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# Endogenizing the Reservation Value in Models of Land Development Over Time and Under Uncertainty[1]

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## Abstract

The notion of a *reservation value* is a key feature of most contemporary dynamic and stochastic models of land development. It is clear that the magnitude of the reservation value has a fundamental bearing on the decision to develop or preserve land. This notwithstanding, many papers that analyze land development in a dynamic and stochastic setting treat a landowner's reservation value as an exogenous variable. Therefore, the purpose of this paper is to *endogenize* the reservation value in the context of a model of land development over time and under uncertainty. Our analysis shows that the optimal reservation value is the solution to a specific maximization problem. In addition, we also show that there exist theoretical circumstances in which the optimal reservation value is unique.

Keywords: Dynamics, Endogenous Reservation Value, Land Development, Uncertainty

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## 1. Introduction

The question of land development in an intertemporal setting has interested economists and regional scientists at least since Weisbrod (1964). Since then, researchers such as Markusen and Scheffman (1978), Arnott and Lewis (1979), and Capozza and Helsley (1989) have studied various aspects of the land development question in a deterministic environment. However, we now know that when the land development decision is irreversible, the use of a certainty framework will bias results about when land ought to be developed. In fact, as we have learned from the investment under uncertainty literature,[4] uncertainty will generally impart an option value to undeveloped land and delay the development of land from, say, agricultural to urban use. Therefore, if we are to really understand when land ought to be developed in the presence of an irreversibility, it is essential that we explicitly account for uncertainty.

Recently, Capozza and Helsley (1990), Batabyal (2003), Batabyal and Yoo (2003) and others have examined the question of land development over time and under uncertainty. In the context of a "first hitting time" problem,[5] Capozza and Helsley (1990) show that land ought to be converted from rural to urban use at the first instance in which the land rent exceeds the reservation rent. Batabyal (2003) first supposes that a landowner has a reservation value in mind, say $\$A$, below which he will not agree to develop his land. Batabyal then shows that this landowner's decision rule is to accept the first bid to develop land that exceeds $\$A$. Batabyal and Yoo (2003) analyze the properties of a decision rule that calls for land development as long as the dollar value of a bid exceeds a

[4] For more on this literature, see Pindyck (1991), Dixit and Pindyck (1994), and Hubbard (1994).

[5] For additional details on this, see Dixit and Pindyck (1994, pp. 83-84) and Ross (1996, pp. 363-366).

stochastic reservation level of revenue. These authors show that although the likelihood of developing land with the above decision rule is always positive, on average, a landowner who uses this decision rule will always end up preserving his land.

As this brief review of the theoretical literature shows, many models of land development over time and under uncertainty have utilized the notion of a reservation value. In addition, the work of Barnard and Butcher (1989), Tavernier and Li (1995), and Tavernier *et al.* (1996) tells us that even the empirical literature on land development has made use of the concept of a reservation value. A perusal of these theoretical and empirical papers tells us that the magnitude of a landowner's reservation value has a significant impact on the decision to develop or preserve land. This notwithstanding, in most of the papers that we have just discussed, the reservation value is exogenous to the analysis. Consequently, we use a theoretical model of land development over time and under uncertainty to *endogenize* the reservation value. Our subsequent analysis will demonstrate that a landowner's optimal reservation value is the solution to a particular maximization problem. We shall also show that there exist theoretical circumstances in which this optimal reservation value is unique.

The rest of this paper is organized as follows. Section 2.1 provides a detailed description of the theoretical framework. Section 2.2 uses this framework to set up a maximization problem for our landowner. Section 2.3 shows that the optimal reservation value is the solution to the above maximization problem. Section 2.4 presents a numerical example and discusses the dependence of our results on the underlying assumptions. Section 3 concludes and offers suggestions for future research on the subject of this paper.

## 2. Land Development Over Time and Under Uncertainty

### *2.1. The Theoretical Framework*

Our model is based on the discussion in Batabyal (2003), Batabyal and Yoo (2003), and Ross (2003, pp. 288-301). Consider a landowner who owns a plot of land. The decision problem faced by this owner concerns when to develop his plot of land. Consistent with the analysis in Batabyal (2003) and in Batabyal and Yoo (2003), we suppose that the development decision is indivisible. In other words, the possibility of partial development of the plot is excluded. The landowner solves his problem in a dynamic and stochastic setting. The setting is stochastic because the decision to develop depends fundamentally on the receipt of non-negative and dollar-valued bids to develop land. Following Batabyal (2004), we suppose that these bids are received in accordance with a Poisson process with rate $\varphi$.[6] The decision making framework of our paper is dynamic in the sense that this framework requires the landowner to decide when land ought to be developed on the basis of his observations—over time—of the Poisson bid receipt process.

To keep the subsequent analysis interesting, we suppose that each bid to develop land is the value of a continuous random variable with density function $h(b)$. Now, once a bid is received by our landowner, he must decide whether to accept it (agree to develop his land) or reject it (preserve his land) and wait for additional bids. When our landowner decides to preserve his land, he incurs benefits and costs. The benefits arise from things like the preservation of the option to develop land later and the costs arise from things like the need to prevent encroachment and the need to maintain the plot of land under study. As such, when a decision to preserve land has been made, our landowner incurs net costs (in \$) at a rate of $c$ per unit time until the land is developed.

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In addition to the literature on land development over time and under uncertainty, the Poisson process has been widely used in the natural resource economics and mathematical ecology literatures. For a more detailed corroboration of this claim, see Uhler and Bradley (1970), Peilou (1977), Arrow and Chang (1980), and Batabyal (2004). For lucid textbook accounts of the Poisson process, see Ross (1996, pp. 59-97) or Ross (2003, pp. 269-348).

Our landowner's objective is to maximize his expected total profit where the total profit equals the dollar amount received upon acceptance of a bid less the net cost incurred. Now, consistent with the approach taken in Cappoza and Helsley (1990), in Batabyal (2003) and in Batabyal and Yoo (2003), we suppose that our landowner's reservation value is $r$ and that this landowner will accept the first bid that exceeds $r$ in dollar terms. The task before us now is to endogenously determine the optimal value of this reservation value $r$.

***2.2. The Maximization Problem***

To determine the optimal value of $r$, we shall first calculate our landowner's expected total profit when the decision rule described in the previous paragraph is used and then we shall choose the value of $r$ that maximizes the expression for our landowner's expected total profit.

Let $B$ denote the value of an arbitrary bid and let $H^c(b)$ denote the tail distribution of this bid. In symbols, we have $H^c(b) = Prob\{B > b\} = \int_b^\infty h(w)dw$. Now note that each bid will exceed the reservation value $r$ with probability $H^c(r)$. Therefore, we can tell that these sorts of bids will be received by our landowner in accordance with a Poisson process with rate $\varphi H^c(r)$. Accordingly, the time until a particular bid is accepted by our landowner is an exponentially distributed random variable with rate $\varphi H^c(r)$.

Now let us denote the total profit from the decision rule that involves accepting the first bid that exceeds $r$ by $\Pi(r)$. Then it should be clear to the reader that the expectation of this total profit is $E[\Pi(r)] = E[\textit{accepted bid}] - E[c(\textit{time until bid accepted})]$. Mathematically, the equation we get is

$$E[\Pi(r)] = E[B/B > r] - \frac{c}{\varphi H^c(r)}. \qquad (1)$$

The conditional expectation on the right-hand-side (RHS) of equation (1) can be simplified further by using the notion of a conditional density function. This simplification yields

$$E[B/B>r] = \int_0^{\infty} b h_{B/B>r}(b)db = \int_r^{\infty} b \frac{h(b)}{H^c(r)} db. \qquad (2)$$

Using equation (2), we can now rewrite our landowner's objective function, i.e., equation (1). We get

$$E[\Pi(r)] = \frac{\int_r^{\infty} bh(b)db - c\varphi^{-1}}{H^c(r)}. \qquad (3)$$

We now have our landowner's objective, i.e., expected total profit in the form in which we would like. To determine the optimal reservation value, our landowner will need to choose $r$ to maximize the RHS of equation (3). We now turn to this task.

### *2.3. The Optimal Reservation Value*

As indicated in the previous section, to compute the optimal $r$, our landowner solves

$$\max_{\{r\}}\left[\frac{\int_r^{\infty} bh(b)db - c\varphi^{-1}}{H^c(r)}\right]. \qquad (4)$$

Taking the derivative of the maximand in equation (4) and then setting it equal to zero gives us the first order necessary condition for an optimum. We get

$$\int_r^{\infty} bh(b)db - \frac{c}{\varphi} = rH^c(r). \qquad (5)$$

Now using the fact that $rH^{c}(r)=r\int_{r}^{\infty}h(b)db,$ we can simplify equation (5). This simplification gives

$$\int_{r}^{\infty}(b-r)h(b)db=\frac{c}{\varphi}. \tag{6}$$

The landowner's optimal reservation value, $r^{*},$ is the solution to equation (6).

Is the above solution unique? To answer this question, let us investigate this solution in somewhat greater detail. To this end, let us define $k^{+}$ to be equal to $k$ if $k>0$ and to be equal to 0 otherwise. With this definition in place, note that the left-hand-side (LHS) of equation (6) can be written as

$$\int_{r}^{\infty}(b-r)h(b)db=E[(B-r)^{+}]. \tag{7}$$

Now observe that $(B-r)^{+}$ is a *non-increasing* function of $r$. Therefore, from well known properties of the expectation operator,[7] it follows that $E[(B-r)^{+}]$ is also a non-increasing function of $r$. This last result tells us that the LHS of equation (6) is a *non-increasing* function of $r$. Given this line of reasoning, we can now see that if $c/\varphi>E[B],$ then there is *no* solution to equation (6) and it is optimal for our landowner to agree to develop his land upon receipt of any bid. In contrast, if $c/\varphi\leq E[B],$ then the optimal reservation value $r^{*}$ is the *unique* solution to equation (6). The reader will note that there is nothing in our model that would suggest that the condition $c/\varphi\leq E[B]$ is unreasonable. Consequently, we conclude that reasonable theoretical circumstances exist in which the landowner's

[7] See Ross (2003, pp. 97-179).

optimal reservation value is unique.

### *2.4. A Numerical Example*

We now illustrate the working of our model with a numerical example. For the purpose of this example, we suppose that $\varphi=2$, $c=\$3$, and that $h(\cdot)$ is uniform over the range from \$0 to \$100. This tells us that $H^c(r)=(100-r)/100$. Substituting these values in equation (3) and then simplifying the resulting expression, we get

$$E[\Pi(r)]=\frac{9700-r^2}{200-2r}. \tag{8}$$

Now maximizing the right-hand-side of equation (8) with respect to $r$ yields a quadratic equation in $r$ and that equation is $2r^2-400r+19400=0$. The solutions to this equation are $r_1^*=117.32$ and $r_2^*=82.68$. Hence it is clear that in this particular example, the landowner's optimal reservation value is $r^*=\$82.68$.

The results of the analysis of a mathematical model typically depend on the underlying assumptions employed in this model and our paper is no exception to this generalization. Having said this, the reader should note that two important functions in our analysis, i.e., the $h(\cdot)$ function and the $H^c(\cdot)$ function are general. The only specific assumption that we have employed in our analysis is to model the bid receipt process with a Poisson process. However, as indicated in footnote 6, the Poisson process has been widely used previously to model natural resource and related phenomena. Therefore, our results are quite general. We now conclude this section by pointing out that the analysis in this paper can be made even more general by modeling the bid receipt process with a renewal process.

## 3. Conclusions

The decision to develop or preserve land is fundamentally contingent on the magnitude of a landowner's reservation value in many contemporary models of land development over time and under uncertainty. This notwithstanding, the reservation value concept is typically an exogenous variable in present-day analyses of the land development problem. As such, we used an intertemporal and probabilistic framework to show that the reservation value concept can be usefully *endogenized*. We first showed that the optimal reservation value $r^*$ is the solution to a particular maximization problem. We then pointed out that reasonable theoretical circumstances exist in which this optimal reservation value is unique.

The analysis in this paper can be extended in a number of directions. In what follows, we suggest two possible extensions. First, the reader will note that we studied a situation in which a landowner knows that the stochastic bid receipt process is a Poisson process. As pointed out in section 2.1, this is a routinely used stochastic process in the land development literature in particular and in the natural resource economics literature in general. Even so, as discussed in section 2.4, it would be useful to see how the underlying analysis changes when the bid receipt process is a (more general) renewal process. Second, it would be useful to determine what happens to the optimal reservation value when the net cost per unit incurred by a landowner is not constant but varying over time. Studies that analyze these aspects of the problem will provide additional insights into the role that endogenous reservation values play in the development of land over time and under uncertainty.

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