Explaining the Election Results in Portugal
A spatial econometrics point of view¤

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Abstract

The great majority of the theoretical analysis about electoral cycles has considered the national space as the territory of interest for the study of the economic consequences of an electoralist behaviour by the central government. This fact, in conjunction to the nature of the data most commonly available, has lead many authors to empirical studies which, by the use of more or less sophisticated econometric techniques, intend to verify the empirical evidence of electoral cycles whether in their political versions or in their partisan versions. Given that the election results for the main parties, at least for Portugal, clearly re‡ect some spatial localization we …nd rather intriguing to verify that so very few of those empirical studies use spatial econometrics techniques. This being said, the main objective of the paper is to analyse the results corresponding to the last legislative election that took place in Portugal, from the partisan viewpoint, by the use of well-known techniques of spatial econometrics. The confrontation of the results with the ones obtained ignoring the spatial localization of the data will lead us to the nature and extent of the improvement on the results obtained by spatial econometrics techniques in what concerns the detection of empirical evidence supporting the existence of a link between voters' ideology and the election results obtained by the two main parties in Portugal.

Keywords: Elections, Partisan Models, Portugal, Spatial Econometrics, Voters

JEL Classification: C31, E32, R12

1 Introduction and Motivation

The existence of democratic elections is often associated with the question of an electoral cycle created by the incumbents.[1] As is well-known, the electoral cycle

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[1] The terminology 'electoral cycles' will be used to designate the general case, that is when the distinction between political and partisan cycles is not important. Following this terminology, a possible confusion between a particular case (i.e. the political case à la Nordhaus) and the general case – as easily happens when the terminology 'political business cycles' is used to designate the general case – is avoided.

literature has developed in two clearly distinct phases. The …rst, which took place in the mid-seventies, considered the existence of non-rational (naive) voters. See Nordhaus (1975) and Hibbs (1977). Following the rational expectations revolution, in the late eighties the second generation of models considered fully rational voters. See Alesina (1987), Rogo¤ and Sibert (1988) and Person and Tabellini (1990).

In fact, in a seminal paper in the mid-seventies, Nordhaus took a decisive step in the development of the theory and practice of political business cycles. Besides assuming that the sole objective of an opportunistic government is to maximise the number of votes at the following election, Nordhaus (1975) also considered myopic and retrospective voters, i.e. an electorate that does not take into account the evolution of the economy beyond the (next) election date. The hypothesised behaviour of government and voters then leads to a typical political business cycle, i.e. recessions at the beginning of the term of every government and in‡ationary expansions at the end of the term.

A critique of political business cycles à la Nordhaus was made by Hibbs (1977), who suggested the partisan approach to the electoral cycles literature. The author considered a di¤erent form of action by the party in power as well as di¤erent behaviour by the voters. In particular, Hibbs (1977) viewed parties as representing social classes with di¤erent political preferences. In this context, electoral victories are not an objective per se, but rather the necessary means to implement the best policy for the class the party represents. Voters, on the other hand, vote for the party which best defends their political ideology. Thus, assuming a partisan approach, it is considered that the political parties exploit di¤erent points on the Phillips curve in order to provide welfare gains to their core constituencies.[2]

The empirical literature generated by the initial theoretical studies on electoral cycles was not conclusive about their consistency with reality. Partly as a reaction to these empirical studies and partly in response to the rational expectations revolution, a new generation of models of (rational) electoral cycles emerged in the late eighties. These 'second-generation' models depart from their predecessors in the behaviour assumed by the private sector, in general, and, in particular, by voters. The assumption of voters rationality reduces the possibility of regular electoral cycles, although it does not eliminate them completely, as will be emphasised below. In fact, if the parties are signi…cantly di¤erent, then voters can rationally anticipate those di¤erences. By allowing parties to be di¤erent in their preferences for economic policies and targets, on the grounds that they represent di¤erent voters who hold di¤erent interests or ideologies, Alesina (1987) concluded that two parties with di¤erent optimal policies have di¤erent incentives to implement economic policies that di¤er from the ones announced. Considering the uncertainty regarding electoral outcomes, Alesina (1987) showed that the consequent uncertainty about future partisan policies generates business ‡uctuations. In this case, the model shows that, at the beginning of a right-wing (resp. left-wing) government's term, income/output will be below (resp. above) its natural level and unemployment above (resp. below). Once expectations and prices are adjusted, output and unemployment converge to their natural level.

[2] In Minford and Peel (1982) and Minford (1990), an interesting 'alternative' is considered, which we could classify as intermediate between the opportunistic and partisan approaches. It assumes that parties are supported by their core constituencies and some ‡oating voters, who determine the election outcome. In this case, the maximisation of some objective function leads to the best trade-o¤ between the chances of winning elections and assuring the loyalty of their support constituencies.

This means that economic activity after the adjustment should be independent of who is in power. Regarding the rate of in‡ation, its level will always be higher during the mandate of a left-wing party, even after the level of economic activity has converged to its natural level.

A …nal point must be made concerning empirical evidence. The great majority of the theoretical analysis about electoral cycles has considered the national space as the territory of interest for the study of the economic consequences of an electoralist behaviour by the central government. This fact, in conjunction to the nature of the data most commonly available, has lead many authors to empirical studies which, by the use of more or less sophisticated econometric techniques, aimed at testing for the existence of this kind of cycle in its various forms. Given that the election results for the main parties, at least for Portugal, clearly re‡ect some spatial localization we …nd rather intriguing to verify that so very few of those empirical studies use spatial econometrics techniques. For instance, a causal observation on the data concerning the (Portuguese) election results over space shows that the results obtained by the incumbent, at a regional level, should not be considered completely independent of the party ruling the distinct municipalities distributed over the national territory. These issues gain particular importance if, from a partisan point of view, one intends to analyse the election results as consequences from decisions taken by an ideological voters, i.e. by an electorate that votes in a party which best represents its political ideology. In other words, if one wants to understand the percentage of votes obtained by the parties at the elections as the result of voting decisions taken by an ideological electorate which, in turn, at least in Portugal, re‡ects some geographical distribution in space, it is apparent that spatial econometrics techniques should be used.

This being said, the main objective of the paper is to analyse the results corresponding to the last legislative election that took place in Portugal, from the partisan viewpoint, by the use of well-known techniques of spatial econometrics. The confrontation of the results with the ones obtained ignoring the spatial localization of the data will lead us to the nature and extent of the improvement on the results obtained by spatial econometrics techniques in what concerns the detection of empirical evidence supporting the existence of a link between voters' ideology and the election results obtained by the two main parties in Portugal.

The paper is structured as follows. Section 2 o¤ers the political-economic model that re‡ects the economic policies and outcomes associated with partisan behaviour by the incumbent. In accordance to those policies and outcomes, ideological voters take voting decisions. This process is shown in section 3. In order to test the hypothesis that the election results re‡ect voters' ideology, section 4 presents, in the …rst place, the non-spatial econometric results and , in the second place, the spatial econometrics results. Section 5 concludes.

2 The Political-Economic Model of Government Behaviour

As is quite common in the literature, concerning the government's objective function, we assume that the discounted disutility – at a rate $\bar{} > 0$ – during a mandate that ends at $t = T$, results from quadratic deviations of in‡ation, $¼_t$, and output (measured in logarithms), q_t, from their desired values which, for the sake of simplicity, are

assumed to be 0 and $\theta > 0$, respectively.[3] In other words, while the desired value for the inflation rate is zero the government also wants to stabilise output at a level above the natural one.[4] Thus,

$$L = \sum_{t=1}^{\infty} \beta^{t-1} \left[(q_t - \theta)^2 + \mu \pi_t^2 \right] \tag{1}$$

represents the government's loss function where,

$$q_t = \alpha (\pi_t - \pi_t^e); \tag{2}$$

and π_t^e is the expected inflation for period t, at time $t - 1$, given by

$$\pi_t^e = E[\pi_t | I_{t-1}]. \tag{3}$$

Concerning the policy instrument, we assume that the government, at period t, selects inflation π_t to minimise its disutility, given by (1), subject to the structure of the economy given by (2) and (3).

By their nature, elections are a source of uncertainty, as a change in the government, and hence a change in policies, may result from an election.[5] Generally speaking, the fact that election results can be considered news is of decisive importance to the partisan vision of electoral cycles. Let us consider two possible kinds of governments, $i = L, R$; which differ in their relative concern about inflation in that their objective functions are given by

$$L = \sum_{t=0}^{\infty} \beta^{t} \left[(q_t - \theta)^2 + \mu^i \pi_t^2 \right]; \tag{4}$$

where $\mu^L < \mu^R$. In other words, right-wing governments ($i = R$) favour less inflationary results than left-wing governments ($i = L$).

Taking expected inflation, π_t^e, as given in the optimisation of (4) subject to (2) and (3), the incumbent determines time-consistent inflation rates π_t^i as follows. If t is not an election period, $\pi_t^e = \pi_t$ which means

$$\pi_t^i = \frac{\alpha}{\mu^i}\theta \quad \text{and} \quad q_t = 0; \qquad i = L, R. \tag{5}$$

Voters who are rational and forward-looking know the two governments' objective functions and hence can compute their two optimal inflation rates (5). Although these policies are known, since it is assumed that the distribution of voters' preferences is unknown, the electoral results are unknown; see Alesina et al. (1997), p. 55. Hence, if t is an election period, expected inflation π_t^e is an average of $\frac{\alpha}{\mu^L}\theta$ and $\frac{\alpha}{\mu^R}\theta$ weighted

[3] Output q_t is measured from the natural level. In other words, $q_t = y_t - y^*$; where y_t is output and y^* is its value at the natural rate of unemployment. Some authors prefer to normalise the natural level of output Y_t^* to 1 such that, in logs, $y^* = 0$. In this case, one can follow the model, taking q_t to be the level of output.

[4] This naturally means that the government does not want output to be infinitely large. If, indeed, that corresponded to the government's objective, then a quadratic (in inflation) linear (in output) objective function would be appropriate. See, for instance, Gärtner (2000), pp. 3-5.

[5] This is also true for some results considering endogenous uncertainty. For instance, it is well known that the results concerning reputation change once that kind of uncertainty is considered.

by the probabilities that each type of government will be in office.[6] That said, if t is an election period, the expected inflation will not coincide with the effective inflation rate as

$$\pi_t^e = p_t \pi_t^L + (1 - p_t)\pi_t^R;$$

where p_t is defined to be the probability of a left-wing electoral victory in period t.

Given that $\mu^L < \mu^R$; a term of office beginning in period t will be characterised by

$$\begin{aligned}
&\pi_t^e < \pi_t^L \Rightarrow q_t = \alpha(1 - p_t)\left[\pi_t^L - \pi_t^R\right] > 0 \Rightarrow y_t > \bar{y} && \text{if L is in office after } t;\\
&\pi_t^e > \pi_t^R \Rightarrow q_t = \alpha p_t\left[\pi_t^R - \pi_t^L\right] < 0 \Rightarrow y_t < \bar{y} && \text{if R is in office after } t;
\end{aligned}$$

and

$$\begin{aligned}
&\pi_{t+1}^e = \pi_t^L \Rightarrow q_t = 0\ ,\ y_t = \bar{y} && \text{if L is in office after } t;\\
&\pi_{t+1}^e = \pi_t^R \Rightarrow q_t = 0\ ,\ y_t = \bar{y} && \text{if R is in office after } t:
\end{aligned}$$

In other words, output is above (resp. below) its natural level in the first period of a left (resp. right)-wing government.[7] Every other period until the next election, as expectations perfectly adjust, output will be at its natural level independently of the kind of incumbent. Inflation, in turn, will always be higher during left-wing governments. Hence, in this case, decreasing (resp. increasing) the electoral period length will, on average and in the case of power rotation, create a higher (resp. lower) volatility (costly fluctuations) of output and inflation rates.[8]

3 The Model of Voting Decisions

As is well-known, the partisan approach to electoral cycles considers that parties implement policies that reflect the preferences of parties' support constituencies. In this

[6] As in most of the literature, we will assume exogenous probabilities. Ellis (1991) is, to the best of our knowledge, the only study to consider endogenous probabilities.

[7] This is also known as the partisan effect.

[8] This means that there may be intermediate values of inflation rates,

$$\pi^R < \pi^{\alpha} < \pi^L$$

such that both types of governments would be better off if both implement π^{α} rather than their preferred policies π^L and π^R because the sub-optimality introduced by fluctuations in inflation and output is eliminated. As we have just noted, one way of reducing these fluctuations on average (but which does not necessarily mean a loss reduction) is to increase the electoral period length.

Alesina (1987) shows that π^{α} is decreasing (resp. increasing) with the probability of a right (resp. left)-wing electoral victory because the increase in the bargaining power of each type of government will make π^{α} closer to their own preferred policies, π^L or π^R. For this mechanism to be effective, i.e. considered credible, one naturally has to assume a sufficiently long time horizon for both types of governments and a sufficiently low discount of future. Almost the same argument is used in the strategic use of budget deficits literature; see Milesi-Ferretti and Spolaore (1994).

sense, the previous analysis may be complemented by the study of its consequences for the voters' set, characterised by di¤erent preferences. Let us, then, consider that

$$L_t^j = (q_t - \phi)^2 + \mu^j \gamma_t^2$$

represents the one-period preferences of voter j. For this voter the cost, during a term starting at t = 1 and …nishing at t = T; depends on the type of incumbent.

The discounted cost (at a rate ½) associated with a left-wing electoral victory would be

$$\frac{1}{2}\Big\{\underbrace{\alpha(1-p_1)\left[\left(\gamma_1^L-\gamma_1^R-\phi\right)^2+\mu^j\left(\gamma_1^L\right)^2\right]}_{\text{partisan effect}}+\sum_{t=2}^{\infty}\frac{1}{2^t}\left[(\phi)^2+\mu^j\left(\gamma_t^L\right)^2\right]\Big\}; \quad (6)$$

whereas in the case of a right-wing electoral victory, this cost would be

$$\frac{1}{2}\Big\{\underbrace{\alpha p_1\left[\left(\gamma_1^R-\gamma_1^L-\phi\right)^2+\mu^j\left(\gamma_1^R\right)^2\right]}_{\text{partisan effect}}+\sum_{t=2}^{\infty}\frac{1}{2^t}\left[(\phi)^2+\mu^j\left(\gamma_t^R\right)^2\right]\Big\}: \quad (7)$$

Following rational behaviour, voters decide to vote on the left or the right after comparing (6) with (7): This gives them the trade-o¤ that they face at the election day. Straightforwardly, the di¤erence in costs (6) ¡ (7) will be

$$\left(\gamma_t^L-\gamma_t^R\right)\frac{\frac{1}{2}\alpha^2(1-\frac{1}{2})(1-2p_t)\left(\gamma_t^L-\gamma_t^R\right)+\mu^j\left[\frac{1}{2}-\frac{1}{2^{T+1}}\right]\left(\gamma_t^L+\gamma_t^R\right)-2\frac{1}{2}\alpha(1-\frac{1}{2})\phi}{1-\frac{1}{2}}-\phi:$$

The previous expression thus gives the criterion used by voter i to vote for the left-wing party or for the right-wing party. Voter j strictly prefers the left-wing candidate if ¢ < 0 and, naturally, strictly prefers the right-wing candidate if ¢ > 0: Moreover, it allows us to con…rm that:

- The more costly in‡ation is to voter j, that is, the higher μ^j is, the higher the costs of having left-wing governments instead of right-wing ones.[9]
- The higher the probability is of a left-wing electoral victory p_t, the higher are the incentives to vote for this kind of party.[10]

4 The Econometric Results

Let us proceed with the detection of empirical evidence supporting the existence of a link between voters' ideology and the election results obtained by the two main parties in Portugal. As is well-known, the two main parties in Portugal are the Socialist Party (PS) and the Social Democrat Party (PSD), which have been in power almost ever since the re-implantation of democracy in Portugal in 1974. Their importance in political terms is such that a good electoral result for one the parties

[9] In fact, $\frac{\partial \phi}{\partial \mu} = \left[\left(\gamma_t^L-\gamma_t^R\right)\left(\gamma_t^L+\gamma_t^R\right)\frac{\frac{1}{2}-\frac{1}{2^{T+1}}}{1-\frac{1}{2}}\right] > 0$:

[10] In fact, $\frac{\partial \phi}{\partial p} = -2\frac{1}{2}\alpha^2\left[\left(\gamma_t^L-\gamma_t^R\right)\right]^2 < 0$:

represents, whatever the type of election, a bad electoral result for the other. Moreover, it is apparent that the Socialist Party represents a (more) 'leftist' part of the electorate whereas the Social Democrat Party represents a (more) 'rightist' part of the electorate.

As is clear, the previous section linked the voters' ideology and the decisions to vote through the relative degree of in‡ation aversion. Unfortunately there is no data for in‡ation su¢ciently disaggregated not to destroy the geographical nature of the observations. Fortunately, the last demographic census that took place in Portugal provided data su¢ciently disaggregated, i.e. at the level of the 278 municipalities, for the unemployment rate.[11] Clearly, assuming a high(er) degree of in‡ation aversion can be considered synonymous of a low(er) degree of unemployment aversion. We therefore proceed the study based on the data for the unemployment rate.

4.1 A non spatial econometrics point of view

A causal observation on the data concerning the (Portuguese) election results over space shows that the results obtained by the incumbent, at a regional level, should not be considered completely independent of the party ruling the distinct municipalities distributed over the national territory. So, the model that will be considered is the following:

$$PS = \bar{}_{0;PS} X_0 + \bar{}_{1;PS} MunPS + \bar{}_{2;PS} UnempRate + u_{PS}; \quad (8)$$

$$PSD = \bar{}_{0;PSD} X_0 + \bar{}_{1;PSD} MunPSD + \bar{}_{2;PSD} UnempRate + u_{PSD}; \quad (9)$$

where:

- PS and PSD represent the percentage of votes obtained by the Socialist and Social Democrats, by municipalities, at the legislative election that took place in Portugal in 2002;
- X_0 denotes a constant;
- MunPS and MunPSD are dummy variables that take the value 1 if the party ruling the municipality is, respectively, the Socialist Party and the Social Democratic Party, and 0 otherwise;
- UnempRate represents the unemployment rate of the municipality as determined by the demographic census;
- u_{PS} and u_{PSD} are residuals supposed to be in accordance to the usual assumptions.

The estimation of (8) and (9) by ordinary least squares resulted in:[12]

$$\widehat{PS} = \underset{(22:047)}{0:3078} + \underset{(7:597)}{0:0714}\, MunPS + \underset{(2:819)}{0:5140}\, UnempRate; \quad (10)$$

$$\widehat{PSD} = \underset{(20:057)}{0:4306} + \underset{(10:416)}{0:1325}\, MunPSD - \underset{(-4:356)}{1:1054}\, UnempRate: \quad (11)$$

[11] Plainly, we are excluding the municipalities for the Azores and Madeira islands.

[12] The t ratios are indicated in parentheses..

Plainly, (10) and (11) con…rm the prediction of the model saying that voters located in municipalities characterised by higher (resp. lower) levels of unemployment, that meaning a higher (lower) level of unemployment aversion decided to give electoral support to the party expected to be characterised also by a higher (lower) level of unemployment aversion.[13] Furthermore, the results con…rm the intuitively clear importance of the party ruling the distinct municipalities on the results obtained by each party at the legislative elections, which is specially evident for the Social Democratic Party.

The fact that the election results obtained by both parties are not independent of the ideology of the party in power at the municipalities re-emphasises the idea that the geographical localisation of the observations should be taken into account. This is strikingly evident from the representation over the national territory of the residuals associated with (10) and (11). See the following …gures.

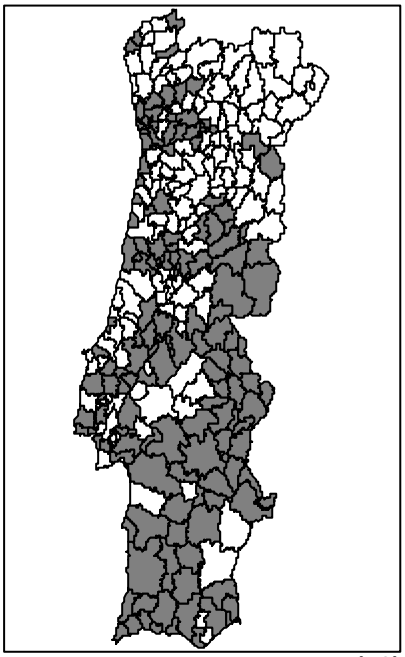

Figure 1: Residuals from (10) – PS.

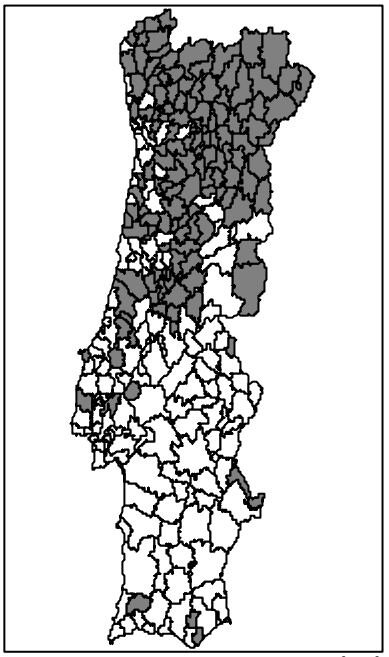

Figure 2: Residuals from (11) – PSD.

Note: Gray/white municipalities mean positive/negative residuals

The previous …gures clearly indicate that ordinary least square residuals are spatially autocorrelated at least in the sense that positive (resp. negative) residuals tend to occur in contiguous municipalities. This phenomenon seams to be specially evident for the Social Democratic Party model. This evidence is indeed con…rmed by the regressions of the residuals on the residuals of the nearest municipality, as the …gures 6 and 7 in the Annex show.[14] In fact, the correlation coe¢cient between the residuals of each municipality and the residuals for the closest municipality are

[13] The regressions made also for the Communist Party (on the left wing) and for the Popular Party (on the right ring), in a way, con…rmed this result. In fact, a positive in‡uence of the unemployment rate on the results obtained by the Communist Party and a negative in‡uence of the unemployment rate on the results obtained by the Popular Party, were revealed, both in signi…cative terms.

[14] The nearest neighbour municipality was obtained through the use of a MATLAB toolbox available at www.spatial-statistics.com due to Kelley Pace. Most of the spatial econometrics results were

around 52%, for the PS model, and 61% for the PSD model.[15] These results provide, in our opinion, clear evidence of spatial autocorrelation. The independence of ordinary least squares residuals is thus violated.

To sum up, the previous results, on the one hand, con…rmed the theoretical predictions of the model but, on the other hand clearly indicate that one should not ignore the localization of the observations over space. In order to explicitly take into account the in‡uence that each observation may exert upon the nearby observations, one has to adopt a spatial econometrics point of view.

4.2 A spatial econometrics point of view

As is relatively well-known, one says that spatial autocorrelation occurs when values of a variable concerning nearby locations are more similar than those concerning more distant locations. For instance, if a high level of unemployment in a municipality makes its presence in neighbouring municipalities more (resp. less) likely, we say that the phenomenon exhibits positive (resp. negative) spatial autocorrelation. The existence of spatial autocorrelation in the data may lead to spatial correlation of OLS residuals, given that positive/negative residuals tend to occur together. This seems indeed to be the case under study. As a consequence, the assumption about the independence of residuals is clearly violated under spatial autocorrelation. To put it formally, when the residuals, u, are spatially autocorrelated

$$E\left[uu^T\right] = \sigma^2 K = \Sigma; \tag{12}$$

where the variance-covariance matrix Σ contains non-zero o¤-diagonal elements. When residuals are not i.i.d., the OLS estimators are unbiased but ine¢cient as the estimated standard errors are biased downwards. Hence, the main e¤ect is the in‡ation of the value of tests statistic, which increases the chance of incorrectly rejecting the null hypothesis of non-signi…cance. See, among others, Dubin et al. (1999).

Despite being possible to use some form of generalised least squares, the fact is that spatial estimators are usually obtained through maximum likelihood methods. In particular, the so-called simultaneous (SAR) or conditional (CAR) approaches have been used quite commonly as speci…cations of spatial autoregressions.[16] Generally speaking, SAR models correspond to modeling the square root of the inverse of the variance-covariance matrix whereas CAR models correspond to modeling the inverse of the variance-covariance matrix. To be more precise, in the SAR models

$$\Sigma^{-\frac{1}{2}} = (I - \rho D);$$

where D is a n by n spatial weights (not necessarily symmetric) matrix with zeros on the diagonal and non-negative values o¤-diagonals, and ρ represents the spatial autoregressive coe¢cient, whereas in the CAR models

$$\Sigma^{-1} = (I - \lambda C)$$

obtained by the use of this Spatial Statistics Toolbox. See also the site www.spatial-econometrics.com from James LeSage.

[15] A parallel exercise showed also an interesting relation between the unemployment rate for each municipality and the unmeployment rate of the nearest neighbour. An ordinary least squares estimation resulted in $\widehat{UnemRate} = \underset{(8:46)}{0:0354} + \underset{(9:17)}{0:52}\, UnemRate_{NearestNeighnour}$.

[16] In fact, some other approaches gain importance as the, so-called, mixed regressive-spatial autoregressive models or even Bayesian methods. See LeSage (1999).

where C is a n by n symmetric weighting matrix with zeros on the diagonal and non-negative values off-diagonals and ρ is a spatial autoregressive coefficient. Usually the C and D matrices are normalized such that their rows sum to 1. As illustrations how to construct the C and D matrices see Pace and Barry (1997b) and Pace and Barry (1997c).[17] Naturally, a non-zero entry in the jth column of the ith row of any of these matrices indicates that the jth observation will be used to adjust the prediction of the ith observation ($i \neq j$). As the particular observation cannot predict itself, the diagonal elements are all zero. This being said, it is apparent that spatial estimators rely upon the examination of the n^2 possible relations between n observations. But, as we reasonably assume that the direct influence of sufficiently distant observations upon a particular observation decay to 0, those matrices are usually sparse and that may be used to overcome obvious computational difficulties.

The conventional approach to estimate CAR models hence should implicate solving the CAR normal equations

$$\left(X^T (I - \rho C) X\right) \beta = X^T (I - \rho C) y;$$

which obviously depend upon the unobservable parameter ρ. As pointed out before, maximum likelihood has been the preferred way to determine the optimal value of ρ. An iterative process is then used to determine, over a grid of ρ; which one maximises the likelihood function.

For the case of SAR models, the spatial estimator corrects the model

$$y = X\beta + \varepsilon;$$

by a weighted average of the values on nearby observations, Dy; such that

$$y = \alpha D y + X\beta + \varepsilon. \tag{13}$$

If so

$$y = (I - \alpha D)^{-1} X\beta + (I - \alpha D)^{-1} \varepsilon.$$

Under the usual conditions, $(I - \alpha D)^{-1} = I + \alpha D + \alpha^2 D^2 + \cdots$. See Anselin (2001). It is therefore clear that each observation is determined by the values of all exogenous variables at all locations through a distance decay function, $(I - \alpha D)^{-1}$, which acts as a spatial multiplier.

The profile likelihood function for the SAR model (13)

$$y - \alpha D y = X\beta + \varepsilon$$

is then

$$L\left(\beta; \alpha; \sigma^2\right) = c + \ln |I - \alpha D| - \frac{n}{2} \ln (SSE);$$

where c represents a constant and SSE denotes the sum-of-squared errors.

[17] Usually, one can choose weighting matrices in accordance to Delaunay triangles or to nearest neighbours. In the first case, the spatial weighting matrix leads to a variance-covariance matrix that depends upon only one parameter, i.e. the spatial autoregressive coefficient. In the second case, the spatial weighting matrix leads to a variance-covariance matrix that depends upon three parameters, i.e. the spatial autoregressive coefficient, the number of neighbours and the rate of decay of the influence of neighbours.

After this brief exposition of the econometric issues we now proceed with the estimation of CAR and SAR models for each of the two parties, PS and PSD. For both models, the locational coordinates of the Portuguese municipalities, as shown by …gure 8 in the Annex, are of crucial importance. From those spatial coordinates it is then possible to obtain the spatial weight matrix based on Delaunay triangularization.[18] The following …gure plots the non-zero elements of that Delaunay weight matrix and it clearly shows how sparse is the matrix.

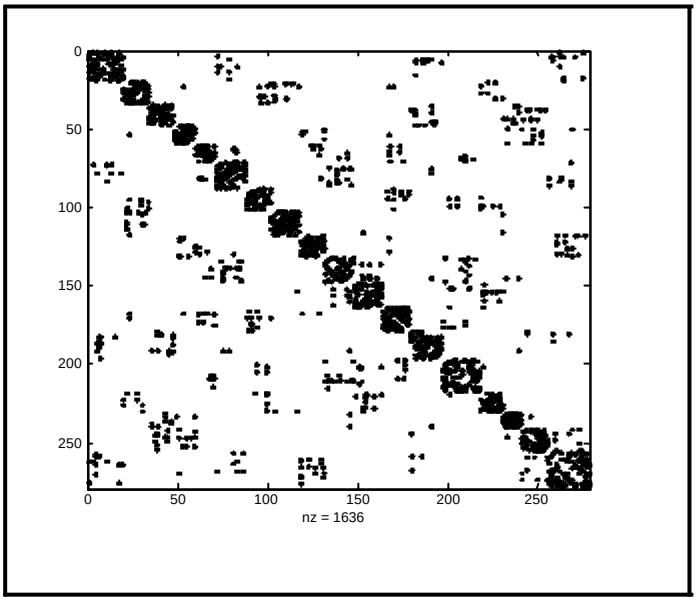

Figure 3: Plot of the Spatial Weight Matrix

The following table shows the estimatives of the coe¢cients for the CAR and SAR models and makes the comparison with the OLS counterparts.

	OLS		CAR		SAR	
	PS	PSD	PS	PSD	PS	PSD
Constant	0.3078	0.4306	0.3202	0.4507	0.3259	0.4434
Mun.	0.0714	0.1325	0.0527	0.0632	0.0510	0.0595
UnempRate	0.5140	-1.1054	0.3939	-0.8363	0.3671	-0.7596

From the results, three facts should be highlighted. In the …rst place, all the three methods lead to estimatives of the same sign.[19] In this sense, the explicit consideration of spatial issues do not contradict the predictions of the model supporting the decision of voters. In the second place, the results for the spatial models are, somehow, similar but, according to the CAR approach, the non-constant elements, i.e. the e¤ect of the party ruling the municipality and the e¤ect of the unemployment rate, exert, in absolute terms, a stronger e¤ect on the percentage of votes obtained by the two main parties in Portugal than that corresponding to the SAR approach. In the third place, both spatial approaches lead to results, in quantitative terms, distinct from those obtained through the non-spatial OLS. This is so because, in fact,

[18] Again, almost all of the following results were obtained thorough the use of a Spatial Statistics Toolbox for MATLAB, due to Kelley Pace.

[19] Moreover, an inspection of the log-likelihood ratio tests associated with each variable reveals signi…cance for all of the results.

for both models, the estimated autoregressive coe¢cients assume values quite large, clearly indicating that spatially lagged variables, which the non-spatial OLS omits, leads this methods towards other values for the estimatives. Quite remarkably the estimated autoregressive coe¢cients are as follows: 0:98 (CAR - PS model), 0:99 (CAR - PSD model), 0:77 (SAR - PS model) and 0:86 (SAR - PSD model). These maximum likelihood values are easily con…rmed by the …gures of the pro…le log-likelihoods by variable which, as an illustration for the SAR models, plotted below.

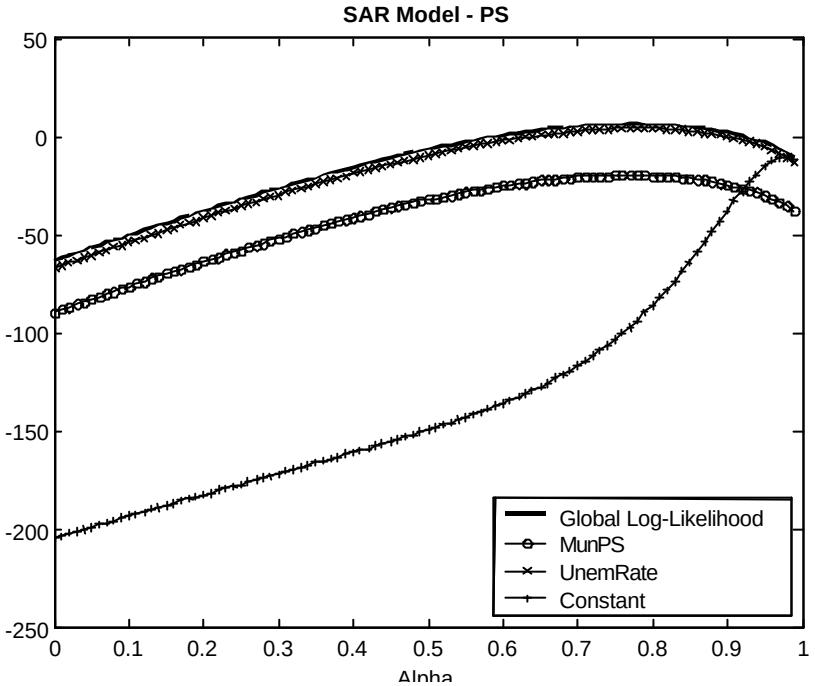

Figure 4: Pro…le Log-Likelihood by Variable

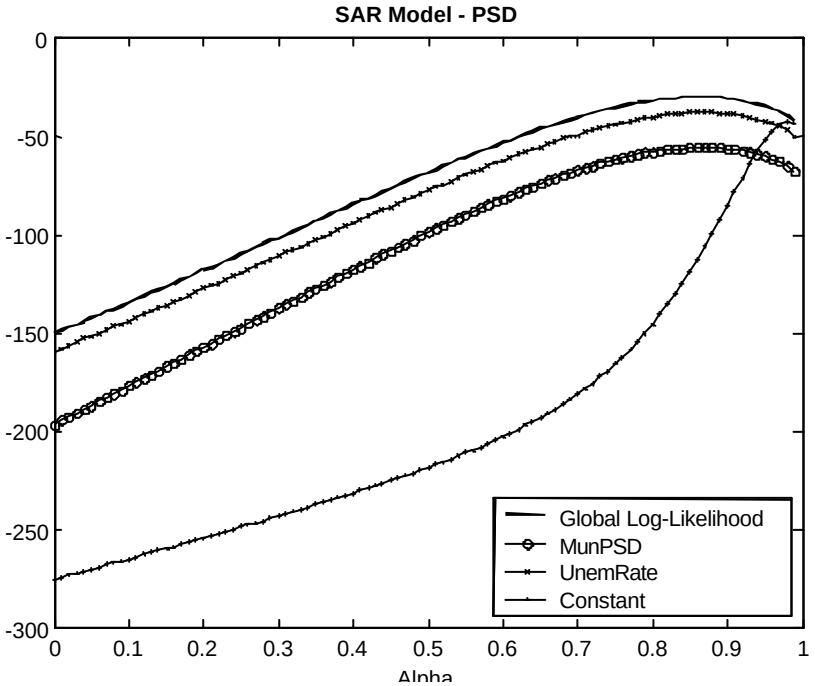

Figure 5: Pro…le Log-Likelihood by Variable

A natural way of assessing the increase on the quality of the results obtained through the application of spatial econometrics techniques, in comparison with non-spatial ones, is the examination of residuals. Clearly, as …gures 1 and 2 show, the OLS models present clear evidence of spatial autocorrelation on the residuals. See also …gures 6 and 7 in the annex. This is clearly (and obviously) not the case for the residuals corresponding to the CAR and SAR models. See …gures 9, 10, 11 and 12 in the annex. In fact, the regression of the estimated residuals on the estimated residuals of the nearest neighbour municipality whose results were

$$\hat{u}_t = \underset{(0:0017)}{5:2414E\text{-}06} \; \underset{(¡\,1:8762)}{¡\,0:1082}\; \hat{u}_t NearestNeighbour \quad \text{CAR-PS model}$$

$$\hat{u}_t = \underset{(0:0027)}{8:0424E\text{-}05} \; \underset{(¡\,1:3518)}{¡\,0:0820}\; \hat{u}_t NearestNeighbour \quad \text{CAR-PSD model}$$

$$\hat{u}_t = \underset{(¡\,0:0025)}{¡\,8:0631E\text{-}06} \; \underset{(0:2248)}{+\,0:0131}\; \hat{u}_t NearestNeighbour \quad \text{SAR-PS model}$$

$$\hat{u}_t = \underset{(¡\,0:0040)}{¡\,1:4523E\text{-}05} \; \underset{(0:3602)}{+\,0:0219}\; \hat{u}_t NearestNeighbour \quad \text{SAR-PSD model}$$

show no signi…cative evidence of spatial autocorrelation.

5 Conclusion

The paper performed an exercise on spatial econometrics to validate the theoretical predictions of a model explaining the decisions to vote in accordance to the degree of unemployment aversion. To do that, we used data for the Portuguese municipalities and the election results obtained by the two main parties at the last legislative election that took place in Portugal. Besides con…rming those predictions, the econometrics results showed clear evidence that, indeed, spatial issues should be taken into account when trying to understand the Portuguese election results. Clearly, the large spatial autoregressive coe¢cients that were obtained indicate the importance of geographically correlated variables which are simply omitted by non-spatial econometrics methods. In this sense, it is not surprising that geographically correlated variables contributed for a substantial increase on the overall …t. Despite not being an objective to obtain a model with the maximum predictive power, it is of striking importance to highlight that the $\overline{R}^2$ increased from the 0:21 (PS) and 0:36 (PSD) for OLS to values substantially larger as 0:61 (CAR - PS model) and 0:79 (CAR - PSD model) or to 0:58 (SAR - PS model) and 0:78 (SAR - PSD model). To sum up, in our opinion, the paper certainly reveals results that deserve to be further explored in future occasions.

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6 Annex

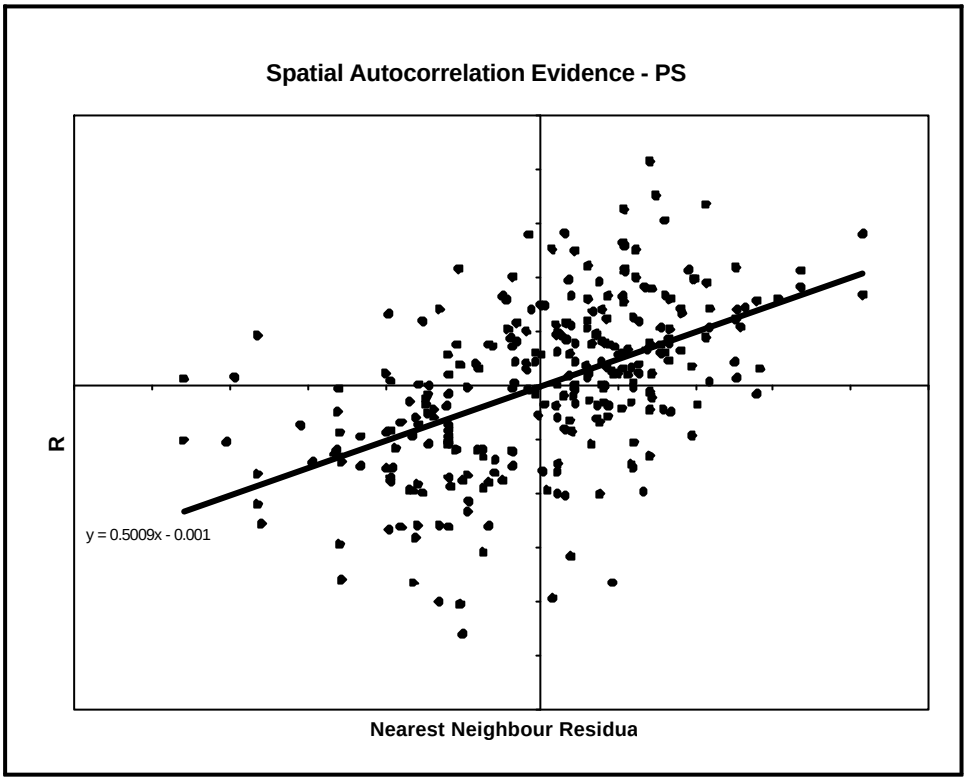

Figure 6: The regression of $\hat{u}_{PS}$ on $\hat{u}_{PS}$ (nearest neighbour)

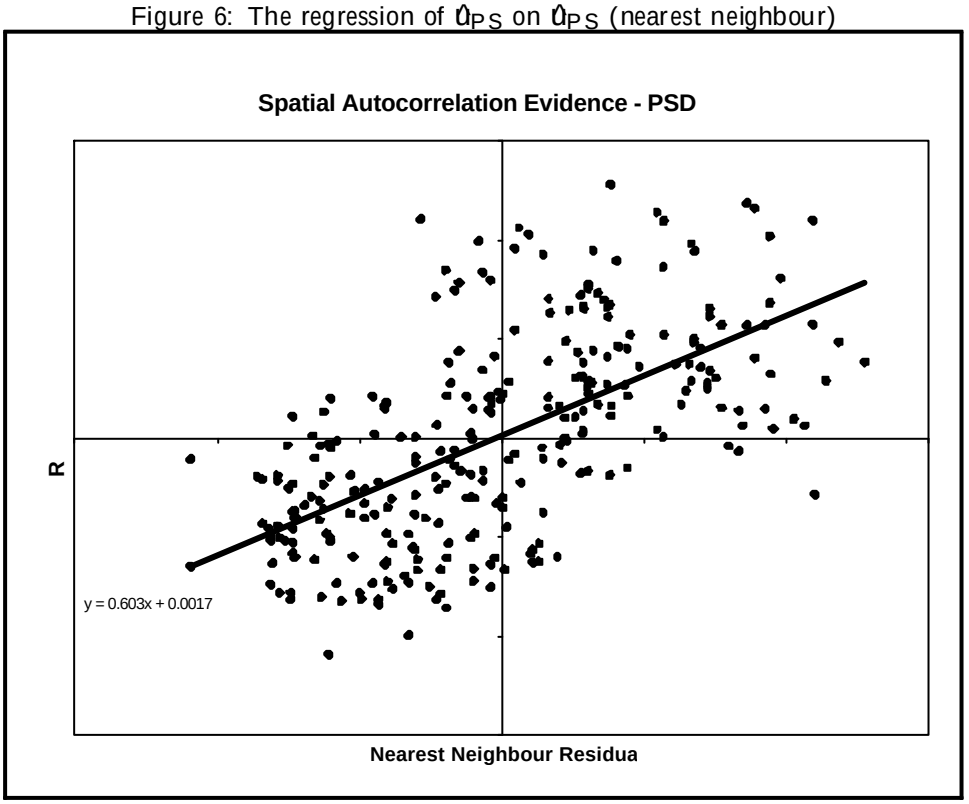

Figure 7: The regression of $\hat{u}_{PSD}$ on $\hat{u}_{PSD}$ (nearest neighbour)

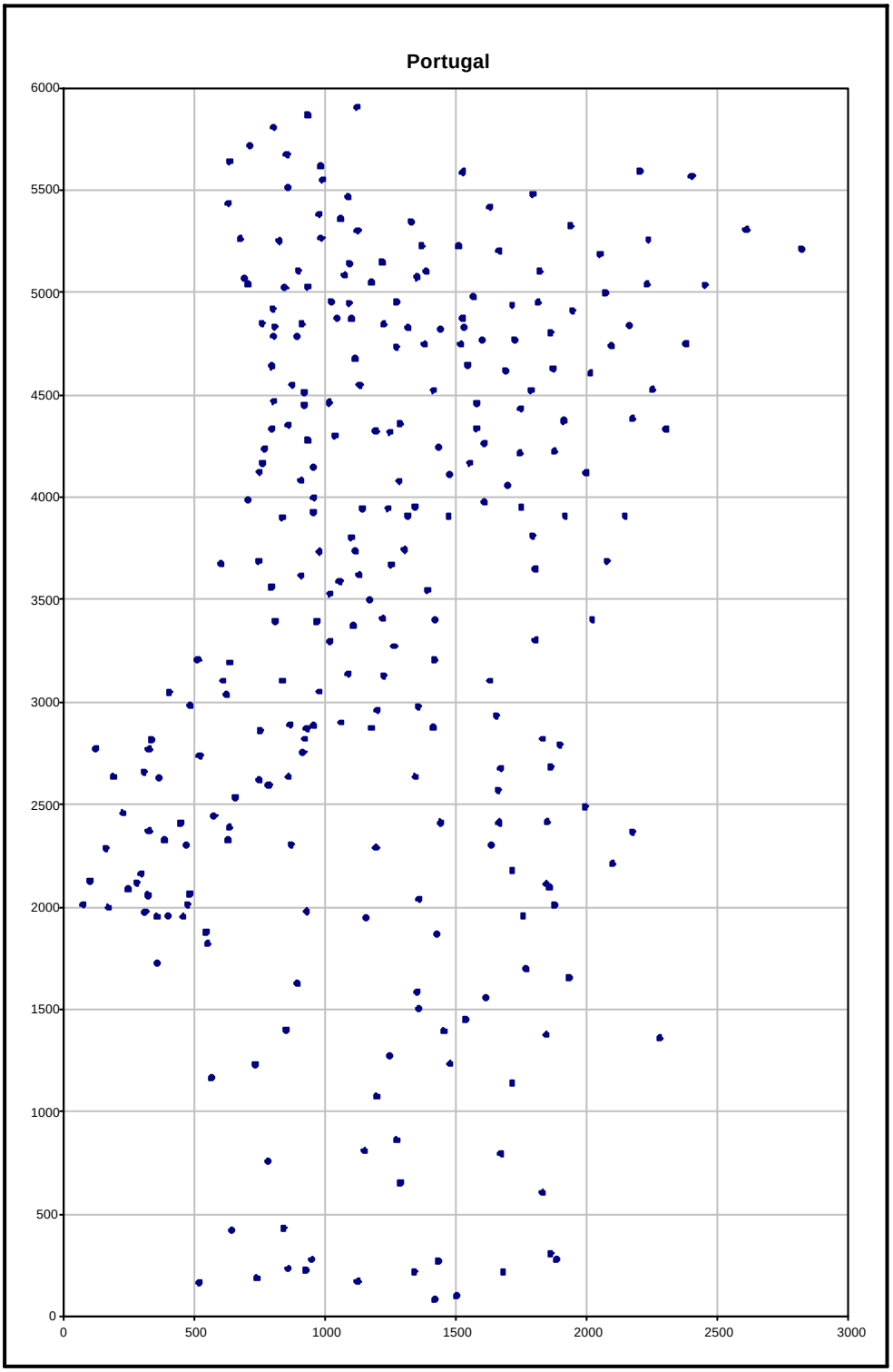

Figure 8: The geographical coordinates of the Portuguese municipalities

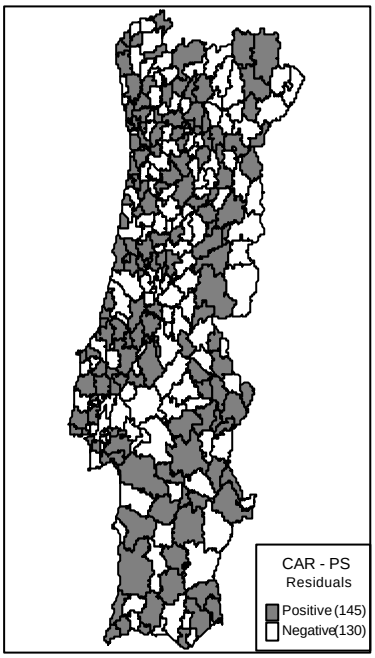

Figure 9: Residuals of the CAR–PS model

Figure 10: Residuals of the CAR–PSD model

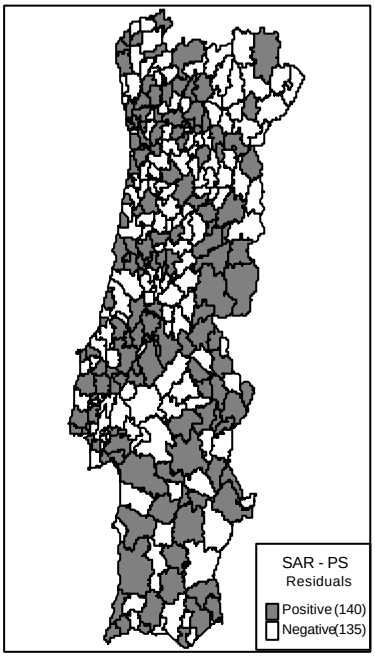

Figure 11: Residuals of the SAR–PS model

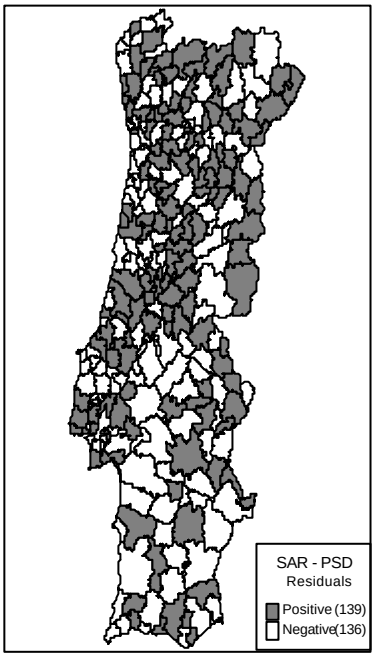

Figure 12: Residuals of the SAR–PSD model