local demand is met by imports from other regions and abroad in addition to local production (cell Si). Through a trade model exports to other regions and production for the region itself is determined. Adding export abroad, gross output by commodity is determined (cell Pi). Through a reverse Make matrix the cycle returns to production by sector (cell Pj).

4.3 The analytical solution to the general interregional quantity model

The model can now be solved by straightforward insertion. By inserting equation (24) into equation (25), and equation (23) into the modified equation (25) and so on, gross output by sector is a function of itself multiplied by two coefficient matrices, one of which reflects the indirect effects and the other the induced effects (see equation 1 in appendix 2). By using the Leontief and Miyazawa solution techniques, the following result is obtained:

$$
\begin{aligned}
x_j^P = \Big(& I - D^P T_i^{R,S}(i - d_i^{S,F}) \circ B_{i,IC}^{P,S}(i - b_{i,IC}^{P,F,f}) \circ B_{IC}^P b_{IC}^P \\
& - D^P T_i^{R,S}(i - d_i^{S,F}) \circ B_{i,CP}^{R,S}(i - b_{i,CP}^{R,F,f}) \circ B_{CP}^R (pu_{h,CP}^R)^{-1} \circ J^R \\
& \big(h_g^{R,F} + J_g^{Q,T} J_g^{P,Q}(i - j_g^{P,F}) \circ J^P ph_j^P \circ (i - b_{IC}^P) \big) \Big)^{-1} \\
& \Big(D^P z_i^{P,F,f} \\
& + D^P T_i^{P,S,f}(i - d_i^{S,F,f}) \circ (u_{i,IC}^{S,F,f} + u_{i,CP}^{S,F,f} + u_{i,CO}^{S,f} + u_{i,IR}^{S,f}) \Big) \quad \text{.........(11)}
\end{aligned}
$$

The solution includes a multiplier (the first 3 lines in the expression) and the exogenous demand in line 4-5. The multiplier can be decomposed into a line showing the indirect effects (line 1) and the induced effects (line 2-3). The exogenous demand can be divided into impacts from foreign exports (line 4) and from commodities for intermediate consumption sold to abroad, foreign tourist consumption, governmental consumption and investment. In appendix 2, the analytical solution has been reformulated using the Miyazawa techniques, dividing the effects into the indirect effects and the induced effects, including the interrelational multiplier which shows the impacts from the consumption of one group in one region on the consumption of another group in any region.

The analytical solution can be used to refine and document a) the multiplier effects of the economic activity in a local area on the local area itself and b) the spill-over effects from economic activities in other local areas on the local area. The general a priori result is that the multiplier becomes smaller the smaller the area, but also that the local area becomes increasingly dependent upon economic activity in other local areas, especially the neighbouring areas. The extreme example is of course the case where a local area only consists of one production unit. In this case the internal multiplier effect on demand from the production unit itself becomes small, whereas the economic dependency on economic activity in all other local areas becomes very important.

Another aspect is that the analytical solution shown in equation 11 is determined from the perspective of the place of production and industrial sector. If the subject was instead effects on income by place of residence (,which is relevant from a place or residence and type of institution perspective), the impacts arising from changes in exogenous demand would be smaller. Alternatively, if the perspective was the effects on economic activity at the place of commodity market (such as retailing activities), then the impacts would be even smaller.

Using the analytical solution, a list of factors determining the level of production at the place of production can be drawn up, the sign in brackets showing the expected impacts on gross output of positive change in the factor:

- Intermediate consumption
  - Share of gross output (?)
  - Purchases abroad (-)
  - Purchases in other local areas (-)
  - Purchases from other local areas (+)
  - Purchases from abroad (+)
- Commuting
  - Place of residence abroad (-)
  - Place of residence in other local areas (-)
  - Place of production in other areas (+)
  - Place of production abroad (+)
- Local private consumption (shopping)
  - Propensity to consume (+)
  - Private consumption abroad, such as tourism abroad (-)
  - Shopping in other local areas, including domestic tourism (-)
  - Shopping from other local areas, including domestic tourism (+)
  - Private consumption from abroad, such as one-day tourism and conventional tourism (+)
- Trade
  - Import from abroad (-)
  - Import from other local areas (-)
  - Export to other local areas (+)
  - Export broad (+)

As can be seen the above list includes factors which involve interaction between the local area itself, other regions and abroad. Other exogenous variables affecting the composition of demand and supply in the commodity market and in the market for production factors also influence economic activity in the local area. Impacts of such changes should be modelled with other types of interregional models, which include impacts from changes in costs and prices.

The list can be used to identify different ideal types of local area. Each group is a pure type, whilst in reality a local area is a mix of different types. The definition relies upon the interaction balance, net

- areas based upon local production
  - primary products (trade balance and intermediate commodity-purchasing surplus in primary products)
  - secondary products (trade balance and intermediate commodity - purchasing surplus in secondary products)
  - advanced services (trade balance and intermediate commodity -purchasing surplus in tertiary products)
- residential areas
  - high level of outward commuting and low level of inward commuting
- areas based upon shopping
  - high level retailing services (local private consumption: shopping surplus)
  - conventional tourist areas (surplus in conventional tourist balance)
    - urban (surplus in conventional tourist balance)
    - rural (surplus in conventional tourist balance)
    - ecological (surplus in conventional tourist balance for ecological tourist type)
  - one day tourist areas
    - cultural (surplus in one-day tourist balance)
    - retailing (surplus in one-day tourist balance)

5. The LINE model and the general interregional quantity model

LINE is an interregional general equilibrium model constructed for Danish municipalities (Madsen et al 2001 and Madsen & Jensen-Butler (2004)). The spatial *two-by-two-by-two* principle described above has been the guiding principle for the construction of the model and the interregional social accounting matrix, SAM-K (Madsen et al 2001 and Madsen & Jensen-Butler (2005)), which serves as the database for LINE. Both LINE and SAM-K are designed on the basis of the structure shown in figure 1, using the double spatial entry principle or extended regional accounts (*two-by-two-by-two*), rather than non-spatial regional accounting principles (*two-by-two*). However, there are some differences between LINE and a model based upon a pure two-by-two-by-two principle.

The structure of LINE follows the basic interregional general equilibrium model shown in figure 5.1 with:

- Factor markets and commodity markets
- Demand and supply in both markets
- Origins and destinations in all interactions

Some simplifications and extensions are however incorporated. The general model is adjusted in order to take into account the nature of the available data and the structure of the regional economy. In some respects the model is developed, whilst in other respects it is simplified.

First, the concept of the market place for factors does not correspond in general to reality. In practice, the place of residence of the production factor (such as labour) can be interpreted as both place of residence and the market place for factors. Only in very few cases does a geographically defined factor market exist. From a data collection point of view, only registration of place of residence and place of production in the factor market is possible. Therefore, the market place for factors has been excluded from LINE.

Second, only factor income from labour receives a full treatment. Regional data on capital income only exist by place of production in Denmark. Data on interregional commuting of capital income is still lacking, which makes a comparable treatment to commuting flows of labour income impossible and identification of a market place for capital income difficult to develop. In the present version of LINE capital income enters exogenously at the place of residence without any information on its spatial origin. Future developments with respect to savings and investments and identification of market places for these could include the use of pooling methods or identification of gross flows, referred to above.

Third, there is a need to keep track of economic interactions at the place of residence between factor groups and between institutions. Interaction between households and the governmental sector is important in order to describe the economic strength of households, for example measured by disposable income of households including income transfers from government and the subtraction of taxes. Interactions between factor groups, household and governmental sectors are therefore included in LINE.

Fourth, consumption by institutions (households) both from a decision-making or a behavioural point of view must be divided into two nested steps. First, consumption is determined at a high level of aggregation, for example food, clothing, transport etc. and in market prices. In the next step, the consumption bundles are further divided into specific commodities, transformed into basic prices, and distributed into domestic and foreign markets and among producing regions. From a decision-making point of view both the first and second steps are a part of the household decision problem, the sellers (the retailing sector) reflecting the demand from the households. The same is the case for intermediate consumption and for other types of final demand, such as governmental consumption and gross capital formation, where decisions are taken in two steps: First, at the place of residence deciding expenditure on aggregate commodity, such as expenditure on schools and second in the institution at the place of commodity market and the place of production, where decisions on type of commodity, by domestic and foreign market and by supplying regions are taken.

Fifth, private consumption has been divided into local private consumption and domestic tourism. This division has been relevant in studies of tourism impacts, where in LINE it is possible to distinguish between tourism by foreigners, domestic tourism and tourism abroad, all divided into either one-day visits or visits involving overnight stays.

Sixth, different price concepts are included in the model, reflecting the fact that different variables for economic activity use different price concepts. For goods and services, total expenditures at the place of commodity market are measured in market prices. Supply of commodities entering the goods and services market is modelled in basic prices. Basic prices are defined as the value of production at the factory, not including net commodity taxes paid by the producer. Going from market/buyers prices to basic prices involves subtraction of commodity taxes and trade margins, where trade margins also are part of the commodity account. Interregional trade is measured in basic prices both seen from at the place of production and place of commodity market point of view. At the place of commodity market commodity prices are transformed from basic prices to market prices.

Finally, LINE is based upon two interrelated circles: a real circuit described above and a dual cost-price circuit. Figure 1 shows the general model structure, based upon the real circle employed in LINE. The two circles are linked together with a link from real economic activities to formation of cost and prices (mainly a weighting system for determining costs and prices) and from the costs and prices to real economic activity. This last link includes the effects of cost and price changes on demand, the transformation of disposable income in current prices to fixed prices and the effects on exports and imports prices in turn determining exports and imports. Part of the model uses fixed prices (the demand and supply of commodities) and part of the model uses current prices (earned income, taxes, transfer incomes and disposable income).

Here only a brief comparison of LINE and the general interregional quantity model is made. The full LINE model and its equations are described in Madsen et al. (2001) and Madsen & Jensen-Butler (2004). LINE has been constructed as a flexible on a number of key dimensions. For any application of LINE the model and the associated database are aggregated in order to capture the special requirements of each case. Thus in any version of LINE the model configuration is specific. One example of such an application is provided by Madsen & Jensen-Butler (2004), where the following dimensions were used:

Sectors
21 sectors aggregated from the 133 sectors used in the national accounts.

Factors
7 age, 2 sex and 5 education groups.

Households
4 types, based upon household composition.

Needs
For private consumption and governmental individual consumption 13 components, aggregated from the 72 components in the detailed national accounts. For governmental consumption, 8 groups. For gross fixed capital formation, 10 components.

Commodities
27 commodities, aggregated from 131 commodities used in the national accounts.

Regions
277 municipalities, including one state-owned island and one unit for extra-regional activities, this being the lowest level of spatial disaggregation. Regions are defined either as place of production, place of residence or as place of commodity market. In this version of LINE the (277) municipalities have been aggregated into 16 regional units, including one unit for extra-regional activities.

6. Summary

In this paper the general interregional quantity model for local or urban economies has been presented. The model represents an extension and integration of the interregional Leontief quantity model including the indirect effects and the interregional Miyazawa quantity model including the induced effects. The general interregional quantity model is based upon the two-by-two-by-two principle including a) markets for commodities and factors, b) production units and institutional units and c) origin and destination for the demand and supply in the two markets. The general interregional quantity model is formulated including a foreign sector and the analytical solution to the model is presented. The basic structure embedded in the general interregional quantity model was used to identify a new typology for local areas according to their specialization in the intra- and interregional interaction. The general interregional quantity model is compared generally with LINE, which is a local economic model for Danish regions with a structure similar to the structure of the general interregional quantity model for local and urban economies. Differences reflecting data limitations and the need to include interaction between households and governmental sector and including a distinction between market prices and basic prices are examples of the deviations between the idealised general interregional quantity model and operationalised models.

Appendix 1

The equations for the general interregional quantity model for local and urban economies in structural form

Variables in the quantity model
The variables in the general interregional quantity model are denoted in the following way:

Variables
x: gross output
B, b: Use coefficient matrix or coefficient vector of demand
U, u: Use value matrix or vector of demand
D: Make coefficient matrix
V: Make value matrix
T, t: Trade coefficient matrix or vector
J, j: Income transformation coefficient matrix or vector
H, h: Income transformation coefficient matrix or vector
pu: Price index vector for demand
px: Price index for gross output
pz: Price index for trade flows

Superscripts
P: Place of production (regional axes)
Q: Place of commodity market (regional axes)
R: Place of residence (regional axes)
S: Place of commodity market (regional axes)
O: Interregional
F: Rest of the world
f: Fixed prices

Subscripts
SAM-axes
j: Sector (SAM-axis)
g: Groups of factors (SAM-axis)
h: Type of institution (SAM-axis)
i: Commodity (SAM-axis)
IC: Intermediate consumption
CP: private consumption
CO: Governmental consumption
IR: Investments

The equations in structural form

$\mathbf{u}^{P,f}_{j,IC} = \mathbf{b}^{P}_{j,IC} \circ \mathbf{x}^{P,f}_{j}$ ....(1)

$\mathbf{u}^{P,f}_{i,IC} = \mathbf{B}^{P}_{IC}\mathbf{u}^{P,f}_{j,IC}$ ....(2) **from Pj to Pi**

$\mathbf{u}^{P,F,f}_{i,IC} = \mathbf{b}^{P,F}_{IC} \circ \mathbf{U}^{P,f}_{i,IC}$ ....(3)

$\mathbf{u}^{P,D,f}_{i,IC} = \mathbf{u}^{P,f}_{i,IC} - \mathbf{u}^{P,F,f}_{i,IC}$ ....(4)

$\mathbf{u}^{S,D,f}_{i,IC} = \mathbf{B}^{P,S,D}_{i,IC}\mathbf{u}^{P,D,f}_{i,IC}$ ....(5) **from Pi to Si**

$\mathbf{u}^{S,f}_{i,IC} = \mathbf{u}^{S,D,f}_{i,IC} + \mathbf{u}^{S,F,f}_{i,IC}$ ....(6)

$\mathbf{h}^{P}_{j} = \mathbf{ph}^{P}_{j} \circ (\mathbf{i}_{j} - \mathbf{b}^{P}_{j,IC}) \circ \mathbf{X}^{P,f}_{j}$ ....(7)

$\mathbf{h}^{P}_{g} = \mathbf{J}^{P}_{g,j}\mathbf{h}^{P}_{j}$ ....(8) **from Pj to Pg**

$\mathbf{h}^{P,F}_{g} = \mathbf{j}^{P,F}_{g} \circ \mathbf{h}^{P,L}_{g}$ ....(9)

$\mathbf{h}^{P,D}_{g} = \mathbf{h}^{P}_{g} - \mathbf{h}^{P,F}_{g}$ ....(10)

$\mathbf{h}^{Q,D}_{g} = \mathbf{J}^{P,Q}_{g}\mathbf{h}^{P,D}_{g}$ ....(11) **from Rg to Qg**

$\mathbf{h}^{R,D}_{g} = \mathbf{J}^{Q,S}_{g}\mathbf{h}^{Q,D}_{g}$ ....(12) **from Qg to Tg**

$\mathbf{h}^{R}_{g} = \mathbf{h}^{R,D}_{g} + \mathbf{h}^{R,F}_{g}$ ....(13)

$\mathbf{h}^{R}_{h} = \mathbf{J}^{R}_{g,h}\mathbf{h}^{R}_{g}$ ....(14) **from Rg to Rh**

$\mathbf{u}^{R,f}_{h,CP} = \mathbf{b}^{R,f}_{h,CP} \circ \mathbf{h}^{R}_{h} / \mathbf{pu}^{R}_{h,CP}$ ....(15)

$\mathbf{u}^{R,f}_{i,CP} = \mathbf{B}^{R,f}_{CP}\mathbf{u}^{R,f}_{h,CP}$ ....(16) **from Th to Ti**

$\mathbf{u}^{R,F,f}_{i,CP} = \mathbf{b}^{R,F,f}_{CP} \circ \mathbf{u}^{R,f}_{i,CP}$ ....(17)

$\mathbf{u}^{R,D,f}_{i,CP} = \mathbf{u}^{R,f}_{h,CP} - \mathbf{u}^{R,F,f}_{h,CP}$ ....(18)

$\mathbf{u}^{S,D,f}_{i,CP} = \mathbf{B}^{R,S}_{i,CP}\mathbf{u}^{R,D,f}_{i,CP}$ ....(19) **from Ti to Si**

$\mathbf{u}^{S,f}_{i} = \mathbf{u}^{S,D,f}_{i,CP} + \mathbf{u}^{S,F,f}_{i,CP}$ ....(20)

$\mathbf{u}^{S,f}_{i} = \mathbf{u}^{S,f}_{i,IC} + \mathbf{u}^{S,f}_{i,CP} + \mathbf{u}^{S,f}_{i,CO} + \mathbf{u}^{S,f}_{i,IR}$ ....(21)

$\mathbf{z}^{S,D,f}_{i} = (\mathbf{i} - \mathbf{d}^{S,F}_{i}) \circ \mathbf{u}^{S,f}_{i}$ ....(22)

$\mathbf{z}^{P,D,f}_{i} = \mathbf{i}^{S} \cdot \mathbf{T}^{R,P}_{i} \circ \mathbf{z}^{S,D,f}_{i}$ ....(23) **from Si to Pi**

$\mathbf{q}^{P,f}_{i} = \mathbf{z}^{P,D,f}_{i} + \mathbf{z}^{P,F,f}_{i}$ ....(24)

$\mathbf{x}^{P}_{j} = \mathbf{D}^{P}\mathbf{q}^{P,f}_{i}$ ....(25) **from Pi to Pj**

Appendix 2

The analytical solution for the general interregional quantity model for local and urban economies

$$\begin{aligned}\mathbf{x}_j^{P} &= \mathbf{D}^{P}\mathbf{T}_i^{R,S}(\mathbf{i}-\mathbf{d}_i^{S,F})\circ\mathbf{B}_{i,IC}^{P,S}(\mathbf{i}-\mathbf{b}_{i,IC}^{P,F,f})\circ\mathbf{B}_{IC}^{P}\mathbf{b}_{IC}^{P}\circ\mathbf{x}_j^{P}\\ &\quad+\mathbf{D}^{P}\mathbf{T}_i^{R,S}(\mathbf{i}-\mathbf{d}_i^{S,F})\circ\mathbf{B}_{i,CP}^{R,S}(\mathbf{i}-\mathbf{b}_{i,CP}^{R,F,f})\circ\mathbf{B}_{CP}^{R}(\mathbf{pu}_{h,CP}^{R})^{-1}\circ\mathbf{J}^{R}\\ &\qquad\Big(\mathbf{h}_g^{R,F}+\mathbf{J}_g^{Q,T}\mathbf{J}_g^{P,Q}(\mathbf{i}-\mathbf{j}_g^{P,F})\circ\mathbf{J}^{P}\mathbf{ph}_j^{P}\circ(\mathbf{i}-\mathbf{b}_{IC}^{P})\Big)\circ\mathbf{x}_j^{P}\\ &\quad+\mathbf{D}^{P}\mathbf{z}_i^{P,F,f}\\ &\quad+\mathbf{D}^{P}\mathbf{T}_i^{P,S,f}(\mathbf{i}-\mathbf{d}_i^{S,F,f})\circ(\mathbf{u}_{i,IC}^{S,F,f}+\mathbf{u}_{i,CP}^{S,F,f}+\mathbf{u}_{i,CO}^{S,f}+\mathbf{u}_{i,IR}^{S,f})\ldots\ldots\ldots\ldots\ldots(1)\end{aligned}$$

$$\begin{aligned}\mathbf{x}_j^{P} &= \Big(\mathbf{I}-\mathbf{D}^{P}\mathbf{T}_i^{R,S}(\mathbf{i}-\mathbf{d}_i^{S,F})\circ\mathbf{B}_{i,IC}^{P,S}(\mathbf{i}-\mathbf{b}_{i,IC}^{P,F,f})\circ\mathbf{B}_{IC}^{P}\mathbf{b}_{IC}^{P}\\ &\quad-\mathbf{D}^{P}\mathbf{T}_i^{R,S}(\mathbf{i}-\mathbf{d}_i^{S,F})\circ\mathbf{B}_{i,CP}^{R,S}(\mathbf{i}-\mathbf{b}_{i,CP}^{R,F,f})\circ\mathbf{B}_{CP}^{R}(\mathbf{pu}_{h,CP}^{R})^{-1}\circ\mathbf{J}^{R}\\ &\qquad\Big(\mathbf{h}_g^{R,F}+\mathbf{J}_g^{Q,T}\mathbf{J}_g^{P,Q}(\mathbf{i}-\mathbf{j}_g^{P,F})\circ\mathbf{J}^{P}\mathbf{ph}_j^{P}\circ(\mathbf{i}-\mathbf{b}_{IC}^{P})\Big)\Big)^{-1}\\ &\quad\Big(\mathbf{D}^{P}\mathbf{z}_i^{P,F,f}\\ &\quad+\mathbf{D}^{P}\mathbf{T}_i^{P,S,f}(\mathbf{i}-\mathbf{d}_i^{S,F,f})\circ(\mathbf{u}_{i,IC}^{S,F,f}+\mathbf{u}_{i,CP}^{S,F,f}+\mathbf{u}_{i,CO}^{S,f}+\mathbf{u}_{i,IR}^{S,f})\Big)\ldots\ldots(2)\end{aligned}$$

$$\begin{aligned}\mathbf{X} &= \mathbf{AX}+\mathbf{CVX}+\mathbf{f}\\ &= \mathbf{AX}+\mathbf{CVX}+\mathbf{DT}(\mathbf{i}-\mathbf{D}^{For})\mathbf{f}^{Dom}+\mathbf{Df}^{For}\end{aligned}$$

$$\mathbf{X} = \left[\mathbf{I} - \mathbf{A} - \mathbf{CV}\right]^{-1}\mathbf{f}$$

$$\left[\mathbf{I} - \mathbf{A} - \mathbf{CV}\right]^{-1}(\mathbf{DT}(\mathbf{i} - \mathbf{D}^{\mathbf{For}})\mathbf{f}^{\mathbf{Dom}} + \mathbf{Df}^{\mathbf{For}})$$

$$= \mathbf{B}\left[\mathbf{I} - \mathbf{CVB}\right]^{-1}\mathbf{f} \quad = \mathbf{B}\left[\mathbf{I} - \mathbf{CVB}\right]^{-1}(\mathbf{DT}(\mathbf{i} - \mathbf{D}^{\mathbf{For}})\mathbf{f}^{\mathbf{Dom}} + \mathbf{Df}^{\mathbf{For}})$$

$$= \mathbf{B}\left[\mathbf{I} + \mathbf{CKVB}\right]\mathbf{f} = \mathbf{B}\left[\mathbf{I} + \mathbf{CKVB}\right](\mathbf{DT}(\mathbf{i} - \mathbf{D}^{\mathbf{For}})\mathbf{f}^{\mathbf{Dom}} + \mathbf{Df}^{\mathbf{For}})$$

where

$$\mathbf{K} = \left[\mathbf{I} - \mathbf{L}\right]^{-1}$$

and

$$\mathbf{L} = \mathbf{VBC}$$

$$\mathbf{A} = \mathbf{D}^{\mathbf{P}}\mathbf{T}_{\mathbf{i}}^{\mathbf{R,S}}(\mathbf{i} - \mathbf{d}_{\mathbf{i}}^{\mathbf{S,F}}) \circ \mathbf{B}_{\mathbf{i,IC}}^{\mathbf{P,S}}(\mathbf{i} - \mathbf{B}_{\mathbf{i}}^{\mathbf{P,F,f}}) \circ \mathbf{B}_{\mathbf{IC}}^{\mathbf{P}}\mathbf{b}_{\mathbf{e,IC}}^{\mathbf{P}}$$

$$\mathbf{C} = \mathbf{D}^{\mathbf{P}}\mathbf{T}_{\mathbf{i}}^{\mathbf{R,S}}(\mathbf{i} - \mathbf{d}_{\mathbf{i}}^{\mathbf{S,F}}) \circ \mathbf{B}_{\mathbf{i,CP}}^{\mathbf{R,S}}(\mathbf{i} - \mathbf{B}_{\mathbf{i,CP}}^{\mathbf{R,F,f}})$$

$$\mathbf{V} = \mathbf{b}_{\mathbf{CP}}^{\mathbf{R}}(\mathbf{pu}_{\mathbf{h,CP}}^{\mathbf{R}})^{-1} \circ \mathbf{J}^{\mathbf{R}}\left(\mathbf{h}_{\mathbf{g}}^{\mathbf{R,F}} + \mathbf{J}_{\mathbf{g}}^{\mathbf{Q,T}}\mathbf{J}_{\mathbf{g}}^{\mathbf{P,Q}}(\mathbf{i} - \mathbf{j}_{\mathbf{g}}^{\mathbf{P,F}}) \circ \mathbf{J}^{\mathbf{P}}\mathbf{ph}_{\mathbf{j}}^{\mathbf{P}} \circ (\mathbf{i} - \mathbf{b}_{\mathbf{j,IC}}^{\mathbf{P}})\right)$$

$$\mathbf{f} = \mathbf{D}^{\mathbf{P}}\mathbf{z}_{\mathbf{i}}^{\mathbf{P,F,f}} + \mathbf{D}^{\mathbf{P}}\mathbf{T}_{\mathbf{i}}^{\mathbf{P,S}}(\mathbf{i} - \mathbf{d}_{\mathbf{i}}^{\mathbf{S,F,f}}) \circ (\mathbf{u}_{\mathbf{i,IC}}^{\mathbf{S,F,f}} + \mathbf{u}_{\mathbf{i,CP}}^{\mathbf{S,F,f}} + \mathbf{u}_{\mathbf{i,CO}}^{\mathbf{S,f}} + \mathbf{u}_{\mathbf{i,IR}}^{\mathbf{S,f}})$$

$$\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$$

$$\mathbf{D} = \mathbf{i}_{\mathbf{i}} \cdot \mathbf{D}^{\mathbf{P}}$$

$$\mathbf{f}^{\mathbf{For}} = \mathbf{z}_{\mathbf{i}}^{\mathbf{P,F,f}}$$

$$\mathbf{D}^{\mathbf{For}} = \mathbf{D}_{\mathbf{i}}^{\mathbf{S,F,f}}$$

$$\mathbf{T} = \mathbf{T}_{\mathbf{i}}^{\mathbf{P,S}}$$

$$\mathbf{f}^{\mathbf{Dom}} = \mathbf{u}_{\mathbf{i,IC}}^{\mathbf{S,F,f}} + \mathbf{u}_{\mathbf{i,CP}}^{\mathbf{S,F,f}} + \mathbf{u}_{\mathbf{i,CO}}^{\mathbf{S,f}} + \mathbf{u}_{\mathbf{i,IR}}^{\mathbf{S,f}}$$

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Notes

1. In principle, the model could be formulated with separate trade models for intermediate consumption and final demand. But because this information is not normally available and because problems with differences in trade patterns can be solved by further disaggregation of commodities, combining intermediate consumption and final demand as total demand at the place of commodity market is proposed.

2. In this paper only the real circle of the general model is presented. A similar structure applies in the dual model, which is the cost price circle in the general model.