An Intertemporal Urban Economic Model with Natural Environment

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Abstract

As the human society has been developing, many environment problems have occurred. Especially the cities, where a half of the world population lives in, have more serious environment problem than rural areas, because there are more car transportation and energy consumption in urban areas. As we know, natural environment is a very important element for social development. We must face and deal with it properly. Since the industrial revolution, the current economic system has been achieved by mass production, mass consumption, and mass disposal of waste in the past. In order to attain a sustainable society, we must realign our present urban system. Compact city is considered as one of the solutions to attain a sustainable urban system. Many papers have been published in agreement, in opposite, and in intermediate standpoint, since Dantzig and Saaty proposed the concept of a compact city. In this paper, we construct an intertemporal urban economic model internalizing the natural environment. This model is characterized as a social optimization problem. And we also simulate and analyze a possibility of formation of a compact city with natural environment.

***Keywords*:** Intertemporal Urban Economic Model, Interzonal Urban Economic Model, Compact City, Natural Environment, Social Optimization

***JEL codes*:** R11 and Q20

1. Introduction:

As the human society has been developing, many environment problems have occurred. Especially the cities, where a half of the world population lives in, have more serious environment problem than rural areas, because there are more car transportation and energy consumption in urban areas.

As we know, natural environment is a very important element for social development. We must face and deal with it properly. Since the industrial revolution, the current economic system has been achieved by mass production, mass consumption, and mass disposal of waste in the past. In order to attain a sustainable society, we must realign our present urban system.

Compact city is considered as one of the solutions to attain a sustainable urban system. Many papers have been published in agreement, in opposite, and in intermediate standpoint, since Dantzig and Saaty proposed the concept of a compact city [3]. Burton, Williams, and Jenks [2] collected study commentaries, surveillance studies, and introduction of city plans, etc. Jenks, Williams, and Burton [6] tried to make a definition of elements of a sustainable city. Jenks and Burgess [4] also studied interests of compact cities in the developing countries in Africa, Asia, and Latin America. Following these backgrounds, the authors have already published a paper [9] which considers the concept of a compact city [3] that has been reexamined mainly in European counties and the United States. In that paper, some simulations show a possibility of formation of a compact city. However, there is a shortcoming that the natural environment, which is important in the environmental symbiosis oriented city, is not incorporated. Therefore, this article aims to take into account the natural environment as an endogenous public good and to develop a new dynamic urban model.

About the theoretical analysis of a dynamic model, one can find some contributions in the urban economics literature. Kanemoto [5] examined the urban growth and the optimality, and Miyao [7] analyzed the dynamics of urban boundary. Palivos and Wang [8] developed an endogenous dynamic urban model and they derived the optimal population by incorporating human capital into the dynamic mono-centric urban framework. Black and Henderson [1] further extended a dynamic urban model into a dynamic system of cities which involves two types of cities with human capital accumulation.

In this paper, we construct an intertemporal urban economic model internalizing the natural environment. This model is characterized as a social optimization problem. And we also simulate and analyze a possibility of formation of a compact city with natural environment.

2. Outline of the Model

In this study, our urban simulation model is specified as a dynamic social optimal problem. Space is divided into two parts, metropolitan area and non-metropolitan area. The metropolitan area is subdivided by zone. Z denotes a zone index in the metropolitan area. The zones are located in a two-dimensional space.

The primary production factors in this economy are labor, land, and capital. The total available land in the metropolitan area is given. However, the land use structure such as housing, industrial, and agricultural areas is endogenously determined. In this model, to examine the compactness of the city, regulations are imposed on land use. We also assume that capital is immobile among industries and zones, and it is classified into industrial, housing, transportation, and natural environmental capitals.

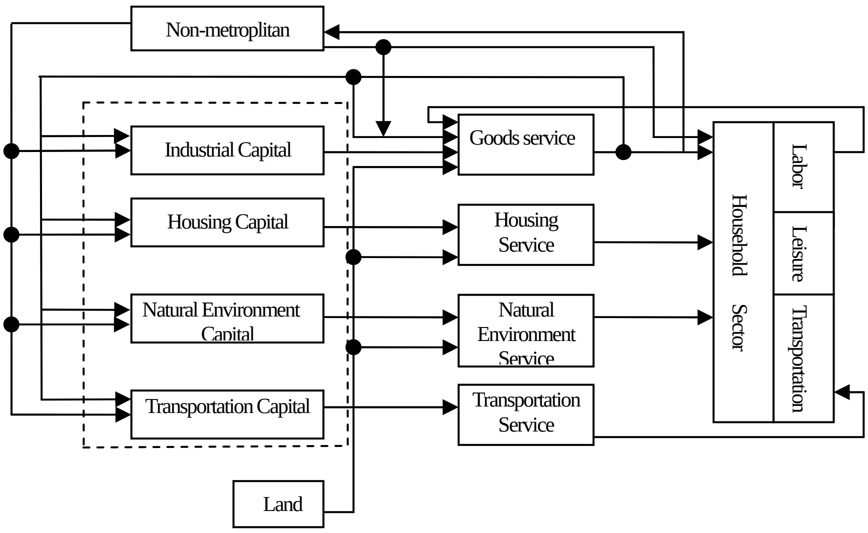

Figure 1 Structure of the Model

The total population in the metropolitan area is exogenously given. We assume that labor is mobile among industries and zones. The urban infrastructure industry is classified into industrial, housing, transportation, and natural environmental services. The housing services are differentiated by the housing type. Q denotes an index of housing. The natural environmental services are also differentiated by the degree of its public nature. E is an index of natural environmental services.

The general industry is classified into agricultural sector, industrial sector, and service sector. Each index of sectors is denoted by I_A, I_I, and I_S. All the sectors in the general industry is defined by an index $I \equiv \{I_A \cup I_I \cup I_S\}$. For example, a sector in the industrial sector is referred as $i \in I_I$.

We assume that tradable commodities between metropolitan and non-metropolitan areas are differentiated. In the metropolitan area, tradable commodities among zones are not differentiated irrespective of origin and destination. The transportation network is given by node and link. An index of transport link is denoted by ML. In this paper, we assume that commuting behavior of household depends on the structure of transportation network, i.e. the minimum distance between home and office. Commuting costs are paid by urban residences. However, for simplicity, transportation costs of goods are ruled out in this model.

3. The Social Optimality

3.1 The objective function

The objective function is the sum of household utilities in the metropolitan area among planning periods and the assets value at the final period. It is a welfare function which is defined as;

$$W = \sum_{t=1}^{T} (u_t^E) \frac{1}{(1+\rho)^t} + \phi(\mathbf{K}_{T+1}, \mathbf{H}_{T+1}, \mathbf{R}_{T+1}, \mathbf{E}_{T+1}) \frac{1}{(1+\rho)^{T+1}} \tag{1}$$

where u_t^E is an equilibrium utility level in the metropolitan area. Parameter ρ is the social discount rate. $\phi(\bullet)$ is a function of assets value and it evaluates the values of capital stocks at the final period. It depends on vectors of industrial capital $\mathbf{K}$, housing capital $\mathbf{H}$, transport capital $\mathbf{R}$, and natural environmental stocks $\mathbf{E}$.

3.2 Constraints

(1) General Industries

The flow condition of general goods and services $i \in I$ in the metropolitan area is given as;

$$J_{Fit} \equiv \sum_{z \in Z} Y_{Fi}^{z}(l_{Fit}^{z}, K_{Fit}^{z}, A_{Fit}^{z}, \mathbf{X}_{Fit}^{z}) - \sum_{z \in Z}\sum_{k \in I} X_{Fikt}^{Dz} - \sum_{z \in Z} c_{it}^{Dz} n_{t}^{z} - \sum_{k \in I} \mu_{ikt}^{K} \sum_{z \in Z} c_{Ki}(I_{Kkt}^{z})$$
$$- \sum_{k \in Q} \mu_{ikt}^{H} \sum_{z \in Z} c_{Hi}(I_{Hkt}^{z}) - \sum_{k \in E} \mu_{ikt}^{E} \sum_{z \in Z} c_{Ei}(I_{Ekt}^{z}) - \mu_{i}^{R} \sum_{s \in ML} c_{Ri}(I_{Rst}) - \overline{O}_{it} \geq 0 \tag{2}$$

where J_{Fit} is the excess supply function of general goods and services, and it must be non-negative. The first term is the total supply of goods and services i in the metropolitan area. $Y_{Fi}^{z}(\bullet)$ is a production function in zone z and it is the function of labor l_{Ft}^{z}, industrial capital K_{Fit}^{z}, land A_{Fit}^{z}, and a vector of intermediate inputs $\mathbf{X}_{Fit}^{z}$. The second term is the intermediate demand in the metropolitan area. The third term is the consumption of households. From the forth term to the last term are demands for investment goods of general industry, housing, natural environmental, and transport investments.

X_{Fikt}^{Dz} is the intermediate demand of good i of industry k in zone z. c_{it}^{Dz} is the household consumption of good i in zone z. I_{Kkt}^{z} is the investment of industry k in zone z. I_{Hkt}^{z} is the investment of type k housing. I_{Ekt}^{z} is the investment of type k natural environmental services. I_{Rst} is the transport investment on link s. Parameters μ_{ikt}^{K}, μ_{ikt}^{H}, μ_{ikt}^{E}, and μ_{ikt}^{R} are capital coefficient matrices, respectively. n_{t}^{z} is the number of households in zone z. Z is the set of zones. We assume that those investments involve the adjustment costs. Functions $c_{Ki}(\bullet)$, $c_{Hi}(\bullet)$, $c_{Ei}(\bullet)$, and $c_{Ri}(\bullet)$ denote adjustment costs, respectively. $\overline{O}_{it}$ is the export of good i from the metropolitan to outside and it is exiguously given.

The flow condition of imported general goods and services $i \in I$ is given as;

$$J_{Mit} \equiv \overline{I}_{Mit} - D_{IMit} \geq 0 \tag{3}$$

where $\overline{I}_{Mit}$ is the supply of the imported goods i from outside to the metropolitan area and it is exogenously given. D_{IMit} is the demand for imported goods and it is defined as;

$$D_{IMit} \equiv \sum_{z \in Z}\sum_{k \in I} X_{Fikt}^{Iz} + \sum_{z \in Z} c_{it}^{Iz} n_{t}^{z} + \sum_{k \in I} \mu_{ikt}^{IK} \sum_{z \in Z} c_{Ki}(I_{Kkt}^{z}) + \sum_{k \in Q} \mu_{ikt}^{IH} \sum_{z \in Z} c_{Hi}(I_{Hkt}^{z}) + \sum_{k \in E} \mu_{ikt}^{IE} \sum_{z \in Z} c_{Ei}(I_{Ekt}^{z})$$
$$+ \mu_{it}^{IR} \sum_{s \in ML} c_{Ri}(I_{Rst}) \tag{4}$$

similarly, X_{Fikt}^{Iz} is the demand for imported goods i. c_{it}^{Iz} is the household consumption of imported goods i. Parameters μ_{ikt}^{IK}, μ_{ikt}^{IH}, μ_{ikt}^{IE}, and μ_{ikt}^{IR} are capital coefficient matrices related to the imported goods, respectively.

(2) Urban Infrastructures

The flow condition of housing services is given as;

$$J_{Hkt}^{z} \equiv Y_{Hk}^{z}\left(H_{kt}^{z}, A_{Hkt}^{z}\right) - h_{kt}^{z} n_{t}^{z} \geq 0 \qquad \left(k \in Q, z \in Z\right) \tag{5}$$

where $Y_{Hk}^{z}(\bullet)$ is a production function of housing services and denotes the supply of type k housing services in zone z. It depends on housing capital stock H_{kt}^{z} and land input A_{Hkt}^{z}. The demand is calculated from housing services per household h_{kt}^{z}.

The flow condition of transportation services is given as;

$$J_{Rst} \equiv Y_{R}(R_{st}) - \sum_{z \in Zs} g_{Ht} \omega_{st}^{z} n_{t}^{z} - \sum_{(o,d) \in MOOD(s)} g_{Lt} L_{SFt}^{od} n_{t}^{o} \geq 0 \qquad \left(s \in M\right) \tag{6}$$

where $Y_R(\bullet)$ means the supply of transportation services on link s, and it is a function of the capital stock R_{st}. The second term is the demand related with the number of households in each zone and the third term is the derived demand of commuting. Parameter g_{Ht} is an average transportation services for urban activities per household in each zone, and g_{Lt} is an average transportation services for commuting. Parameter ω_{st}^z means a weight of link s in zone z if the link is set between different zones. Otherwise, ω_{st}^z=1. Notation $(o,d) \in MOOD(s)$ represents a set of OD pair that the route uses link s. Although there are some routes from origin zone and destination zone, the shortest route is chosen. Zs is the index of zone that relate to s.

The flow condition of natural environmental services is given as;

$$J_{Eit}^z = Y_{Ei}^z(E_{it}^z, A_{Eit}^z) - e_{it}^z(n_t^z)^{\theta_i} \geq 0 \qquad (i \in E, z \in Z) \tag{7}$$

where $Y_{Ei}^z(\bullet)$ represents the supply of type i natural environmental services in zone z and it is a function of the stock E_{it}^z and land input A_{Eit}^z. e_{it}^z is the demand of households. In general, these services have public characteristics. The degree of publicity is parameterized by θ_i $(0 \leq \theta_i \leq 1)$. If $\theta_i = 0$ then, the service is public goods and if $\theta_i = 1$ then, it is private goods. In this paper, we assume that the natural environment is a public good, i.e. $\theta_i = 0$.

(3) Time Allocation

The time allocation constraint on households in zone z is given as;

$$J_{Tt}^z \equiv T_H - f_t^z - \sum_{d \in Z} L_{SFt}^{zd} - \sum_{d \in Z} D_t^{zd} L_{SFt}^{zd} = 0 \tag{8}$$

Parameter T_H is the available time and it is allocated to leisure, labor, and commuting. f_t^z is leisure. L_{SFt}^{zd} is the labor supply and a household living in zone z offers labor to firms in zone d. The total labor supply in zone z is given by summing up these by zone d. Parameter D_t^{zd} is an average commuting time per labor and it depends on the distance from zone z to zone d.

(4) Labor

The labor constrain is defined in each OD zone as;

$$J_{Lt}^{od} \equiv L_{SFt}^{od} n_t^o - \sum_{i \in I} L_{Fit}^{od} \geq 0 \tag{9}$$

The first term is the labor supply of households n_t^o in zone o and they offer their labor to firms in zone d. The second term is the labor demand of firms in zone d.

(5) Land Constraints

The land constraint in each zone is given as;

$$J_{At}^z \equiv A_t^z - \sum_{k \in Q} A_{Hkt}^z - \sum_{i \in I} A_{Fit}^z - \sum_{i \in E} A_{Eit}^z \geq 0 \tag{10}$$

The first term A_t^z is the available land supply and it is exogenously given. The other terms are demands of housing, industry, and natural environmental services, respectively. According to the purpose of land use in each zone, we further impose additional constraints in each zone.

Housing:
$$J_{AHt}^z \equiv \overline{A}_{Ht}^z - \sum_{k \in Q} A_{Hkt}^z \geq 0 \tag{11}$$

Agriculture:
$$J_{AAt}^z \equiv \overline{A}_{At}^z - \sum_{i \in I_A} A_{Fit}^z \geq 0 \tag{12}$$

Industry: $$J^z_{AFt} \equiv \overline{A}^z_{Ft} - \sum_{i \in I_I \cup I_s} A^z_{Fit} \geq 0 \tag{13}$$

Natural Environment: $$J^z_{AEt} \equiv \overline{A}^z_{Et} - \sum_{i \in E} A^z_{Eit} \geq 0 \tag{14}$$

We impose any restriction on land use. Parameters $\overline{A}^z_{Ht}, \overline{A}^z_{At}$, $\overline{A}^z_{Ft}$, and $\overline{A}^z_{Et}$ imply the upper limits of housing, agriculture, industry, and natural environmental land uses, respectively.

(6) Population

In our framework, we assume that the population denoting the labor supply in the metropolitan area is exogenously given at any time t. The constraint on the population is given as;

$$J_{Nt} \equiv \sum_{z \in Z} n^z_t - N_t = 0 \tag{15}$$

where N_t is the number of population of the metropolitan area and it is given exogenously.

(7) Land Arbitrage Condition

$$J^z_{ut} \equiv u^z\left(f^z_t, \mathbf{c}^z_t, \mathbf{h}^z_t, \mathbf{e}^z_t\right) - u^E_t = 0 \tag{16}$$

$u^z(\bullet)$ is the instantaneous utility of a household in zone z and it is a function of a vector of consumption of general goods and services (tradable and non-tradable) $\mathbf{c}^z_t$, a vector of housing services $\mathbf{h}^z_t$, leisure f^z_t, and a vector of natural environmental services $\mathbf{e}^z_t$. u^E_t is the equilibrium utility level in the metropolitan area.

(8) Capital Accumulation

Capital accumulation equations are defined as;

General goods and services: $$I^z_{Kit} + (1 - \delta_{Kit})K^z_{it} = K^z_{Ki,t+1} \tag{17}$$

Housing: $$I^z_{Hti} + (1 - \delta_{Hit})H^z_{it} = H^z_{i,t+1} \tag{18}$$

Natural Environment: $$I^z_{Eit} + [1 - (\delta_{Ei0t} + \delta_{Ei1t} n^z_t)]E^z_{it} = E^z_{i,t+1} \tag{19}$$

Transportation: $$I_{Rst} + (1 - \delta_{Rst})R_{st} = R_{s,t+1} \tag{20}$$

where parameters $\delta_{Kit}, \delta_{Hit}, \delta_{Ei0t}, \delta_{Ei1t}$, and δ_{Rst} are depreciation rates.

The initial conditions of stock variables are also given as;
$K^z_{i1} = \overline{K}^z_i, H^z_{i1} = \overline{H}^z_i, E^z_{i1} = \overline{E}^z_i$, and $R_{s1} = \overline{R}_s$.

4. Optimal Control

4.1 The Hamiltonian Function

Applying the optimal control theory, we have the following Hamiltonian function in discrete time.

$$H_t = \frac{u^E_t}{(1+\rho)^t} + \sum_{z \in Z}\sum_{i \in I} q^z_{Kit}\left[I^z_{Kit} - \delta_{Kit} K^z_{it} \right] + \sum_{z \in Z}\sum_{i \in Q} q^z_{Hit}\left[I^z_{Hit} - \delta_{Hit} H^z_{it} \right]$$
$$+ \sum_{i \in E}\sum_{z \in Z} q^z_{Eit}\left[I^z_{Eit} - (\delta_{Ei0t} + \delta_{Ei1t} n^z_t) E^z_{it} \right] + \sum_{s \in M} q_{Rst}\left[I_{Rst} - \delta_{Rst} R_{st} \right] \tag{21}$$

where q^z_{Kit}, q^z_{Hit} , q^z_{Eit} ,and q^z_{Rst} are costate variables.

The Lagrangian (Generalized Hamiltonian) function is given as;

$$\begin{aligned} L = H_t &+ \sum_{i\in I} p_{Fit} J_{Fit} + \sum_{i\in I} p^{IM}_{Fit} J_{Mit} + \sum_{k\in Q}\sum_{z\in Z} p^z_{Hkt} J^z_{Hkt} + \sum_{i\in E}\sum_{z\in Z} p^z_{Eit} J^z_{Eit} + \sum_{s\in ML} P_{Rst} J_{Rst} \\ &+ \sum_{z\in Z} P^z_{Tt} J^z_{Tt} + \sum_{o\in Z}\sum_{d\in Z} w^{od}_t J^{od}_{Lt} + \sum_{z\in Z} p^z_{HAt} J^z_{AHt} + \sum_{z\in Z} p^z_{AAt} J^z_{AAt} \\ &+ \sum_{z\in Z} p^z_{FAt} J^z_{AFt} + \sum_{i\in E}\sum_{z\in Z} p^z_{EAt} J^z_{EAt} + \sum_{z\in Z} p^z_{At} J^z_{At} + p_{dt} J_{Nt} + \sum_{z\in Z} v^z_t J^z_{ut} \end{aligned} \tag{22}$$

where variables p_{Fit}, p^{IM}_{Fit} , p^z_{Hkt} , p^z_{Eit} , p_{Rst}, p^z_{Tt} , w^{od}_t , p^z_{HAt} , p^z_{AAt} , p^z_{FAt}, p^z_{EAt}, p^z_{At}, p_{dt}, and v^z_t are the Lagrangian multipliers usually interpreted as shadow prices. p_{Fit} is the price of the non-tradable goods and services i, and p^{IM}_{Fit} is the price of the tradable goods and services i. p^z_{Hkt}, p^z_{Eit}, and P_{Rst} are the prices of type k housing services, type i natural environmental services, and the price of transportation services on link s, respectively. p^z_{Tt} is the value of time in zone z and w^{od}_t is the wage rate that firms in zone d pay to households in zone o. p^z_{At} is the land rent when additional land use constraints are not bound. Variables p^z_{HAt}, p^z_{AAt}, p^z_{FAt}, and p^z_{EAt} mean land values associated with additional constraints of housing, industry, and natural environmental land uses. p_{dt} is the value of household (population) and v^z_t is an economic value associated with specific zone characteristics.

4.2 Optimum Conditions

For the optimality, at any planning time $t = 1, 2, \cdots, T$, we have the following necessary conditions $(z \in Z)$.

$$K^z_{i,t+1} - K^z_{it} = \frac{\partial L}{\partial q^z_{Kit}} \ (i \in I) \tag{23}$$

$$H^z_{i,t+1} - H^z_{it} = \frac{\partial L}{\partial q^z_{Hit}} \ (i \in Q) \tag{24}$$

$$E^z_{i,t+1} - E^z_{it} = \frac{\partial L}{\partial q^z_{Eit}} \ (i \in E) \tag{25}$$

$$R_{s,t+1} - R_{st} = \frac{\partial L}{\partial q_{Rst}} \ (s \in M) \tag{26}$$

These equations mean the stock accumulation equations.

$$q^z_{Kit} - q^z_{Ki,t-1} = -\frac{\partial L}{\partial K^z_{it}} \ (i \in I) \tag{27}$$

$$q^z_{Hit} - q^z_{Hi,t-1} = -\frac{\partial L}{\partial H^z_{it}} \ (i \in Q) \tag{28}$$

$$q^z_{Eit} - q^z_{Ei,t-1} = -\frac{\partial L}{\partial E^z_{it}} \ (i \in E) \tag{29}$$

$$q_{Rst} - q_{Rs,t-1} = -\frac{\partial L}{\partial R_{st}} \ (s \in M) \tag{30}$$

These equations are dynamic equations of costate variables.

$$\frac{\partial L}{\partial X^{Dz}_{Fij}} = 0 \quad (i, j \in I) \tag{31}$$

$$\frac{\partial L}{\partial X^{Iz}_{Fij}} = 0 \quad (i, j \in I) \tag{32}$$

$$\frac{\partial L}{\partial L^{od}_{Fi}} = 0 \quad (i \in I, o, d \in Z) \tag{33}$$

$$\frac{\partial L}{\partial A^z_{Fij}} = 0 \quad (i \in I) \tag{34}$$

$$\frac{\partial L}{\partial A^z_{Hk}} = 0 \quad (k \in Q) \tag{35}$$

$$\frac{\partial L}{\partial h^z_k} = 0 \quad (k \in Q) \tag{36}$$

$$\frac{\partial L}{\partial A_{Ei}^{z}} = 0 \qquad (i \in E) \qquad (37)$$

$$\frac{\partial L}{\partial c_{i}^{Dz}} = 0 \qquad (i \in I) \qquad (38)$$

$$\frac{\partial L}{\partial c_{i}^{Iz}} = 0 \qquad (i \in I) \qquad (39)$$

$$\frac{\partial L}{\partial f^{z}} = 0 \qquad (40)$$

$$\frac{\partial L}{\partial e_{i}^{z}} = 0 \qquad (i \in E) \qquad (40)$$

$$\frac{\partial L}{\partial L_{SF}^{zd}} = 0 \qquad (42)$$

$$\frac{\partial L}{\partial I_{Ki}^{z}} = 0 \qquad (i \in I) \qquad (41)$$

$$\frac{\partial L}{\partial I_{Hi}^{z}} = 0 \qquad (44)$$

$$\frac{\partial L}{\partial I_{Ei}^{z}} = 0 \qquad (i \in E) \qquad (45)$$

$$\frac{\partial L}{\partial I_{Rs}^{z}} = 0 \qquad (s \in M) \qquad (46)$$

$$\frac{\partial L}{\partial n^{z}} = 0 \qquad (47)$$

$$\frac{\partial L}{\partial u^{E}} = 0 \qquad (48)$$

These equations are the marginal conditions for the optimality. From inequality conditions, assuming that the Lagrangian multipliers are non-negative, then we have;

$$p_{Fit} J_{Fit} = 0 \qquad (i \in I) \qquad (49)$$

$$p_{Fit}^{IM} J_{Mit} = 0 \qquad (i \in I) \qquad (50)$$

$$p_{Hit}^{z} J_{Hit}^{z} = 0 \qquad (i \in Q) \qquad (51)$$

$$p_{Eit}^{z} J_{Eit}^{z} = 0 \qquad (i \in E) \qquad (52)$$

$$p_{Rst} J_{Rst} = 0 \qquad (s \in M) \qquad (53)$$

$$p_{Tt}^{z} J_{Tt}^{z} = 0 \qquad (54)$$

$$w_{t}^{od} J_{Lt}^{od} = 0 \qquad (o, d \in Z) \qquad (55)$$

$$p_{HAt}^{z} J_{AHt}^{z} = 0 \qquad (56)$$

$$p_{AAt}^{z} J_{AAt}^{z} = 0 \qquad (57)$$

$$p_{FAt}^{z} J_{AFt}^{z} = 0 \qquad (58)$$

$$p_{EAt}^{z} J_{EAt}^{z} = 0 \qquad (59)$$

$$p_{At}^{z} J_{At}^{z} = 0 \qquad (60)$$

$$p_{dt} J_{Nt} = 0 \qquad (61)$$

$$v_{t}^{z} J_{ut}^{z} = 0 \qquad (62)$$

These equations correspond with market clearing conditions for general goods and services, social infrastructure services, and labor at the market solution. If the demand is equal to the supply, then the shadow price has a positive value. If there is an excess supply, it must be zero.

At the final period, the transervality conditions are given by;

$$\frac{\partial \phi_{T+1}}{\partial K_{i,T+1}^{z}} = \eta_{Ki,T+1}^{z} \qquad (63)$$

$$\frac{\partial \phi_{T+1}}{\partial H_{i,T+1}^{z}} = \eta_{Hi,T+1}^{z} \qquad (64)$$

$$\frac{\partial \phi_{T+1}}{\partial E_{i,T+1}^{z}} = \eta_{Ei,T+1}^{z} \qquad (65)$$

$$\frac{\partial \phi_{T+1}}{\partial R_{s,T+1}} = \eta_{Rs,T+1} \qquad (66)$$

where the right hand sides represent the marginal values of capital stock at period $T+1$ respectively. These correspond with shadow prices of capital stocks at period T.

5. Commuting Cost and Income

From the optimum conditions, we have some implications about household behavior.

(1) Wage and Commuting Costs

From $\frac{\partial L}{\partial L_{SF}^{zd}} = 0$, we have;

$$w_t^{zd} = p_{Tt}^z + p_{Tt}^z D_t^{zd} + \sum_{s \in M(z,d)} p_{Rst} g_{Lt} \tag{67}$$

where $M(z,d)$ represents that a set of links which are used by households who commute from zone z to zone d. This explains the relation between wage and commuting costs. The wage is the sum of the time value of labor (the first term in the right hand side), the time value of commuting (the second term), and the transportation cost of infrastructure (the third term). This means that the transportation costs are paid by firms and the wage contains transportation costs.

(2) Income

The income constraint of household is given as from $\frac{\partial L}{\partial n^z} = 0$, and $\frac{\partial L}{\partial u^E} = 0$.

$$\sum_{d \in D} w_t^{zd} L_{St}^{zd} + p_{dt} - h_{et} = 0 \tag{68}$$

The first term in the left hand side is the wage. The second term represents revenues from capital income, and redistributed income from firms and government being denoted by p_{dt}. The right hand side is the expenditure of household and it is given as

$$h_{et} = \sum_{i \in I} p_{Fit} c_{it}^{Dz} + \sum_{i \in I} p_{Fit}^{IM} c_{it}^{Iz} + \sum_{i \in I} p_{Hkt}^z h_{kt}^z + \sum_{s \in S} p_{Rst} g_{Ht} \omega_{st}^z \tag{69}$$

The fist and second terms mean the consumption of general goods and services (non-traded and traded). The third term is the cost for housing services. The fourth term is commuting costs and the last term is the expenditure for natural environmental services.

6. Assumptions for the Simulation

To solve the above optimization problem, we specify the utility, production, costs adjustment, and assets functions (see Appendix). And then the numerical computation method of optimal control theory is applied. Some numerical experiments are conduced to evaluate the compactness of the city under the following conditions.

(1) Period, Zone, and Sector

The planning period is set as $T = 5$ (25 years). The city is assumed to have two dimensions and it is subdivided into 25 zones. The economy is classified into three general sectors (agriculture, industry, and services). The housing services have two types, i.e. low-rise and high-rise houses. The natural environmental service is only one type (as local public goods). The transportation services for goods are ruled out, but the commuting is implemented into the model.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Figure 2 Location of Zones

(2) Initial Conditions of the Urban Spatial Structure

In this model, the urban spatial structure strongly depends on the initial conditions of capital stocks. As a benchmark, we consider a situation of urban dispersed activities, i.e. 'urban sprawl'. This is a typical phenomenon in developed countries at the present. Based on the flowing scenarios, the initial values of capital stocks are determined exogenously. For example, assuming that the weights of initial values of services sector set relatively large around the central and suburban areas, the recent tendency of the location of large shopping center would be reproduced.

(3) Scenarios of the Simulation

In our simulation, we assume two simulation cases.

In case 0, we assume that the natural environmental resources mainly exist in the suburban areas, but a part of those exist in the center of the city as well. The industrial and services sectors are located in both central area and suburban areas. Most of the housing services is low-rise in the metropolitan area. In the near future, the dispersion tendency will continue.

In case 1, we simulate the urban configuration where the dispersed economic activities are concentrated in the central area. This case presents an example of a formation of a compact city.

The difference of these cases is expressed by only changing the parameters in the asset value function at the end of the period.

7. Results of the Simulation

(1) Population

Changes in the population are shown in Figure 3. The population density is set very high in the central area of the city at the first period.

At the final period in case 0, the population in the center of the city decreases, and the increasing tendency in the population of suburban areas are observed in Figure 3.

At the final period in case 1, the populations of the center of the city and its surrounding areas show a large increase. Hence, the dispersion tendency is suppressed as compared with case 0. The reason of this fact is interpreted as that the people prefer the short commuting time when the economic activities concentrate in the central area and its surrounding areas. Moreover, it can be said that these reproduce the phenomenon of the recurrence of the senior citizen to the central area of the city.

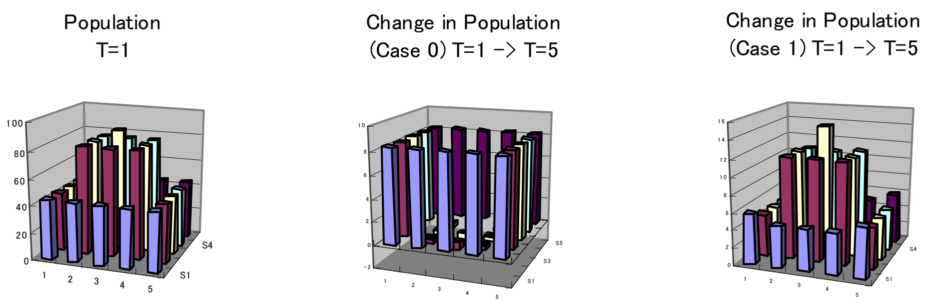

Figure 3　Population

(2) Demand for Housing Services

Figure 4 shows changes in the amount of housing service demand. The amount of the low rise housing service demand is set more than the high rise housing at the beginning of the period.

At the final period in case 0, the amount of the low rise housing demand decreases, and the amount of the high rise house demand shows an increase.

At the final period in case 1, the amount of the low rise housing demand decreases in all zones as compared with that in the case 0. The increase rate of high rise housing demand is higher in the central area of the city than in suburban areas. These can be interpreted as that the low rise houses were converted to the high rise houses to use land effectively when the population concentrates in the central area.

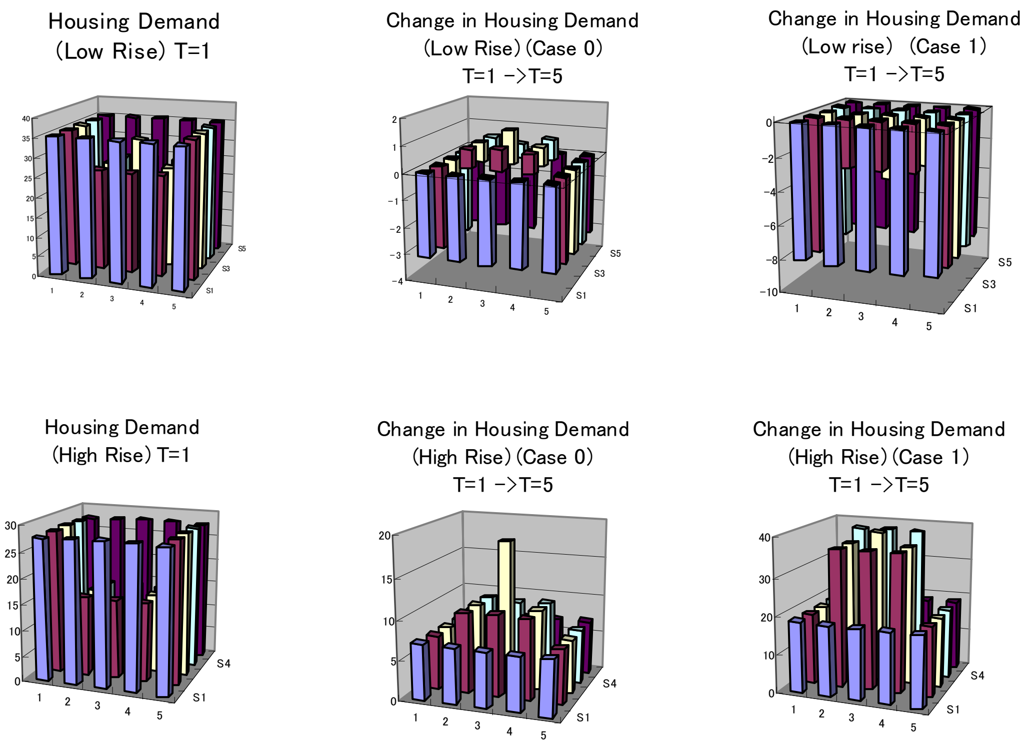

Figure 4 Amount of Housing Service Demand

(3) General Industrial Capital Stock

Figure 5 shows changes in the capital stocks of the general industries. The initial values of the industrial capital stocks determine the future urban configuration.

At the final period in case 0, each general industry in each zone increases its industrial capital stock.

At the final period in case 1, the accumulation of capital stocks of industrial and service sectors section is remarkable around the central area. This implies that these sectors concentrate into the central area.

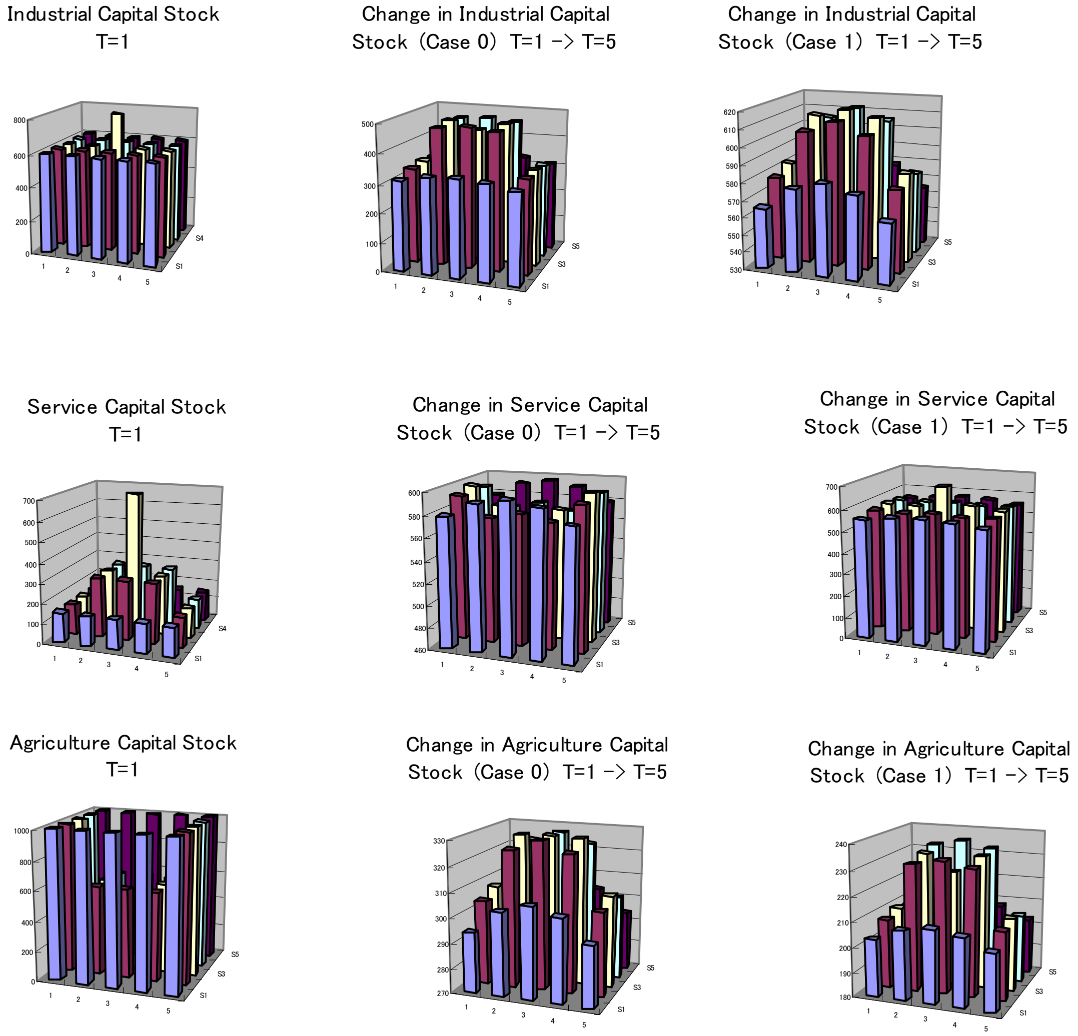

Figure 5 General Industrial Capital Stock

(4) Natural Environmental Capital Stock

Figure 6 depicts changes in the natural environment capital stock. As for the natural environmental capital stock at the initial period, the central area of the city and its surrounding areas are set with lower environmental capital stocks than in the suburban areas.

At the final period in case 0, the natural environmental capital stock in the central area is greater than in the suburban areas.

At the final period in case 1, comparing with case 0, the natural environmental capital stock increases in the central area, and it gets lower in surrounding areas than in the suburban areas.

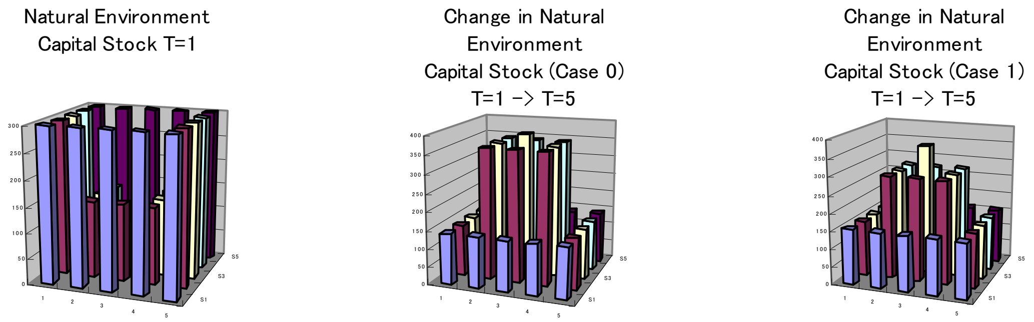

Figure 6 Natural Environment Capital Stock

(5) Equilibrium Utility Level

Figure 7 presents changes in the equilibrium utility level. In both of two cases, the equilibrium utility level shows an increase tendency. Particularly, the equilibrium utility level in case 1 is higher than that in case 0. This is interpreted as that the increases in leisure time and the decreases in commuting time make the equilibrium utility level higher, when the population is concentrated in the central area of the city. That is, a formation of a compact city gets higher the equilibrium utility level. This point is highlighted in this study.

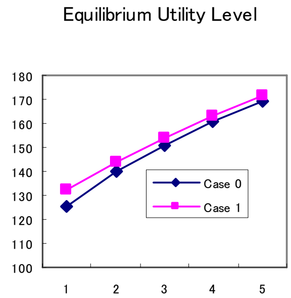

Figure 7 Equilibrium Utility Level

8. Concluding Remarks

In this paper, an intertemporal urban economic model with natural environment has been developed, and the numerical simulations of the two cases have been carried out. We can have obtained an interesting result such as that the household instantaneous equilibrium utility increases as the city gets more compact. In addition, it has also been obtained that the introduction of natural environment contributes to the formation of a compact city. As for the natural environment, a more realistic specification may be necessary, e.g. incorporating the dynamic interaction between different types of natural environment. Areas worth examining in the future include introduction of material recycling, agglomeration economy and diseconomy, and reproduction of natural resources. Finally, this study is financially supported by the Grant-in-Aid for Scientific Research (C)(2) of the Minister of Education, the Government of Japan (No. 16510021).

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Appendix: Specification of Functions

Utility Function

$$u^z\left(f_t^z, \mathbf{c}_t^z, \mathbf{h}_t^z, \mathbf{e}_t^z\right) = (f_t^z)^{\alpha_{Ft}} \prod_{i \in I} (c_{it}^z)^{\alpha_{cit}} \prod_{k \in Q} (h_{kt}^z)^{\alpha_{hkt}} \prod_{i \in E} (e_{it}^z)^{\alpha_{ekit}}$$

Composite Consumption

$$c_{it}^z = (c_{it}^{Dz})^{\alpha_{Dt}} (c_{ti}^{Iz})^{\alpha_{It}} \qquad (i \in I)$$

Production Function of General Industry

$$Y_{Fi}^z(l_{Fit}^z, K_{Fit}^z, A_{Fit}^z, \mathbf{X}_{Fi1}^z, \cdots \mathbf{X}_{Fit}^z) = A_{it}^z (l_{Fit}^z)^{\beta_{LFti}^z} (K_{Fit}^z)^{\beta_{KFti}^z} (A_{Fit}^z)^{\beta_{lFit}^z} \prod_{j \in I} (X_{Fijt}^z)^{\beta_{xFijt}^z}$$

Composite Input

$$X_{Fijt}^z = (X_{Fijt}^{Dz})^{\beta_{Dit}^z} (X_{Fijt}^{Iz})^{\beta_{Iit}^z} \qquad (i, j \in I)$$

Composite Labor Input

$$l_{Fi}^z \equiv \prod_{o \in z} \left(L_{Fi}^{oz}\right)^{\xi_{ot}}$$

Production Function of Housing Services

$$Y_{Hk}^z\left(H_{kt}^z, A_{Hkt}^z\right) = \overline{A}_{Hkt}^z (H_{it}^z)^{\beta_{KHit}^z} (A_{Hit}^z)^{\beta_{lHit}^z} \qquad (i \in H)$$

Production Function of Natural Environmental Services

$$Y_{Ei}^z(E_{it}^z, A_{Eit}^z) = \overline{A}_{Eit}^z (E_{it}^z)^{\beta_{KEit}^z} (A_{Eit}^z)^{\beta_{lEit}^z} \qquad (i \in E)$$

Assets Function

$$\phi(T+1) \equiv \sum_{z \in Z} \sum_{i \in I} \eta_{Ki}^z K_{i,T+1}^z + \sum_{z \in Z} \sum_{i \in Q} \eta_{Qi}^z H_{i,T+1}^z + \sum_{z \in Z} \sum_{i \in E} \eta_{Ei}^z E_{i,T+1}^z + \sum_{s \in ML} \eta_{RS} R_{s,T+1}$$

Adjustment Costs Function

$$c_{Ki}(x) = c_K x^{\varepsilon_K} \qquad (i \in I) \qquad\qquad c_{Hi}(x) = c_H x^{\varepsilon_H} \qquad (i \in I)$$

$$c_{Ei}(x) = c_E x^{\varepsilon_E} \qquad (i \in I) \qquad\qquad c_{Ri}(x) = c_R x^{\varepsilon_R} \qquad (i \in I)$$