

# **Applying Multi-Hour Traffic Assignment to Estimate Trips Distribution between Hours and the Intrinsic Value of Hour of the Day**

Haim Aviram<sup>1</sup>

## **Abstract**

Recent analysis regarding the patterns and composition of trips in morning peak hour revealed that an increasing part of these trips correspond to non-work related trips. In general, these trips are not specifically associated with morning peak. Other studies show that price elasticity of leisure trips is considerably higher than that of commuting and business trips. According to Wardrop's first principle, if the transportation network maintains equilibrium, then no user could improve his situation by selecting alternative travel route. Expanding on that principle, the fact that non-work related trips (such as shopping, family visits, tourism) take place in the morning peak hour, under congestion conditions, means that the specific hour, not only the duration of time and trip purpose, is important to the user. Hence, each hour of the day has its own intrinsic value, named "Intrinsic Hourly Value" (IVH). In order to estimate this value, calibration of a multi-hour equilibrium assignment is suggested to determine the magnitude and characteristics of the IVH.

Several hypotheses have been tested with regard to the IVH. For example, significant positive values were estimated for morning IVH.

The methodology was applied using a symbolic network. A clear method to estimate the IVH is presented. The results can assist public authorities to decide on policies such as activities time rescheduling or the application of congestion tolls.

---

<sup>1</sup> Technion, Israel Institute of Technology, Faculty of Architecture and Town Planning and the Jordan Valley College, Israel

Currently applied transportation models use disaggregate models for the determination of the demand for trips (Ben Akiva and Lerman, 1985). Such models are specifically engaged with the split among vehicle types or modal split. According to these models the user forms his demand choosing among various alternatives. The observed aggregate demand is a function that reflects the decision making process carried out individually by many users.

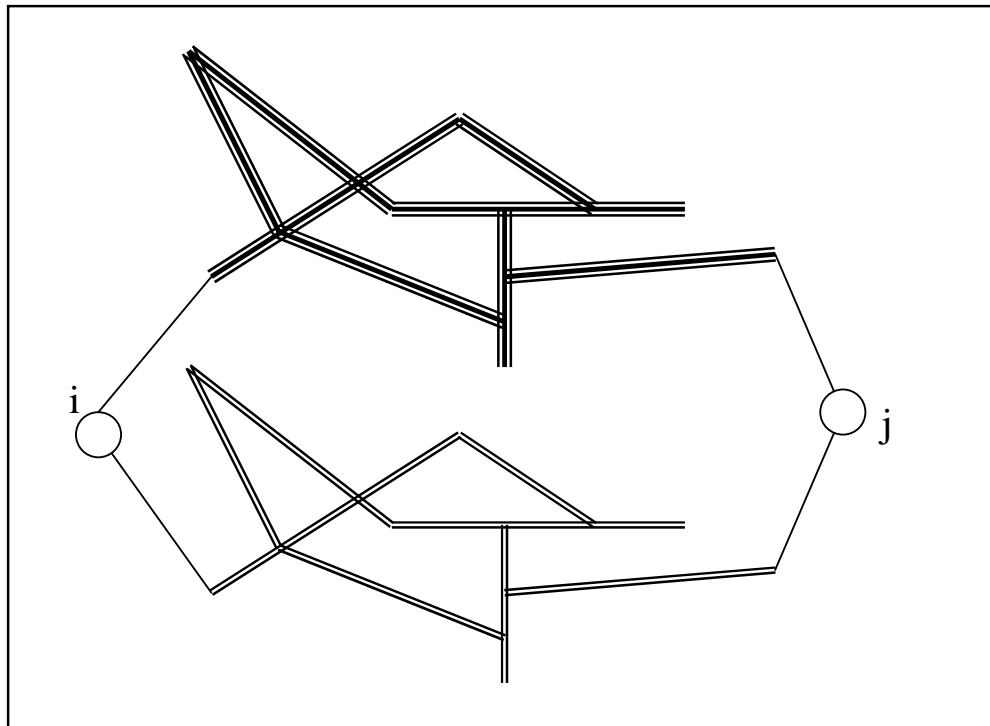
The economic interpretation of “demand” means quantity and price. In transportation, however, many additional variables are involved. Some of those refer to the location of the demand: its origin and destination and its spatial distribution, both – connecting pairs of origin and destination zones and selecting preferred routes. Other variables refer to the transport mode choice. Not much has been developed regarding the time chosen for the trip. This paper will discuss this subject aiming in the determination of time equilibrium conditions and evolving a method to estimate a specific value that should be assigned to the particular time chosen, based on equilibrium of traffic assignment. Our interest in time and value of time is not the classical view drawn by Backer (1965) but the location of time in a specific period of the day.

The basic concept of the model parts from the hypothesis that some people are able to choose their trip hour freely. According to The National Households Survey in Israel, about 25% of morning peak-hour trips are not related to commuting or to education. Even commuters, who travel at morning peak hour, could wake up earlier and drive to work at 6 AM instead of 7 AM. Of course, for some of them it means to waste an hour, but for others this change is a real choice, sometimes a feasible one. This argument is not merely theoretical. In several congested cities a continuous reduction in peak hour portion of daily traffic is observed. Thus, an assumption regarding certain equilibrium between hours, e.g. 6-7 AM and 7-8 AM is supported by evidence and should be accepted. If so, what is the reason for the difference in congestion conditions between off-peak and peak hour. Why can't we discover the same travel time at both periods?

The answer to that question is somehow complicated. If we consider the first Wardrop's principle, there is no obvious reason for that phenomenon. According to

Wardrop(1952), after reaching equilibrium, no user can improve his situation by changing his route (Holden, 1989).

Fig. No. 1: Equilibrium between two periods of time

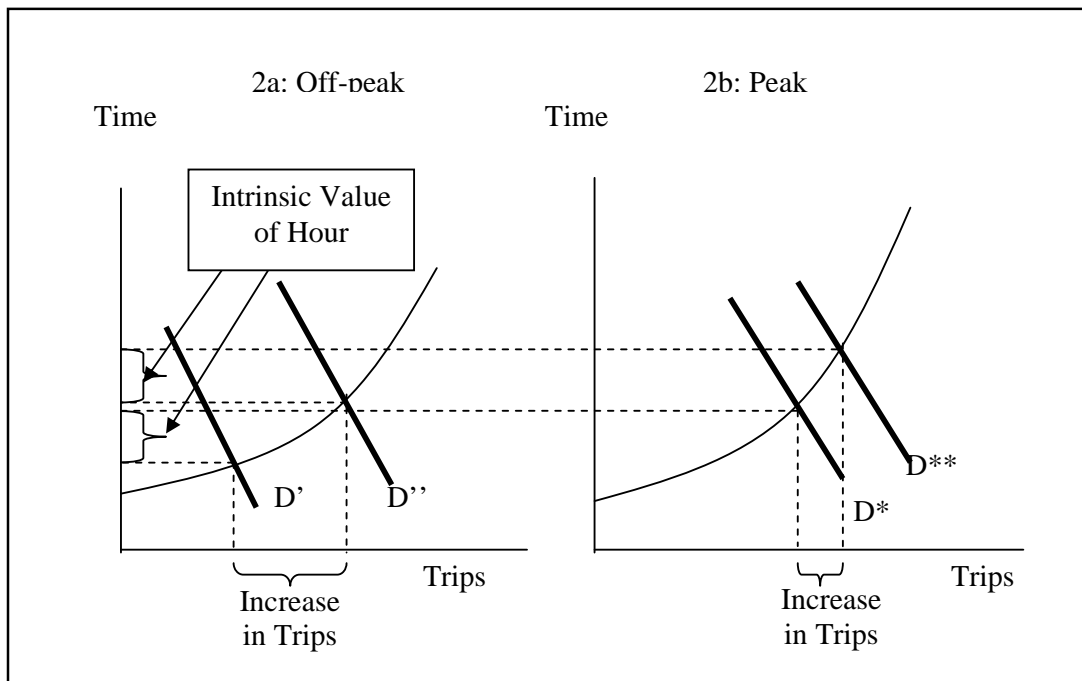


Usually, and it will be discussed later – the simple way of improving user's situation means reducing the trip time. Fig. No. 1. presents a situation where equilibrium is achieved between two hours. Users who travel between zone i and zone j are allowed not only to choose their route but also the period in which they will travel. They might select the first period, which is presented by a triple line network or the second period, through the double line network. As well, they can choose among routes within these networks. It should be noted that all the discussion is limited to the fixed demand hypothesis and no induced traffic is considered (Goodwin, 1996 and Goodwin, 1997). If no further hypotheses are made, it is obvious that each network will carry exactly half of the trips, using the same routes, and both will be loaded equally. However, we know that in reality this phenomenon does not happen, and each network carries different amounts of trips. Is this a contradiction to Wardrop's principle? not necessarily. Remembering that the use of time as the base for the selection of route is

only a simplification of the overall utility of the user where time and money plays a similar role (Leurent, 1994), We can easily replace the target of minimum time by a maximum utility one. In that case – the difference between the loads on the two networks indicates the existence of an additional benefit that justifies traveling on the more congested network. We may denominate this extra benefit “Intrinsic Value of Hour” (denoted IVH), since this value reflects an additional benefit, which refer solely to the hour in which the trip is carried out. It might be interesting to estimate this value, given that it represents the value of the marginal utility to the user in selecting specific period for his trip. Moreover, in order to manage the demand for different periods, this IVH should be studied and estimated. Specific tools, which are sensitive to the choice among periods, may help in combating congestion, using variables that influence the period choice made by the user.

Suppose that the IVH leads to a certain equilibrium. After several years, the congestion level increases and hence a new level of equilibrium is achieved. Due to the non-linearity of the trip time function (decreasing elasticity of the cost function when the number of trips increases), the new equilibrium will shift trips from the congested to the less congested network, or from peak to off-peak hour.

Fig. No. 2: The impact of increasing demand on the equilibrium



In the first year, the demand for trips from zone i to zone j at off-peak is  $D'$  while at peak is  $D^*$ . Marginal users agree to pay the different, equal to the value of the extra time they spend on each trip. This difference is exactly the “Intrinsic Value of Hour” (IVH). After several years, the demand functions increase to  $D''$  at off-peak and  $D^{**}$  at peak. Considering that there is no change in the IVH, the increase in trips at off-peak is greater than at peak. Empirical evidence support this phenomenon.

In order to calibrate the IVH using traffic assignment procedure, we should assign to the link that connects zone i to the network a value that represents the IVH. If the assignment results in too low or too high peak traffic, this value should be changed accordingly, until the equilibrium between hours is achieved. The values that were added to each traffic zone, in order to estimate the new equilibrium are the required IVHs. By this procedure the shift of traffic from peak to off-peak periods can be detected and predicted.

A required condition for the existence of this equilibrium is that at least a certain part of the users have a real choice between different hours.

To show the way of estimating IVH and reaching the above mentioned equilibrium, suppose, first, the simplest network possible, formed merely by a single link, one Km. Long, that connects zones i and j. The capacity of this link is 2000 PCU. Consider also that traffic is calibrated for this year, and a forecast for five years is needed. Let us assume that present off-peak traffic is 1,000 PCU per hour and present peak-hour traffic is 1,800 PCU. The annual rate of traffic increase is 2%. The impedance function used is similar to the BPR one:

$$(1) \quad t = t_0 [1 + 0.15(v/c)]^4$$

Where:  $t$  is the trip time in minutes

$t_0$  is the trip time under free-flow conditions in minutes

$v/c$  is the volume to capacity ratio.

$t_0$  is estimated to be 1. According to the given traffic data, trip time  $t$  will be 1.65 minutes at peak hour and 1.34 minutes at off-peak. Thus, the Intrinsic Value of Hour (IVH) is equal to 0.31 minutes.

After five years total traffic in PCU will increase to 3,091. It will consist of 1,920 PCU at peak hour and 1,171 at off-peak. Trip time at peak will be 1.71 minutes versus 1.40 at off-peak, maintaining the IVH equivalent to 0.31 minutes.

It is worthwhile to focus on the changes in traffic. While the number of trips at peak hour increases in 120 (6.7%), at off-peak it raises by 171 (17.1%). This is a numerical expression of the idea presented in fig. 2.

An extension of the example is presented using a symbolic network that includes several links. Note that in the case of a network the determination of IVH is not necessarily unique, since it is quite probable that the IVH between zones i and j will differ from the IVH between zones i and k. This could result from many different reasons, such as differences in trip purposes, in socioeconomic characteristics of the population etc. Hence, the use of a single value is a restriction that, theoretically, can be easily removed. For every zone a different value is calculated. Technically, these average values, in terms of minutes, were added to the time assigned for all trips that use the off-peak period, resulting in trip times that satisfy the following equation:

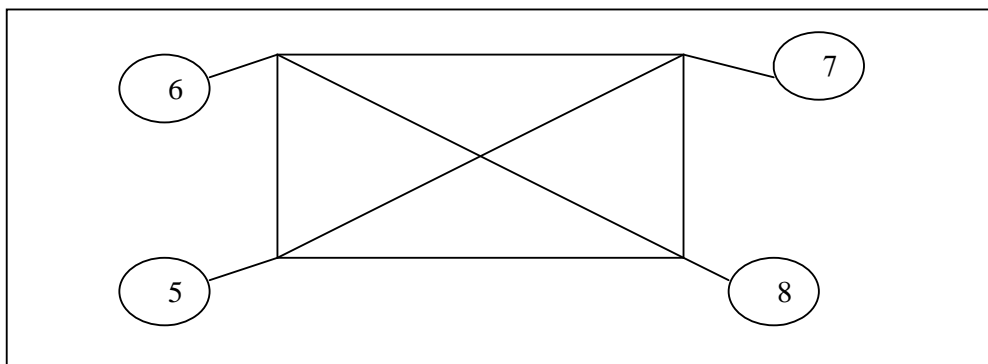
$$(2) \quad IVH_{ik} + VOT_{ik} = IVH_{jk} + VOT_{jk} + e_{ijk} \quad \forall i,j,k$$

Where i and j stand for different periods, k is a zone indicator and  $e_{ij}$  is a statistical error. In other words, the values related to distinct periods of the day compensate for the differences in trip times and hence, by adding those, equilibrium can be achieved. However, for the purpose of the presentation of the idea, a unique simple average was used to represent all values of IVH to all trips originated in zone I at peak hour. In the case of large network equilibrium it can be argued that the composition of trips, the environmental nature and even the socioeconomic attributes of users, should not differ substantially between zones and therefore the index k of the IVH may be omitted. This assumption results in the determination of at most 24 IVH values to calibrate traffic between day periods.

$$(3) \quad IVH_i + VOT_{ik} = IVH_j + VOT_{jk} + e_i \quad \forall i,j,k$$

This argument was further checked on a simplified network, presented in fig. No. 3.

Figure No. 3: A Simplified Network



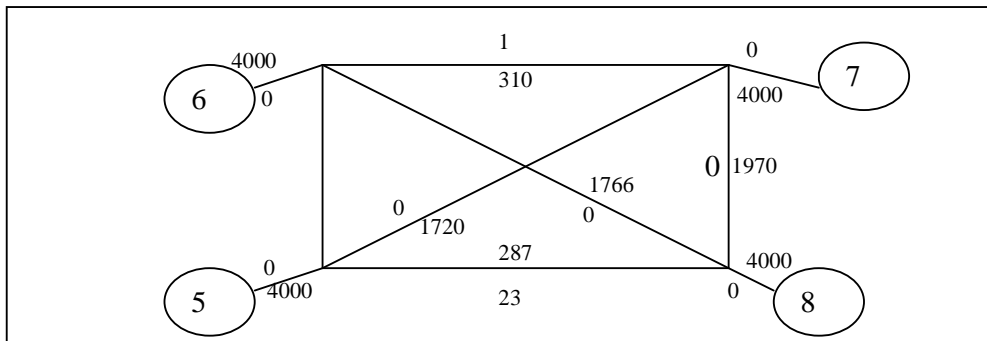
The Origin Destination matrix that describes morning peak flows between transportation zones is the following:

Table No. 1: Origin Destination Trips Matrix (Morning Peak Hour)

Destination Origin	5	6	7	8
5	0	2000	2000	0
6	0	0	0	0
7	0	0	0	0
8	0	2000	2000	0

Trips were assigned on the simplified network using the transportation modeling package TransCad. The first step was the assignment of morning peak hour (7-8 AM) flows. The results are presented in figure No. 4.

Figure No. 4: Peak Hour Assignment Results



Then, a matrix of two hours (7-9 AM) was set up and was assigned on a network composed by two identical parts: one representing 7-8 AM and the other for the 8-9 period. Obviously, since total trips in these two hours is less than double morning peak demand, the traffic volumes obtained on the 7-8 AM links of the network are lower than the correct volumes for that hour. In the other network, belonging to 8-9 AM, flows are over-estimated.

Now, the procedure of calibrating the IVH began. Using trial and error procedure, the impedance values of the centroid connectors of the 8-9 AM matrix were increased until a correct total trips number in morning peak hour was achieved. The procedure continues until the total number of trips in the morning peak part of the doubled

network equals the number of trips in the peak hour sole assignment. The procedure used values of IVH as described in table No. 2.

Table No. 2: IHV Values Calibration

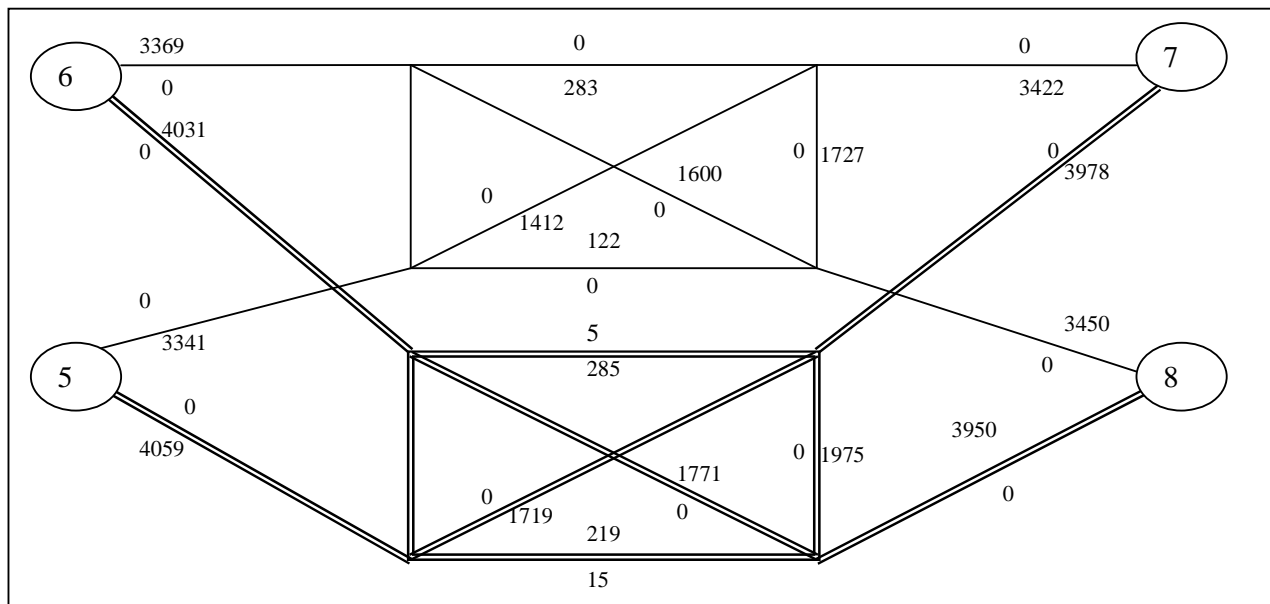
1	2	3	4	5	6	7
Iteration	% of peak traffic	Penalty off-peak connectors (minutes IVH)	Demand on peak hour connectors	Proportion of original peak(%)	Demand on off-peak network	Total traffic on links
0	200	0	8001	92.51	7999	16000
1	185	0	7399	92.49	7401	14800
2	185	8	8220	102.75	6580	14800
3	185	4	7813	97.67	6987	14800
4	185	6	8019	100.24	6781	14800
5	185	5.9	8010	100.12	6790	14800

The left column in table no.1 counts the iterations used through the calibration.

Column 2 indicates the total traffic assigned. The off-peak traffic considered was 85% of peak volume. Penalty (col. 3) is the increase of the impedance assigned to off-peak connectors. At equilibrium, these are the IVHs.

Traffic flows of this assignment are presented in fig. No. 5. Note that the peak hour network is marked with double line while the off-peak with single line.

Figure No. 5: Final Two Hours Assignment With IVH





The procedure assumes unique IVH for all origins. Equal IVHs were introduced to the centroe connectors of the off-peak network for the bi-hour assignment.

In order to revise whether a single factor is suitable and that no specific factors are required on a zone level, two tests were carried out. The first was a comparison between single peak hour assignment and peak hour in the bi-hour assignment. This comparison is presented in table No. 3.

Table No. 3: Comparison Between Traffic Volumes on Links

Peak Hour Assignment	Peak Hour in Bi-Hour Assignment	Difference	Percentage of Difference (%)
4000	4031	-31	-0.77
1	5	-4	-400.00
310	285	25	8.06
4000	3978	22	0.55
1970	1975	-5	-0.25
4000	3950	50	1.25
287	219	68	23.69
23	15	-8	34.78
1720	1719	1	0.06
4000	4059	-59	-1.47
1766	1771	-5	-0.28

The differences between the results of traffic do not seem considerable. However, the simple null hypothesis of equal values, tested by Chi-Square test is rejected at a level of 2.5%. It is worthwhile to mention that the Pearson correlation coefficient between both estimates exceeds 99%.

The second check was an assessment of the ratio between peak and off-peak traffic counts in different links. According to the test matrices, the overall ratio between off-peak and peak was be 85%, hence the expected difference should be 15%. Results show that this assumption is far from being accepted. The comparison of these values is presented in table no. 4.

Table No. 4: Peak to Off-Peak Volumes Ratio in Selected Locations

Peak Hour Volume	Off-Peak Hour Volume	Difference	Percentage of Difference
4031	3341	900	22%
5	0	5	100%
285	283	2	1%
3978	3422	556	14%
1975	1727	248	13%
3950	3450	500	13%
219	122	97	44%
15	0	15	100%
1719	1412	307	18%
4059	3341	718	18%
1771	1600	171	10%

This paper brought up a first attempt to distinguish between the values of time assigned to the same activities in different hours. It was demonstrated that such values exist. However, a simple example has revealed that an overall single value is not capable to cover the different aspects of this issue. Following stages of the research will include tests on real networks as well as the use of diverse IHV.

#### References

- Backer, E.S. (1965), A Theory of the Allocation of Time, The Economic Journal, vol. 75, pp. 493-517
- Ben Akiva, M. and Lerrman, S. R. (1985), Discrete Choice Analysis: Theory and Application to Travel Demand, MIT Press, Cambridge, Ma.
- Goodwin, P. H. (1997), Solving Congestion, Inaugural Lecture for the Professorship of Transport Policy, University College, London.
- Goodwin, P. H. (1996), Empirical Evidence on Induced Traffic, a Review and Synthesis, Transportation, vol. 23, pp. 35-54.
- Holden, D. J.(1989), Wardrop's Third Principle, Journal of Transportation Economics and Policy, pp. 239-262.

Leurent, F. (1994), Cost Versus Time Equilibrium Over a Network, TRB 1443, pp. 84-91.

Wardrop, J. (1952), Some Theoretical Aspects of Road Traffic Research, Proceedings of the 4<sup>th</sup> Congress of the Institute of Civil Engineers, Part II, pp. 325-378.