

# **Revisiting the Role of Common Labeling in a Context of Asymmetric Information: Critique and Extensions**

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## **I. Abstract**

The paper examines results presented by Marette, Crespi and Schiavina in an article entitled *the role of common labeling in a context of asymmetric information* (1999). They show that, given high cost of labeling, a cartel that provides information about product quality may improve overall welfare even if producers collude to reduce quantity competition. This study extends their model and programs it as a mixed complementarity model, to account for demand of low quality products under certification. It is found that the firms differentiate the high and low quality consumers. Unlike the results by Marette, Crespi, and Schiavina, the welfare impact is ambiguous. It is concluded that the model presented here can be further developed in an empirical setting.

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## ***II. Introduction***

Households in the Western Hemisphere are no longer self sufficient in food production. Rather, the majority purchase food from a third party seller, e.g. from a restaurant, retailer, and grocery store etc. Viewing the product only from the shelves makes it difficult for the consumer to gain insight in the production practices and the quality attributes to the product.

Formally, we can describe this as the food products purchased from a grocery store contain less *search characteristic*. Thus, the consumer cannot determine the quality of the product a priori the purchase. Instead the food products are characterized to be more of *experience* (quality is revealed after purchase) or *credence characteristic* (quality is not revealed even after purchase).

Although it is not possible to determine the quality of the packaged food product on the shelves, the issues concerning food product quality are not trivial issues in society. The consumers may boycott not only food that can contain food-borne diseases, but also products that may be considered processed or produced in an unethical or hazardous method for the environment. For example, the linkage between the BSE (Bovine Spongiform Encephalopati) in beef and CJD (Creutzfeld-Jakobs Disease) in humans changed the consumption pattern rapidly in Europe, although not all countries reported occurrence of BSE. Frewer, Risvik and Shifferstein (2001), and Westgren (1999) scrutinize the impact of changing consumers' preferences vis-à-vis the implications of the structure in the food-marketing chain.

Consequently, these issues create incentives for the agribusiness firm to design programs for differentiating food products on basis of perceived quality aspects. Producers supplying products that appeal to the consumers' taste have incitement to differentiate their products by other means than the pricing mechanism. The differentiation process is carried out through implementation of quality policy-, or certification programs. Certification programs and organizations like ISO, USDA, FAIRTRADE, CROP-WATCH, PDO, PGI, and Organic Europe, distinguish the product quality in terms of in production process, origin, or other (in-) tangible characteristics, etc. Several studies have scrutinized the

economic implications of quality, or certification programs in agricultural markets, e.g. see Crespi and Marette (2002, 2001), Hoffman (1997, 2002), and Marette, Crespi, and Schiavina (1999).

### **III. Problem**

When one or several stages in the food chain join to establish specific quality standards, both producers and consumers might reap economic gains through lowered uncertainty and increased efficiency. On the contrary, there is also a probability that the development of quality policy programs may further enhance market power, thus offset the potential social gains of the program. In essence, a certification program used by individual stages in the agribusiness chain may lead to vertical or horizontal cooperation (collusion), thus potentially incurring costs upon their factor and product markets.

From an economist point of view, it is not feasible to rationalize which quality or quantity a firm should strive for, without first analyzing relevant information concerning the production process and the demand situation in the relevant product markets. Amongst others, Marette, Crespi, and Schiavina (1999) observe that agricultural markets are working imperfectly due to asymmetric information, since the consumers lack perfect information about the product quality. The suppliers, on the other hand, have incentives to produce both high and low quality products, although the consumers always prefer the higher quality products.

The authors hypothesize that the societal welfare increases if consumers can distinguish between high quality and low quality products. Marette et al test this hypothesis by developing a partial equilibrium model (henceforth called the base model) under imperfect information in two elaborate scenarios. Topics concerning quality, vertical product differentiation, and signaling are popular topics in the industrial organization literature and it encompasses a broad range of various modeling frameworks. Classical references in this topic are for example Martin (2002), Tirole (1988), and Vives (2002).

The model derived by Marette et al treats labeling in agricultural markets in a delicate way. With the certification scheme in place the consumer are able to distinguish between high and low quality products. However, the certification implies that the high quality producers gain market power. The low quality producers are no longer producing, and the high quality producers can exercise market power by tacitly colluding or acting as Cournot quantity setters. Essentially, they show that the societal welfare increases when high quality producers come together in a certification scheme and eliminate asymmetric information.

Nevertheless, it is crucial to note that the assumptions build in the partial equilibrium framework drives the results. First, the authors choose to use a demand function, which strictly discriminates high quality from low quality products. Second, the authors' assumes that all firms have access to the same technology and have identical marginal cost of production. Third, the certification scheme does not alter the high quality firms' marginal cost.

Thus, it is perceived necessary to extend the model to analyze the same set of issues. Relaxing the assumptions for developing an empirically testable model may show that unlike the results by Marette et al, the welfare impact is ambiguous.

#### **IV. Objective and Outline**

The objective of this study is to analyze certification programs and its impact on the market structure using a programmable mixed complementarity model. This study continues developing the model from Marette et al. Specifically, this study attempts to relief some of the rather restrictive assumptions on consumer and producer behavior that Marette et al have in their paper.

The outline of the paper is as follows. The subsequent sections present the model and the derived results Marette et al. Continuing, a few of the restrictive assumptions are relieved and its implications analyzed further. The paper concludes by discussing the discrepancies and commonalities with the Marette et al's findings, and finally comments suggestions for future research.

## V. Conceptual Framework

The demand structure in the Marette et al paper is specified so that consumption of low quality products incurs a disutility upon the consumers. Under asymmetric information, the low quality producers can sell their low quality produce, although knowingly, the consumers would not be willing to purchase a low quality product *per se*. Consequently, the suppliers have incentives to produce both high and low quality products, although the consumers always prefer the higher quality products.

Numerous studies in vertical product differentiation specify the indirect utility function so that more quality is strictly preferred to less. Vertical product differentiation models are developed and analyzed, although not exclusively, by, Mussa and Rosen (1978), Peitz (1997, 2000), Tirole (1988), and Vives (2001). A classical approach is to specify the consumer's indirect utility-function rather than direct demand:

$$(***) \quad D(k, p) = \theta \cdot k - p,$$

where the quality of the product is described by the parameter  $k$ . Additionally, consumers differ in taste, described by the uniformly distributed parameter  $\theta \in [0,1]$ . The linear indirect utility function can be thought of as a first-order Taylor series expansion of the true underlying indirect utility function. Tirole (1988: p.96), and later Motta (1993: p.115), interpret the parameter  $\theta$  as the marginal rate of substitution between income and quality. Hence, as  $\theta$  approaches its upper limit the marginal utility of income decreases, or alternatively, the income increases.

It may be somewhat dubious to define what the quality parameter  $k$  really represents. The quality parameter may represent a label that ensures the product is free from diseases, genetically modified organisms, organic produce or alike. This study views the parameter as product differentiation parameter. However, due to asymmetric information buyers and other sellers cannot observe the quality level. As in line with most studies, Marette et al set  $k_l$  to zero so that no buyers would knowingly purchase a low-quality product (*this appears to be a rather popular assumption, e.g. see the most recent paper by Hoffman: p.5*).

On the producer side, Marette et al assumes that there are only two high quality firms ( $n_H=2$ ), and five low quality firms ( $n_L=5$ ). The low quality producers cannot provide high quality products. Crespi states that the numbers of high quality producers are arbitrarily chosen:

If we can show that a cartel that colludes in quantities can actually benefit consumers, then it doesn't really matter whether  $n_H=2, 3, 100$ . (2002, personal communication).

Furthermore, the marginal cost of production is identical for all producers and equal to zero. Hotelling, for example argues that the inclusion of cost may be a trivial task:

This condition of no cost is not essential to the existence of such profits. If a constant cost of production per unit had been introduced in the calculations above, it would simply have been added to the prices without affecting the profits. Fixed overhead charges are to be subtracted from  $\pi_1$  and  $\pi_2$ , but may leave a substantial residuum. These gains are not compensation for risk, since they represent a minimum return. (1929: p.51)

This is an analytically tractable approach: numerous studies on product differentiation set the marginal cost of production to zero.

## VI. Solution Practice

The product differentiation model by Marette et.al. is set up as a mixed complementarity problem. Ferris and Pang (1997); Murphy, Sherali, and Soyster (1982); and Bazaraa, Sherali, and Shetty (1993), provide extensive discussions of mixed complementarity methodology relevant for economic problem solvers. The objective of mixed complementarity problems is to solve for stationary points in a Karush-Kuhn-Tucker convex programming context. The primal profit function for a firm is specified as:

$$(***) \quad \pi = p(Q) * q - c(q),$$

where  $p(Q)$  is the inverse demand,  $q$  the firm's output level,  $Q$  the aggregate the output level, and  $c(q)$  the firm's cost function. The first order condition is:

$$(***) \quad \frac{\partial \pi}{\partial q} = p(Q) + q \cdot \frac{\partial p(Q)}{\partial q} - \frac{\partial c(q)}{\partial q} \geq 0,$$

and the complementarity condition is specified as:

$$(***) \quad q \cdot \frac{\partial \pi}{\partial q} = q \cdot \left[ p(Q) + q \cdot \frac{\partial p(Q)}{\partial q} - \frac{\partial c(q)}{\partial q} \right] = 0,$$

where  $q \geq 0, \partial \pi / \partial q \geq 0$  so that when the output level is positive, the first order condition must hold with equality.

The market clearing condition is:

$$(***) \quad \sum_i q_i^* \geq Q(p),$$

that is, aggregate supply must be at least as large as demand. The complementarity problem for the market clearing condition is specified as:

$$(***) \quad p^e \cdot \left[ \sum_i q_i^* - Q(p) \right] = 0,$$

where  $p^e$  is the market clearing price.

The producers can either tacitly collude or set quantity independently in a Cournot game. The solution  $\{q_i^*\}_{i=1..n}$  to the Cournot game constitutes Nash equilibrium. An outcome is Nash equilibrium if no player would find it beneficial to deviate if no players deviate from their strategies played at the Nash outcome. Formally, for all producers  $i$  and  $j$ , this is described as:

$$(***) \quad \pi_i(q_i^*, \bar{q}_{-i}) \geq \pi_i(q_i, \bar{q}_{-i}) \quad \forall_i.$$

The advantage of programming the Nash-Cournot game as a mixed complementarity framework lies on the numerical plane, rather than on the analytical. The mixed complementarity can find a numerical approximation to the market clearing equilibrium for analytically intractable problems.

With this approach, it is also possible to find equilibrium for multiple demands and firms with different production technology (it is assumed initially that each producer has identical costs).



## VII. Nash-Cournot: No Certification

Recall that the base scenario assumes that  $n_L=5$ ,  $n_H=2$ , and that the high quality parameter  $k_H$  is equal to unity, whereas the low quality parameter  $k_L$  is zero. With no certification, the consumers are not able to distinguish between the high and low quality products and base their consumption decision on the expected quality. The expected quality is:  $\bar{k} = 2 * k_h / 2 + n_l$ . The marginal consumer who is indifferent between buying the product or not has the taste parameter:  $\theta_0 = p / \bar{k}$ . Consequently, total demand can be described as:  $Q = 1 - \theta_0 = 1 - p / \bar{k}$ . Derivations follow in appendix.

The formal definition of the first firm's profit is:

$$(***) \quad \pi_1^H = p * q_1^H = \bar{k} [1 - \sum_{i=1}^{n_l} q_i^L - \sum_{j=1}^2 q_j^H] * q_1^H,$$

and the associate first order condition is:

$$(***) \quad \frac{\partial \pi_1^H}{\partial q_1^H} = \bar{k} [1 - Q] - \bar{k} q_1^H = 0.$$

Solve for the optimal quantity,  $q = 1/(3 + n_l) = .125$ . Total output is  $Q = .875$  and the market-clearing price is  $p = .0357$ . The profit for each firm is  $\pi_m^{H*} = .0357$ . Moreover, since it is assumed that the per unit cost is identical for both the high and low quality producers, the output and consequently the profit is equal.

Turning the focus to the consumer surplus:

$$(***) \quad CS = \int_{\theta_0}^1 (\theta \bar{k} - p^e) d\theta = .109.$$

Consequently, the total welfare with no certification is equal to

$$(***) \quad W = CS + \sum_i \pi_i^{H*} + \sum_j \pi_j^{L*} = .141.$$

### VIII. Nash-Cournot with Certification

Marette et al assume that the consumer is able to distinguish between the high-quality and low-quality products when the high quality producers signal their quality in a certification program. Due to the theoretical construct of the demand the consumers never have desire to buy low quality products.

Hence, the certification program also implies that there are no low-quality producers contained in the market. The model is straightforward in that there is no asymmetric information with a certification scheme: all high quality producers have incentive to commit to the higher quality. Consequently, there are only two producers with identical marginal cost they also earn the same profit when playing the Nash-Cournot game.

The marginal consumer can be identified as  $0 = \theta_0 k_h - p \rightarrow \theta_0 = p/k_h$ , and the inverse demand is  $p = k_h[1 - Q]$ .

The profit for each of the two high quality producers is:

$$(***) \quad \pi^h = p * q_1 - C/2 = k_h[1 - Q] * q_1^H - C/2 = k_h[1 - q_1 - q_2]q_1 - C/2,$$

where C is a fixed cost of establishing the certification scheme.

The associated first order condition is:

$$(***) \quad \frac{\partial \pi_1^H}{\partial q_1^H} = k_h[1 - Q] + k_h[q_1^H] * [-1] = 0,$$

and solving for  $q_1$  we get the best response function:  $q_1^*(q_2) = (1 - q_2)/2$ . Since both high quality producers have the same marginal cost, they have identical best response functions. The Nash-Cournot output level is:  $q_1^* = q_2^* = 1/3$ , and the market clearing price is  $p^e = k_h[1 - Q] = k_h/3 = .333$ . Consequently, the profit for each high quality producer is:

$$(***) \quad \pi_1^* = \pi_2^* = 1/9 k_h - C/2.$$

The consumer surplus is:

$$(***) \quad CS = \int_{\theta_0}^1 (\theta k_h - p^e) d\theta = .222$$

Consequently, the total welfare under certification is:

$$(***) \quad W = CS + \pi_1^H + \pi_2^H = 4/9 \cdot k_H - C .$$

## IX. Collusion with Certification

Under collusion, the high-quality sellers collude on quantities and share the cost of certification. Each producer supplies 50 percent of the marketed quantity:  $Q = q_1^* + q_2^*$ .

The cartel's profit is:

$$(***) \quad \pi_{collusion} = p^* Q - C = k_h [1 - Q]^* Q - C ,$$

the F.O.C. is:

$$(***) \quad \frac{\partial \pi}{\partial Q} = k_h [1 - Q] - k_h [Q] = 0 = \underbrace{k_h [1 - Q]}_{\neq 0} - \underbrace{Q}_{=0} ,$$

and the resulting quantities and price are:  $q_1^* = q_2^* = .250$ , and  $p^e = k_h [1 - Q] = .500$ .

The marginal consumer is identified as:

$$(***) \quad \theta_0 = \frac{p}{k_h} = \frac{k_h}{2} \frac{1}{k_h} = .500 .$$

Consequently, the producer profit is:

$$(***) \quad \pi^* = p^e * q^* - C/2 = k_h/8 - C/2 .$$

The consumer surplus is:

$$(***) \quad CS = \int_{\theta_0}^1 (\theta k_h - p^e) d\theta = \frac{1}{8} k_h ,$$

so the total welfare is equal to:

$$(***) \quad W = CS + collusive(\pi_1^H, \pi_2^H) = 3/8 \cdot k_H - C .$$

## X. Base Model: Summary of Scenarios

The table below provides a summary of the base model developed by Marette et al. The model is programmed in GAMS as a mixed complementarity problem (see appendix). The last column presents the numerical estimates of consumer and producer surplus with (fictitious) data for high and low quality parameters;  $k_l=0$  and  $k_h=1$ . In lieu with the Marette et al study, there are only two high quality producers and five low quality producers in the market. The total cost of establishing the certification scheme is set to 0.15.

**Table 1.** Qualitative summary of Marette's paper.

| <i>Strategy</i>  | <i>Output level</i>                  | <i>Market clearing<br/>prices</i> | <i>Consumers' surplus</i> | <i>Producers' profit<br/>(per producer)</i> | <i>Welfare<br/>(numerical)*</i> |
|--|--------------------------------------|-----------------------------------|---------------------------|---|---------------------------------|
| No<br>certification  | $Q = .875$<br>$q^* = .125$           | $p^e = .035$                      | CS=.109                   | $\pi^* = .00446$                            | $W = .141$                      |
| Certification,<br>and Nash<br>Cournot  | $Q = .666$<br>$q_1^* = q_2^* = .333$ | $p^e = .333$                      | CS=.222                   | $\pi_1^* = \pi_2^* = .0361$                 | $W = .295$                      |
| Certification,<br>collusion  | $Q = .500$<br>$q_1^* = q_2^* = .250$ | $p^e = .500$                      | CS=.125                   | $\pi^* = .0500$                             | $W = .225$                      |
| * Base model programmed in GAMS with the following parameter values: $n_h=2$ ; $n_l=5$ ; $k_l=0$ ; $k_h=1$ ; $C=.15$ |                                      |                                   |                           |   |                                 |

Essentially, the certification program internalizes the externalities through labeling, as it eliminates asymmetric information. The consumer surplus' increases from 0.109 to 0.222 units with certification and two Nash-Cournot producers. When the high quality producers collude on quantities, and act as a joint monopolist, the consumer surplus' increases to 0.125 units.

If the high quality firms could construct an enforcing collusive certification scheme, the per-producers profit would increase from 0.036 to 0.05 units when going from a Nash-Cournot behavior to acting as a joint monopolist. For a reasonable cost of certification, C, it holds that:

$$(***) \quad \pi_{Collusion} > \pi_{No Certification} ,$$

and

$$(***) \quad \pi_{NashCournot} = [1/9 k_h - C/2]_{NashCournot} < [1/4 k_h - C/2]_{Collusion} = \pi_{Collusion} .$$

However, since the marginal profit of producing an additional unit for the individual producer is strictly positive, the collusion cannot be Nash equilibrium. Recall the individual firm's first order conditions:

$$(***) \quad \left. \frac{\partial \pi_1^H}{\partial q_1^H} \right|_{q_1^* = 1/4} = k_h [1 - \underbrace{1/2}_{Q_{Collusion}} - 1/4] > 0 ,$$

thus, the high quality producer have a *strictly* positive marginal profit of producing more units than the agreed allotment. Unless there is not a formal treatise that forbids production more than the allotment from the collusion game, this is not a stable equilibrium.

Given the existing numbers of producers (two high quality, and five low quality, respectively), the total welfare ranking is:

$$(***) \quad W_{no} = \frac{9 \cdot k_H}{64} < W_{collusive} = 3/8 \cdot k_H - C < W_{Cert} = 4/9 \cdot k_H - C ,$$

i.e. the welfare under certification is strictly higher than under no certification (and collusion), as long as the cost of certification is lower than  $C \approx 0.304 \cdot k_H$ .

## ***XI. Modification of Demand: No Certification***

Recall the specification of the indirect utility function:

$$(***) \quad D(k, p) = \theta \cdot k - p.$$

The specification raises mainly three concerns whether it is an optimal theoretical representation. First, it is unclear whether the linear approximation is a good representation of consumer behavior for large perturbations of prices and quality. Peitz (1995, 1997, and 2000) make several attempts of deriving an aggregate consumer demand from the perspective of horizontal and vertical product differentiation perspective.

The second issue is whether indirect utility function exhausts all necessary features of vertical product differentiation. The quality variables may not solely assume the extreme points  $0$  and  $1$  in terms of the utility function. Under full information, the simplifying assumption that  $k_l$  is zero drives the low quality producers out of the marketplace. Thus, the theoretical construct could also allow for a dynamic representation, with both low and high quality producers represented on the market under full information. Marette et al note this issue:

The basic model could also be extended to permit low quality [...] greater than zero. In this case, buyers who have a low willingness to pay prefer low-quality products. Results are similar to the basic model because only high-quality sellers have an incentive to certify their products [...]. (1999: p. 174)

However, these results are not elaborated on within the paper. Motta (1993) develops these issues further for the case with two competing producers and find that product differentiation always arises at equilibrium. Hence, in line with Motta's spirit, this study proceeds by deriving conditions for when  $k_l \neq 0$ , i.e. when low quality products do not cause a disutility upon consumption for all consumers. Analogous with previous section, we find the marginal consumer, the associated first order conditions, consumer surplus and producer profit, respectively.

The expected quality of the products is:

$$(***) \quad \bar{k} = (2 * k_h + n_l * k_l) / (2 + n_l).$$

The marginal consumer has the taste parameter  $\theta_0 = p / \bar{k}$ . Consequently, total demand is:

$$(***) \quad Q = 1 - \theta_0 = 1 - p / \bar{k}.$$

Since all seven firms are symmetric, the Nash-Cournot game yields symmetric solutions, i.e. all seven firms have the same profit. The profit is defined as:

$$(***) \quad \pi_m = p \cdot q_m = \bar{k} \cdot [1 - \sum_{i=1}^{n_l} q_i^L - \sum_{j=1}^2 q_j^H] \cdot q_m \quad m \in \forall H, L,$$

and the F.O.C. is:

$$(***) \quad \frac{\partial \pi_m}{\partial q_m} = \underbrace{\bar{k} [1 - Q - q_m]}_{=0},$$

and the output level per firm is  $q = 1/(3 + n_L) = 1/8$ , and the total market output is  $Q = (2 + n_L)/(3 + n_L) = .875$ . The marginal consumer is located at the distance  $\theta_0 = 1 - Q = 1/(3 + n_L) = .125$ . Since consumer have utility of consuming low quality products,  $k_l \neq 0$ , the market-clearing price is somewhat different from the base model:

$$(***) \quad p^e = \frac{5}{56} \cdot k_L + \frac{1}{28} \cdot k_H.$$

The profit for each firm is:

$$(***) \quad \pi_m^* = \frac{5}{448} \cdot k_L + \frac{1}{224} \cdot k_H.$$

Consumer surplus is:

$$(***) \quad CS = \int_{\theta_0}^1 (\theta \bar{k} - p^e) d\theta = \frac{35}{128} \cdot k_L + \frac{7}{64} \cdot k_H.$$

Consequently, the total welfare is:

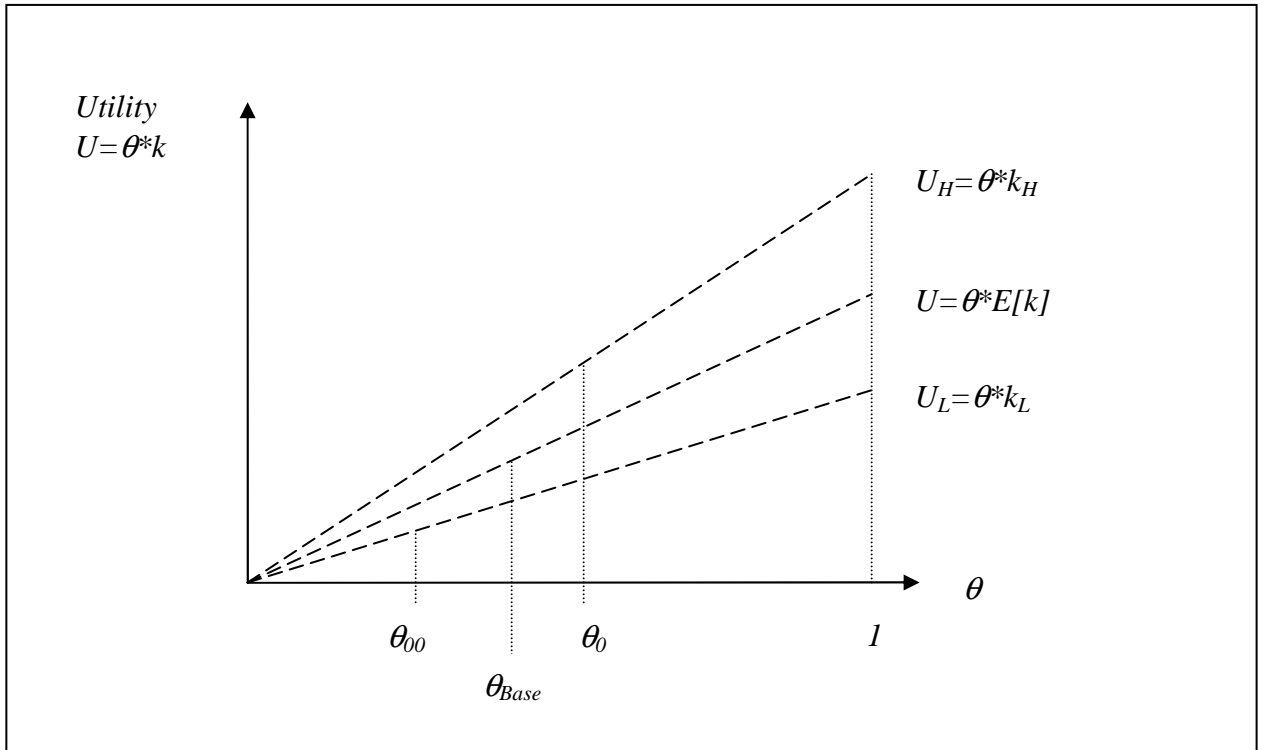
$$(***) \quad W_{NoCert} = \frac{45}{128} \cdot k_L + \frac{9}{64} \cdot k_H .$$

Thus, in the scenario with no certification, but where the consumers have preferences for low quality products yields a higher welfare, than when  $k_L=0$ , ceteris paribus:

$$(***) \quad W(k_H \geq k_L \geq 0) = \frac{45}{128} \cdot k_L + \frac{9}{64} \cdot k_H \geq W(k_H \geq k_L = 0) = \frac{9}{64} \cdot k_H$$

## XII. Modified Demand: Certification

With certification, the two high quality sellers certify the produce, and the low quality sellers serve the fringe demand. Essentially, this situation resembles a traditional product differentiation case, with a slight modification. The figure below depicts the problem:



**Figure 1.** The product differentiation case.



The vertical axis measures consumers' utility, and the horizontal axis the population taste distribution parameter  $\theta$ . The utility for high quality goods are  $U_H = \theta^* k_H$ , and for the low quality goods  $U_L = \theta^* k_L$ . It is easy to be betrayed that the figure resembles an *upward sloping demand curve*. Rather, the figure depicts the utility as a function of the uniformly distributed taste distribution parameter.

Recalling the base model, we note that  $k_l = 0$ , or in words: there is no demand for low quality products. This particular utility is depicted in the figure by the (indirect) utility function  $U = \theta^* E[k]$ , where  $E[k]$  is the expected quality level under no certification.

With certification, the qualitative difference with the base model is that there are effectively two active “demands”, or more precisely utilities associated with consumption:  $U_H$  and  $U_L$ , for differentiated products and one for non-differentiated, respectively.

This particular model has two types of marginal consumers: those indifferent between buying either product, and those indifferent between buying nothing and the low quality product. First, the marginal consumer that is indifferent between consuming either product is described by the taste parameter  $\theta_0$ :

$$(***) \quad \theta_0 k_H - p_H = \theta_0 k_L - p_L,$$

or, alternatively:

$$(***) \quad \theta_0 = \frac{p_H - p_L}{k_H - k_L}.$$

Consequently, the demand for high quality products is:

$$(***) \quad Q_H = 1 - \frac{p_H - p_L}{k_H - k_L}$$

The marginal consumer at  $\theta_{00}$  is indifferent between consuming nothing and the low quality product. Solving for  $\theta_{00}$ :

$$(***) \quad \theta_{0,0} = \frac{p_L}{k_L}$$

The demand for the low quality product is:

$$Q_L = \frac{p_H - p_L}{k_H - k_L} - \frac{p_L}{k_L} \quad (***)$$

The inverse demands are:

$$p_H = k_H - Q_H k_H - Q_L k_L, \quad (***)$$

and

$$p_L = (1 - Q_H - Q_L) k_L. \quad (***)$$

Since  $k_L \neq 0$ , the demands are inversely dependent on the degree of substitutability.

Hence, this case is similar to the product differentiation case.

The optimal output level for each high and low quality producer is found through their respective first order conditions. The profit for the high quality producer is:

$$\pi_1^H = p \cdot q_1^H = (k_H - Q_H k_H - Q_L k_L) q_1^H - C/2, \quad (***)$$

and the first order condition is:

$$\frac{\partial \pi_1^H}{\partial q_1^H} = (k_H - Q_H k_H - Q_L k_L) - k_H q_1^H = 0. \quad (***)$$

Solving for first high quality firm's best response function:

$$q_1^H(q_2^H, q_i^L, i=1..5) = \frac{(k_H - Q_L \cdot k_L - q_2^H \cdot k_H)}{2 \cdot k_H}. \quad (***)$$

By analogy, the profit for the (first) low quality producer is:

$$\pi_1^L = p \cdot q_1^L = (1 - Q_H - Q_L) \cdot k_L \cdot q_1^L, \quad (***)$$

and the F.O.C. is:

$$(***) \quad \frac{\partial \pi_1^L}{\partial q_1^L} = (1 - Q_H k_H - Q_L k_L - q_1^L) \cdot k_L = 0,$$

so the best response function for the (first) low quality producer is:

$$(***) \quad q_1^L(Q^H, q_{-i}^L) = \frac{\left(1 - \sum_{j \neq i} q_j^L - Q^H\right)}{2}.$$

The optimal output levels for each firm is found by solving the systems of best response functions, or by solving the respective first order conditions. Note that  $q_i^{L*} = q_j^{L*} \forall i, j = 1..n$ , so:

$$(***) \quad q_i^{L*} = \frac{k_H}{18 \cdot k_H - 10 \cdot k_L} \forall i \in [1..n]$$

by analogy, the optimal output level for each of the two high-quality firms is:

$$(***) \quad q_i^{H*} = \frac{6 \cdot k_H - 5 \cdot k_L}{18 \cdot k_H - 10 \cdot k_L} \forall i \in [1..2],$$

so the market output levels are:

$$(***) \quad Q^H = \frac{6 \cdot k_H - 5 \cdot k_L}{9 \cdot k_H - 5 \cdot k_L},$$

and

$$(***) \quad Q^L = \frac{5 \cdot k_H}{18 \cdot k_H - 10 \cdot k_L}.$$

The corresponding market clearing price for high quality products with certification is:

$$(***) \quad p_h = \frac{(6 \cdot k_h - 5 \cdot k_l)k_h}{18 \cdot k_H - 10 \cdot k_L},$$

and for the low quality products:

$$(***) \quad p_l = \frac{k_h \cdot k_L}{18 \cdot k_h - 10 \cdot k_l}.$$

Consequently, the marginal consumer between buying the high or low quality product is defined as:

$$(***) \quad \theta_0 = \frac{3 \cdot k_H}{9 \cdot k_H - 5 \cdot k_L},$$

and the marginal consumer between buying the low quality product and nothing is defined as:

$$(***) \quad \theta_{00} = \frac{k_H}{18 \cdot k_H - 10 \cdot k_L}$$

The profit for the high quality producers are:

$$(***) \quad \pi_1^H = \pi_2^H = \frac{1}{4} \frac{(6 \cdot k_h - 5 \cdot k_l)^2 k_h}{(9 \cdot k_H - 5 \cdot k_L)^2} - \frac{C}{2},$$

and for the low quality producer:

$$(***) \quad \pi_i^L = \frac{1}{4} \frac{k_H^2 \cdot k_l}{(9 \cdot k_H - 5 \cdot k_L)^2}, i \in [1..5],$$

The profit ratio between the low and high quality producer is equal to:

$$(***) \quad \pi^L / \pi^H = k_H \cdot k_l / (6 \cdot k_H - 5 \cdot k_L)^2,$$

That is, in the extreme case where the consumer obtains identical utility from consuming the high and low quality products, there is no difference in profit. The subsequent section discusses this result further. The consumers' surplus from consuming the high quality products are:

$$(***) \quad CS_H = \int_{\theta_0}^1 (\theta k_H - p_H) d\theta = \left[ \frac{\theta^2 k_H}{2} - \theta p_H \right]_{\theta_0}^1 = \frac{3 \cdot (k_H)^2 (6 \cdot k_H - 5 \cdot k_L)}{(9 \cdot k_H - 5 \cdot k_L)^2},$$

and from consuming the low quality products:

$$(***) \quad CS_L = \int_{\theta_{0,0}}^{\theta_0} (\theta k_L - p_L) d\theta = \left[ \frac{\theta^2 k_L}{2} - \theta p_L \right]_{\theta_{0,0}}^{\theta_0} = \frac{25}{8} \cdot \frac{(k_H)^2 \cdot k_L}{(9 \cdot k_H - 5 \cdot k_L)^2},$$

so the total consumer surplus is:

$$(***) \quad CS_{TOT} = CS_H + CS_L = \frac{1}{8} \cdot \frac{(k_H)^2 (144 \cdot k_H - 95 \cdot k_L)}{(9 \cdot k_H - 5 \cdot k_L)^2}.$$

The analytical expression for total welfare is:

$$(***) \quad W_{Certification} = CS_{TOT} + \sum_i \pi_i^{H*} + \sum_j \pi_j^{L*} = \frac{1}{8} \cdot \frac{k_H (288 \cdot k_H^2 - 325 \cdot k_L \cdot k_H + 100 \cdot k_L^2)}{(9 \cdot k_H - 5 \cdot k_L)^2} - C.$$

### XIII. Modified Demand: Summary

Numerical estimates for the no-certification and certification scenario are summarized in the table below. Collusive action is omitted, since the producer always has incentive to deviate from collusive quantity due to the existence of positive marginal profit.

**Table 2.** Numerical estimates with the extended Marette model.

| Strategy                        | Numerical estimates*   |                              |  |                                  |               |
|---------------------------------|--|------------------------------|--|----------------------------------|---------------|
|                                 | Output level   | Prices                       | Consumers' surplus   | Producers' profit (per producer) | Total welfare |
| No certification                | $q_L = q_H = .125$<br>$Q = .875$                             | $p^e = .161$                 | $CS = .492$  | $\pi = .020$                     | $W = .633$    |
| Certification, and Nash Cournot | $q_H = .269$<br>$q_L = .077$<br>$Q_H = .538$<br>$Q_L = .385$ | $p_H = .538$<br>$p_L = .077$ | $CS_{Low} = .089$<br>$CS_{High} = .393$<br>$CS_{TOT} = .482$ | $\pi_H = .145$<br>$\pi_L = .006$ | $W = .741$    |

\* Base model programmed in GAMS with the following parameter values:  $n_h=2$ ;  $n_l=5$ ;  $k_l=1$ ;  $k_h=2$ ;  $C=0.15$   
The result from collusive behavior is not presented here, as it is not a Nash equilibrium

Constructing a utility function that permits demand for low quality products yield rather interesting results as both low quality and high quality producers can coexist under certification. The (aggregate) output level increases from .875 to .923 units with certification. In addition, the prices charged are vastly different between the certified and

non-certified product: the high quality products are seven times expensive than the low quality (non-certified) product. Essentially, with certification the consumers' surplus and low quality producers profit decreases, whereas the high quality producers profit increases.

The producer profit high quality producers increases from .02 to .145 units since they produce more units of output to higher price. The low quality producers on the other hand serve the fringe market with relatively small prices, and their profit decreases to .006 units. The consumer on the other hand, loses roughly half of the surplus with the certification scheme.

#### **XIV. Perturbation of Quality Parameter and Cost of Certification**

It is notable that the overall welfare increases from 0.633 to 0.741 units when the high quality producers market their produce under a common labeling. Marette et al emphasize upon this particular point (*c.f. first comment by Crespi*):

The basic model could also be extended to permit low quality,  $k_L$ , greater than zero. In this case, buyers who have a low willingness to pay prefer low-quality products. Results are similar to the basic model [no certification, author's note] because only high-quality sellers have an incentive to certify their products, because of higher profits under perfect information... (1999: p. 174)

However, the results are not stable for larger perturbations of the quality parameters,  $k_L$  and  $k_H$ , and the cost of certification,  $C$ . For example, the society as a whole is indifferent between certification and no certification when:

$$(***) \quad W_{NoC} = W_{Certification},$$

or, alternatively expressed in terms of  $k_L$  and  $k_H$ , and the cost of certification,  $C$ :

$$(***) \quad C = \frac{25}{128} \cdot \frac{(126 \cdot k_H^3 - 289 \cdot k_L \cdot k_H^2 + 208 \cdot k_L^2 \cdot k_H - 45 \cdot k_L^3)}{(9 \cdot k_H - 5 \cdot k_L)^2}.$$

That is, given that  $k_L=1$  and  $k_H=2$ , the highest cost that leaves the society indifferent with certification is 0.25577 units: recall that the current model assumes  $C$  is 0.15 units.

As noted from the expression above, as  $k_L$  approaches  $k_H$ , the welfare impact becomes ambiguous. Given the cost of certification  $C=0.15$  and the high quality parameter  $k_H=2$ , for small perturbations of  $k_L$ , the welfare impact become ambiguous. For example when  $k_L$  is above 1.338 units, there is a negative impact on the society when the high quality sellers certify their produce.

## **XV. Varying the Number of Producers**

This first section explores the case when varying the number of high quality producers, *ceteris paribus* (*n b the cost of certification is set to zero for  $n_H > 10$* ). The table below reports the results of the simulation. Each scenario reports the numerical results of no certification versus certification.

When there are no high quality producers on the market, the low quality producers supply the whole market. As the high quality producers increase in number, the Nash-Cournot equilibrium approaches the competitive market outcome, i.e. the market price approaches the firm's marginal cost. Hence, as the market price approaches zero, each producer supplies an infinite small unit of output, and the total welfare approaches unity.

With certification, there is a clear trend towards the low quality producers becoming fringe suppliers. Although supplier serves a fringe demand, the profit has not necessarily to be lower. However, in this case, the high quality producer's profit is roughly three times higher for all levels of high quality suppliers.

**Table 3.** Varying the number of high quality sellers.

| Number of<br>high quality<br>sellers <sup>#</sup> | Output level<br>(per firm) |       | Prices |       | Consumers'<br>surplus | Profit<br>(per firm) |        | Total welfare |
|---|----------------------------|-------|--------|-------|-----------------------|----------------------|--------|---------------|
|   | High*                      | Low** | High*  | Low** |                       | High*                | Low**  |               |
| 0 <sup>##</sup>                                   |                            | .167  |        | .167  | .347                  | .028                 |        | .486          |
| w/c <sup>###</sup>                                | -                          | .167  | -      | .167  | .347                  | -                    | .028   | .486          |
| 1   |                            | .143  |        | .167  | .429                  | .024                 |        | .571          |
| w/c   | .368                       | .105  | .737   | .105  | .468                  | .122                 | .011   | .645          |
| 2   |                            | .125  |        | .161  | .492                  | .020                 |        | .632          |
| w/c   | .269                       | .077  | .539   | .077  | .571                  | .070                 | .006   | .741          |
| 3   |                            | .111  |        | .153  | .543                  | .017                 |        | .679          |
| w/c   | .212                       | .061  | .424   | .061  | .644                  | .040                 | .004   | .782          |
| 4   |                            | .100  |        | .144  | .585                  | .014                 |        | .715          |
| w/c   | .175                       | .050  | .350   | .050  | .696                  | .024                 | .003   | .804          |
| 5   |                            | .090  |        | .136  | .620                  | .012                 |        | .744          |
| w/c   | .145                       | .043  | .298   | .042  | .736                  | .014                 | .002   | .817          |
| 6   |                            | .083  |        | .129  | .649                  | .011                 |        | .767          |
| w/c   | .130                       | .037  | .259   | .037  | .767                  | .009                 | .001   | .825          |
| 7   |                            | .077  |        | .122  | .675                  | .009                 |        | .799          |
| w/c   | .115                       | .033  | .230   | .033  | .790                  | .005                 | .001   | .830          |
| 8   |                            | .071  |        | .115  | .696                  | .008                 |        | .804          |
| w/c   | .103                       | .030  | .206   | .029  | .810                  | .002                 | .0009  | .834          |
| 9   |                            | .067  |        | .110  | .716                  | .007                 |        | .818          |
| w/c   | .093                       | .027  | .187   | .023  | .823                  | .001                 | .0007  | .837          |
| 10  |                            | .063  |        | .104  | .732                  | .007                 |        | .830          |
| w/c   | .085                       | .024  | .171   | .024  | .840                  | -.0004               | .0006  | .839          |
| w/c***  | .085                       | .024  | .171   | .024  | .840                  | .0146                | .0006  | .989          |
| 20  |                            | .038  |        | .069  | .832                  | .003                 |        | .899          |
| w/c***  | .046                       | .013  | .092   | .013  | .911                  | .004                 | .0002  | .997          |
| 40  |                            | .022  |        | .041  | .904                  | .0009                |        | .944          |
| w/c***  | .024                       | .007  | .048   | .007  | .953                  | .001                 | .00005 | .999          |
| 50  |                            | .018  |        | .034  | .921                  | .0006                |        | .954          |
| w/c***  | .019                       | .006  | .039   | .006  | .962                  | .0007                | .00003 | .999          |
| 100   |                            | .009  |        | .018  | .956                  | .0002                |        | .976          |
| w/c***  | .009                       | .003  | .020   | .003  | .981                  | .0002                | ~0     | ~1            |

Perturbations based on model programmed in Maple; <sup>#</sup> 5 low quality producers; Parameter values:  $k_l=1$ ;  $k_h=2$ ;  $C=0.15$ ; <sup>##</sup> No certification: high and low quality producers have identical output levels; <sup>###</sup> Certification; w/c = scenario with certification; w/c \*\*\* = scenario with certification, but the certification cost is set to zero ; \* High = High quality producers output level, unit price, and profit, respectively; \*\* Low= Low quality producers output level, unit price, and profit, respectively;



Next, the number of low quality producers is varied, ceteris paribus, e.g. number of high quality producers, cost of certification and quality are held constant.

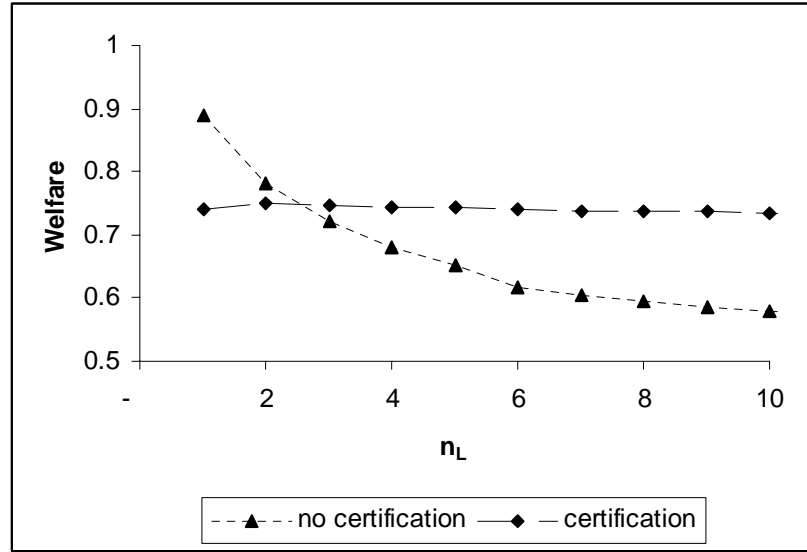
**Table 4.** Varying the number of low-quality sellers.

| <i>Number of<br/>low quality<br/>sellers<sup>#</sup></i> | <i>Output level<br/>(per firm)</i> |              | <i>Prices</i> |              | <i>Consumers'<br/>surplus</i> | <i>Profit<br/>(per firm)</i> |              | <i>Total welfare</i> |
|--|------------------------------------|--------------|---------------|--------------|-------------------------------|------------------------------|--------------|----------------------|
|  | <i>High*</i>                       | <i>Low**</i> | <i>High*</i>  | <i>Low**</i> |                               | <i>High*</i>                 | <i>Low**</i> |                      |
| <i>0<sup>##</sup></i>                                    |                                    | .333         |               | .667         | .444                          | .222                         |              | .889                 |
| <i>w/c<sup>###</sup></i>                                 | .333                               |              | .667          |              | .444                          | .147                         |              | .739                 |
| <i>1</i>   | .25                                |              | .417          |              | .469                          | .104                         |              | .781                 |
| <i>w/c</i>   | .300                               | .200         | .600          | .200         | .500                          | .105                         | .04          | .750                 |
| <i>2</i>   | .200                               |              | .300          |              | .480                          | .060                         |              | .720                 |
| <i>w/c</i>   | .286                               | .143         | .571          | .143         | .531                          | .088                         | .020         | .748                 |
| <i>3</i>   | .167                               |              | .233          |              | .486                          | .389                         |              | .681                 |
| <i>w/c</i>   | .278                               | .111         | .556          | .111         | .549                          | .079                         | .012         | .745                 |
| <i>4</i>   | .143                               |              | .191          |              | .490                          | .027                         |              | .653                 |
| <i>w/c</i>   | .273                               | .091         | .546          | .091         | .562                          | .074                         | .008         | .743                 |
| <i>5</i>   | .125                               |              | .161          |              | .492                          | .020                         |              | .633                 |
| <i>w/c</i>   | .269                               | .077         | .539          | .077         | .571                          | .070                         | .006         | .741                 |
| <i>6</i>   | .111                               |              | .139          |              | .494                          | .015                         |              | .617                 |
| <i>w/c</i>   | .267                               | .067         | .533          | .067         | .578                          | .067                         | .004         | .739                 |
| <i>7</i>   | .10                                |              | .122          |              | .495                          | .012                         |              | .605                 |
| <i>w/c</i>   | .265                               | .059         | .529          | .059         | .583                          | .065                         | .003         | .738                 |
| <i>8</i>   | .091                               |              | .109          |              | .496                          | .010                         |              | .595                 |
| <i>w/c</i>   | .263                               | .053         | .526          | .053         | .587                          | .064                         | .003         | .736                 |
| <i>9</i>   | .083                               |              | .098          |              | .497                          | .008                         |              | .587                 |
| <i>w/c</i>   | .262                               | .048         | .524          | .048         | .591                          | .062                         | .002         | .736                 |
| <i>10</i>  | .077                               |              | .090          |              | .497                          | .007                         |              | .580                 |
| <i>w/c</i>   | .261                               | .043         | .522          | .043         | .594                          | .061                         | .002         | .735                 |
| <i>20</i>  | .04                                |              | .047          |              | .499                          | .002                         |              | .544                 |
| <i>w/c</i>   | .256                               | .023         | .512          | .023         | .608                          | .056                         | .001         | .731                 |
| <i>40</i>  | .023                               |              | .024          |              | ~.500                         | .001                         |              | .524                 |
| <i>w/c</i>   | .253                               | .012         | .506          | .012         | .616                          | .053                         | .0001        | .728                 |
| <i>50</i>  | .019                               |              | .020          |              | ~.500                         | .0004                        |              | .519                 |
| <i>w/c</i>   | .252                               | .010         | .505          | .010         | .618                          | .052                         | .00009       | .727                 |
| <i>100</i>   | .010                               |              | .010          |              | .500                          | .0001                        |              | .510                 |
| <i>w/c</i>   | .251                               | .005         | .503          | .005         | .621                          | .051                         | .00002       | .726                 |

*Perturbations based on model programmed in Maple; <sup>#</sup> 2 high quality producers; Parameter values:  $k_l=1$ ;  $k_h=2$ ;  $C=0.15$ ; <sup>##</sup> No certification: high and low quality producers have identical output levels; <sup>###</sup> Certification; w/c = scenario with certification; \* High = High quality producers output level, unit price, and profit, respectively; \*\* Low= Low quality producers output level, unit price, and profit, respectively;*

When there are two high producers serving the market, the expected quality equals the high quality. Therefore, with certification the high quality producers' profit is lower due to the cost of certification.

As the number of low-quality-firms increase, the consumers' surplus increase, but decreases the each firm's profit. The output for the high quality producer approaches 0.25 as the number of low quality producer increase. The figure below displays how the total welfare changes as the number of firms increase.



**Figure 2.** Welfare impact vs. number of low quality firms.

The figure depicts an interesting pattern. Although the consumers' surplus is strictly higher under certification, the total welfare impact of a certification scheme is ambiguous. For  $n_L \leq n_H = 2$ , the welfare impact of a certification scheme is negative. In addition, the welfare is decreasing with the number of low quality providers.

The qualitative difference between varying the number of high and low quality firms is that the welfare is increasing in the number of high quality producer, whereas the total welfare impact is ambiguous when varying the number of low quality producer.

## XVI. Product Differentiation

The differences in price charged and market quantities warrant further discussion. Recall the relation of the inverse demands for high and low quality products:  $p_H = k_H - Q_H k_H - Q_L k_L$ , and  $p_L = (1 - Q_H - Q_L) k_L$ . Hotelling (1929) noted this essential point and claims that:

It is the gradualness in the shifting of customers from one merchant to another as their prices vary independently which is ignored in the examples worked out by Cournot, Amoroso, and Edgeworth [authors note: the latter consider homogeneous products]. The assumption, implicit in their work, that all buyers deal with the cheapest seller leads to a type of instability which disappears when the quality sold by each is considered as a continuous function of the differences in price. [...] So only in theory of value a market is usually considered as a point in which only one price can obtain; but for some purposes it is better to consider a market as an extended region. (1929: p. 44)

Shy (1995: pp. 136) provide some intuition of the results: using the own and cross quantity terms for each inverse demand, it is possible to analyze the price responsiveness, or the degree of product differentiation. The own and cross terms are  $(k_H, k_L)$ , respectively for the high quality; and  $(k_L)$  for the low quality. Containing the high quality and the low quality prices in the indirect utility function allow consumers to base their decision on quality *and* price. Essentially, consumers of the high quality products perceive that the high- and low-quality products appear being highly differentiated:

$$(***) \quad \delta \equiv \frac{(k_L)^2}{(k_H)^2} = \frac{(1)^2}{(2)^2} = \frac{1}{4},$$

or, in words, as  $\delta \rightarrow 0$ , the change in the price of the low quality product has a negligible impact on the high quality product.

In contrast, consumers buying low quality products perceive that the products are almost homogeneous to the high quality products.

$$(***) \quad \delta \equiv \frac{(k_L)^2}{(k_L)^2} = 1,$$

hence, the low-quality consumers perceive that the price has a stronger impact on both demands. Hence, there are two aggregate types of consumers: one inelastic and another

elastic segment of consumers. The inelastic high quality type has a strictly higher willingness to pay for high quality products. The second type, on the other hand, also likes high quality, but is more sensitive to price changes than the high quality type.

## ***XVII. Further Extensions***

This section outlines further extensions of the product differentiation model developed by Marette et al. The subsequent section discusses the possibility of extending the demand to a non-linear demand type to account for larger perturbations of prices and quantities. The next section discusses re-specification of the production technology and the associate marginal cost. Finally, the last section discusses the implications of relieving some of the restrictive assumptions on market behavior entry/exit.

## **XVIII. Demand Specification II: Non-Linear Demand**

The linear demand curve can be seen as a first order Taylor series expansion of the “true” demand function. However, the linear approximation is a poor representation of reality under large perturbations. An alternative route is to utilize a second-order Taylor-series expansion and specifically model demand as a quadratic, CES or a translog utility function, see Deaton and Muellbauer (1980) or Pollack and Wales (1992). Essentially, a general (non)-linear function of quality and the population distribution parameter can be thought of as:

$$(***) \quad D_L(f(k_L; \theta), h(p_L, p_H; \theta, k_L)),$$

and

$$(***) \quad D_H(f(k_H; \theta), h(p_L, p_H; \theta, k_H)),$$

for the low and high quality products, respectively. Furthermore,  $f(k; \theta)$  is a general utility function of the quality conditional on the population distribution parameter  $\theta$ , and  $h(p, k; \theta)$  are the interaction between the quality and the price term conditional on the population distribution parameter  $\theta$ , so that  $D_H > D_L$  for all  $p_H$ .

It is possible to specify the quality function,  $f(k; \theta)$ , in analogous development with the utility-separability framework. For separability, e.g. see Deaton and Muellbauer (1980: pp. 127,133). Hence, the quality function  $f(k; \theta)$ , could be regarded as a “demand-shift-parameter” (Roberts, Josling and Orden; 1999).

## **XIX. Production Technology**

Historically, studies on product differentiation circumvent the specification of the production technology, by assuming zero marginal cost; see the references made by Martin, Motta, and Tirole. For example, Motta develops a framework for vertical differentiation models, allowing for the existence of a fixed cost associated with the choice of quality. The firms decide qualities in the first period, and set the quantities in the second and final period. In the final period, however, the incurred costs in the first period are already sunk and do not enter into the decision process, e.g. R&D expenditures, advertising, brand promotion campaigns, etc.

Modeling farm production is an inherently challenging task. Heady and Dillon (1961), Shumway et al (1984), and Beattie and Taylor (1993), discusses suitable representations for the agricultural technology. In essence, engaging in a certification scheme (e.g. organic farming), may force the firm to adopt new technology (e.g. substitute fertilizers and chemicals for crop rotation and mechanical weeding).

## XX. Specification of the Marginal Cost Function

Recent studies introduce ownership structures, returns to scale, and cost of production in the vertical differentiation framework: see Amacher et al. (2001), Hoffman, Martin, and Sexton (2000). Essentially, recent studies indicate that the choice of appropriate cost/technology appear provide value for analyzing market behavior in agricultural markets. Thus, it is appropriate to account for the production technology at the production stages. Analytical expressions for farm-level output supply can be derived by using duality theory, e.g. see Lau and Yotopoulos (1972). The advantage with duality theory is that it is possible to find closed form expressions for supply without explicitly specifying the production technology.

Additionally, the certification scheme usually involves more than one stage of the food marketing chain and it may be incorrect to analyze one specific stage in isolation. Sexton provides a conceptual framework for analyzing the dynamics of the food marketing chain. For example, Sexton (2000: p1090) the relation between the farm gate price  $P^f$  and the consumer price,  $P^r$  is:

$$(***) \quad P^r \left( 1 + \frac{\xi}{\eta} \right) = P^f \left( 1 + \frac{\theta}{\varepsilon} \right) + c,$$

where  $\eta$  is the retail price elasticity of demand, epsilon  $\varepsilon$  the farm price elasticity of supply,  $\xi$  measures departures from competition in selling the finished product at retail,  $\xi \in (0,1)$  as approaches 1 the market power increases (monopoly/collusion),  $\theta \in (0,1)$  plays a similar role in terms of procurement of the farm product as theta approaches 1 the market power increases (monopsony/ oligopsony).

Thus, extending the base model with Sexton's framework may be a more realistic representation of the chain. Essentially, individual farm firms may not be able to exercise market power, whereas a larger processor may (Paarlberg et al 1999). Finally, it can also account for the influence of agricultural policy instruments, which may affect the performance of the food marketing chain (Hertel, 1989).

## **XXI. Market Behavior**

Both the base model and the elaborated model showed that the high quality producer's profit increases with certification. Thus, if the quality producer also could switch from producing low to high quality, the profit could potentially increase. An interesting extension is to allow producers producing a range of high and low quality products, e.g. free exit and entry. Marette et al and Motta shows that in general, all high quality producers will label their high quality products, and the rest low quality producers supply the fringe demand. In fact, Motta shows that the equilibrium is characterized by a high degree of product differentiation, independently of the chosen cost structure (1993: p. 114). In practice, Scandinavian farmers may certify a proportion of the farm, or certify the whole production as organic according to the KRAV rules (organic labeling organization).

## ***XXII. Summary and Discussions***

This essay has analyzed the implications of certification on the food marketing chain. A frequently occurring problem with food purchase is that the quality of the food is rarely observed prior to purchase. Rather, since food is an experience or credence characteristic good, there is a mismatch, or asymmetry, between a consumer's desire for quality and what is actually produced by the producer. Therefore, it is hypothesized that there is a positive welfare impact when producers choose to label their products.

The theoretical construct of the Marette et al model shows an unambiguous improvement in welfare, when high quality producers certify their products. On the other hand, the modified model shows that the welfare impact of certification is ambiguous when varying the number of firms providing low quality. When there is less low quality than high quality firm, the welfare impact is negative of a certification scheme. For reasonable parameter values, the welfare decreases as firms providing low quality products increase. However, the welfare impact is strictly increasing when increasing the number of high quality firms, and when allowing for certification.

Since the cost of production is assumed zero for all firms, the driving story behind these latter results is the specification of demand. The market for high quality products is relatively inelastic. Consequently, the high quality producers find it profitable to decrease quantity thus earning higher profits. Although the low quality firms increase, the high-quality producers do not adjust the production level. On the other side is the demand for low products, which is more elastic, and the five low quality producers faces a fringe demand as the number of firms providing high quality increases.

The study proposes by in large three major revisions to the model developed by Marette et al. First, instead of using a linear utility function that serves as a linear approximation to any utility function; it is deemed appropriate to first a concrete representation of consumer behavior using a second order Taylor-series approximation to consumer demand: where consumers' decision parameters include prices for both certified and uncertified products. Second, rather than assuming a zero unit of production, it is deemed appropriate to extend the framework by developing an underlying production technology with associate marginal cost. Third and lastly, the current model setup does not allow low quality producers to supply high quality goods. This is a rather abstract assumption, however, and should be extended to allow producers to interchangeably supply both high and low quality products, based on profit maximizing principles rather than subjectively chosen rules.



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## **XXIV. Appendix**

Appendix contains the analytical derivations of the results with a general number of low quality producers,  $n_L$ , high quality,  $k_H$ , and low quality,  $k_L$ , and two high quality producers. Recall that the base scenario assumes that  $n_L=5$ ,  $n_H=2$ , and that the high quality parameter  $k_H$  is equal to unity, whereas the low quality parameter  $k_L$  is zero.

### **XXV. Nash-Cournot: no certification**

Expected quality with no certification:

$$(***) \quad E[Quality] = \bar{k} = \Pr(high) * k_h + \Pr(low) * k_l = \frac{2 * k_h}{2 + n_l} + \frac{n_l * k_l}{2 + n_l},$$

replace  $k_L=0$ , the expression for the expected quality is:  $\bar{k} = 2 * k_h / (2 + n_l)$ . The marginal consumer has the taste parameter:  $\theta_0 = p / \bar{k}$ . Total demand can be described as:

$$(***) \quad Q = 1 - \theta_0 = 1 - p / \bar{k}.$$

The formal definition of the first firm's profit is:

$$(***) \quad \pi_1^H = p * q_1^H = \bar{k}[1 - Q] * q_1^H = \bar{k}[1 - \sum_{i=1}^{n_l} q_i^L - \sum_{j=1}^2 q_j^H] * q_1^H,$$

FOC:

$$(***) \quad \frac{\partial \pi_1^H}{\partial q_1^H} = \bar{k}[1 - Q] + \bar{k}[q_1^H] * [-1] = 0 = \underbrace{\bar{k}[1 - Q - q_1^H]}_{=0}.$$

Solve for the optimal quantity,  $q_1^H$ :  $q_1^H = 1 - Q = 1 - q(2 + n_l) \Rightarrow q + q(2 + n_l) = 1$ , the optimal output level for each firm is:  $q = 1/(3 + n_l) = 1/8$ , and the total market output is  $Q = (2 + n_l)/(3 + n_l) = 7/8$ . Consequently, the market-clearing price is:

$$\begin{aligned}
 (***) \quad p &= \bar{k}[1-Q] = \bar{k}[1-q(2+n_l)] = (2^*k_H)/(2+n_l) * (1-(2+n_l)/(3+n_l)) = \\
 &= 2^*k_H/((2+n_l)(3+n_l)) = 2^*k_H/56,
 \end{aligned}$$

the profit for each firm:

$$(***) \quad \pi_m^{H*} = p^* q_m^{H*} = \bar{k}[1-Q]^2 = \frac{2^*k_H}{2+n_l}[1-Q]^2 = \frac{2^*k_H}{2+n_l} \left[ 1 - \frac{(2+n_l)}{(3+n_l)} \right]^2,$$

which simplifies to:

$$(***) \quad \pi_m^{H*} = \frac{2^*k_H}{2+n_l} \left[ \frac{(3+n_l-2-n_l)}{(3+n_l)} \right]^2 = \frac{2^*k_H}{(2+n_l)(3+n_l)^2}.$$

Consumer surplus:

(\*\*\*)

$$CS = \int_{\theta_0}^1 (\theta \bar{k} - p^e) d\theta = \left[ \frac{\theta^2 \bar{k}}{2} - \theta p^e \right]_{\theta_0}^1 = \frac{\bar{k}}{2} - p^e - \frac{\theta_0^2 \bar{k}}{2} + \theta_0 p^e =$$

$$\frac{\bar{k}}{2} [1 - \theta_0^2] + p^e [\theta_0 - 1] =$$

$$\frac{\bar{k}}{2} \left[ 1 - \frac{p^2}{\bar{k}^2} \right] + p \left[ \frac{p}{\bar{k}} - 1 \right] = \frac{\bar{k}}{2} - \frac{1}{2} * \frac{p^2}{\bar{k}} + \frac{p^2}{\bar{k}} - p = \frac{\bar{k}}{2} + \frac{1}{2} \frac{p^2}{\bar{k}} - p$$

$$\rightarrow \text{[replace the expression for } \bar{k} = \frac{2^*k_H}{2+n_l} \text{]} \rightarrow$$

$$= \frac{1}{2} \left[ \frac{2^*k_H}{2+n_l} \right] + \frac{1}{2} \left[ \frac{2^*k_H}{2+n_l} \right]^{-1} * p^2 - p =$$

$$\rightarrow \text{[substitute in for } p = \frac{2^*k_H}{(2+n_l)(3+n_l)} \text{]}$$

$$= \frac{1}{2} \left[ \frac{2^*k_H}{2+n_l} \right] + \frac{1}{2} \left[ \frac{2^*k_H}{2+n_l} \right]^{-1} * \left[ \frac{2^*k_H}{(2+n_l)(3+n_l)} \right]^2 - \left[ \frac{2^*k_H}{(2+n_l)(3+n_l)} \right] =$$

$$\left[ \frac{k_H}{2+n_l} \right] + \frac{1}{2} \left[ \frac{2+n_l}{2^*k_H} \right] * \left[ \frac{2^*k_H}{(2+n_l)(3+n_l)} \right]^2 - \left[ \frac{2^*k_H}{(2+n_l)(3+n_l)} \right] =$$

$$\begin{aligned}
& \left[ \frac{k_h}{2+n_l} \right] + \frac{1}{2} \left[ \frac{2^* k_H}{(2+n_l)(3+n_l)^2} \right] - \left[ \frac{2^* k_H}{(2+n_l)(3+n_l)} \right] = \\
& \frac{k_h(3+n_l)^2}{(2+n_l)(3+n_l)^2} + \frac{k_H}{(2+n_l)(3+n_l)^2} - \frac{2^*(3+n_l)k_H}{(2+n_l)(3+n_l)^2} = \\
& \frac{k_H + k_h(3+n_l)^2 - 2^*(3+n_l)k_H}{(2+n_l)(3+n_l)^2} = \\
& \frac{k_H [9+6n_l+n_l^2-6-2n_l+1]}{(2+n_l)(3+n_l)^2} = \frac{k_H [4+4n_l+n_l^2]}{(2+n_l)(3+n_l)^2} = \frac{k_H [2+n_l]^2}{(2+n_l)(3+n_l)^2} = \frac{k_H (2+n_l)}{(3+n_l)^2}.
\end{aligned}$$

Consequently, the total welfare with no certification is equal to

$$(***) \quad W = CS + \sum_i \pi_i^{H*} + \sum_j \pi_j^{L*} = \frac{k_H(4+n_l)}{(3+n_l)^2}.$$

## XXVI. Nash-Cournot with certification

The marginal consumer can be identified as  $0 = \theta_0 k_h - p \rightarrow \theta_0 = p/k_h$ , and the inverse demand is  $p = k_h [1 - Q]$ .

The profit for each of the two high quality producers is:

$$(***) \quad \pi^h = p^* q_1 - C/2 = k_h [1 - Q]^* q_1^H - C/2 = k_h [1 - q_1 - q_2] q_1 - C/2,$$

where C is a fixed cost of establishing the certification scheme.

FOC:

$$(***) \quad \frac{\partial \pi_1^H}{\partial q_1^H} = k_h[1-Q] + k_h[q_1^H]^*[-1] = 0 = \underbrace{k_h[1-Q]}_{\neq 0} - \underbrace{q_1^H}_{=0},$$

and solving for  $q_1$  we get the best response function:  $q_1^*(q_2) = (1 - q_2)/2$ . Since both high quality producers have the same marginal cost, they have identical best response functions. The Nash-Cournot output level is:  $q_1^* = q_2^* = 1/3$ , and the market clearing price is  $p^e = k_h[1-Q] = k_h/3$ . Consequently, the profit for each high quality producer is:

$$(***) \quad \pi_1^* = \pi_2^* = p^* q - \frac{C}{2} = k_h \left[ 1 - \frac{1}{3} - \frac{1}{3} \right] \frac{1}{3} - \frac{C}{2} = 1/9 k_h - C/2.$$

The consumer surplus is:

$$(***) \quad CS = \int_{\theta_0}^1 (\theta k_h - p^e) d\theta = \left[ \frac{\theta^2 k_h}{2} - \theta p^e \right]_{\theta_0}^1 = \frac{k_h}{2} - \frac{k_h}{3} - \frac{k_h}{9 \cdot 2} + \frac{k_h}{3 \cdot 3} =$$

$$k_h \left[ \frac{3-1+2}{18} \right] = \frac{2}{9} k_h$$

Consequently, the total welfare under certification is:

$$(***) \quad W = CS + \pi_1^H + \pi_2^H = 4/9 \cdot k_H - C.$$

## XXVII. Collusion with certification

The cartel's profit:

$$(***) \quad \pi_{collusion} = p^* Q - C = k_h [1 - Q]^* Q - C ,$$

F.O.C.:

$$(***) \quad \frac{\partial \pi}{\partial Q} = k_h [1 - Q] - k_h [Q] = 0 = \underbrace{k_h [1 - Q - Q]}_{\substack{\neq 0 \\ = 0}} ,$$

and the resulting quantities and price are:  $q_1^* = q_2^* = 1/4$ , and  $p^e = k_h [1 - Q] = k_h / 2$ .

The marginal consumer is identified as:

$$(***) \quad \theta_0 = \frac{p}{k_h} = \frac{k_h}{2} \frac{1}{k_h} = \frac{1}{2} .$$

The producer profit is:

$$(***) \quad \pi^* = p^e * q^* - C/2 = k_h / 8 - C/2 .$$

The consumer surplus is:

$$(***) \quad CS = \int_{\theta_0}^1 (\theta k_h - p^e) d\theta = \left[ \frac{\theta^2 k_h}{2} - \theta p^e \right]_{\theta_0}^1 = \frac{k_h}{2} - \frac{k_h}{2} - \frac{1}{4} \frac{k_h}{2} + \frac{1}{2} \frac{k_h}{2} = \frac{1}{8} k_h ,$$

so the total welfare is equal to:  $W = CS + collusive(\pi_1^H, \pi_2^H) = 3/8 \cdot k_H - C$ .



## XXVIII. Base model: summary of scenarios

The table below provides a summary of the base model developed by Marette et al. The model is programmed in GAMS as a mixed complementarity problem (see appendix).

**Table 5.** Qualitative summary of Marette's paper.

| <i>Strategy</i>                 | <i>Output level</i>  | <i>Market clearing prices</i>        | <i>Consumers' surplus</i>           | <i>Producers' profit (per producer)</i>            | <i>Welfare (numerical)*</i>                            |
|---------------------------------|--|--------------------------------------|-------------------------------------|--|--|
| No certification                | $Q = \frac{(2+n_l)}{(3+n_l)}$<br>$q^* = \frac{1}{(3+n_l)}$ | $p^e = \frac{2*k_H}{(2+n_l)(3+n_l)}$ | $CS = \frac{k_H(2+n_l)}{(3+n_l)^2}$ | $\pi^* = \frac{2*k_H}{(2+n_l)(3+n_l)^2}$           | $W = CS + PS$<br>$= 0.109 +$<br>$7*0.004 =$<br>$0.141$ |
| Certification, and Nash Cournot | $Q = \frac{2}{3}$<br>$q_1^* = q_2^* = \frac{1}{3}$         | $p^e = \frac{k_h}{3}$                | $CS = \frac{2}{9}k_h$               | $\pi_1^* = \pi_2^* = \frac{1}{9}k_h - \frac{C}{2}$ | $W = CS + PS$<br>$= 0.222 +$<br>$2*0.036 =$<br>$0.295$ |
| Certification, collusion        | $Q = \frac{1}{2}$<br>$q_1^* = q_2^* = \frac{1}{4}$         | $p^e = \frac{k_h}{2}$                | $CS = \frac{1}{8}k_h$               | $\pi^* = \frac{k_h}{8} - \frac{C}{2}$              | $W = CS + PS$<br>$= 0.125 +$<br>$2*0.05 = 0.225$       |

\* Base model programmed in GAMS with the following parameter values:  $n_h=2$ ;  $n_l=5$ ;  $k_l=0$ ;  $k_h=1$ ;  $C=0.15$

## XXIX. Modification of demand: no certification

Expected quality of the products is:

$$(***) \quad \bar{k} = \Pr(\text{high}) * k_h + \Pr(\text{low}) * k_l = (2 * k_h + n_l * k_l) / (2 + n_l).$$

The marginal consumer has the taste parameter  $\theta_0 = p / \bar{k}$ . Consequently, total demand is:

$$(***) \quad Q = 1 - \theta_0 = 1 - p / \bar{k}.$$

The profit is defined as:

$$(***) \quad \pi_m = p \cdot q_m = \bar{k}[1 - Q] \cdot q_m = \bar{k} \cdot [1 - \sum_{i=1}^{n_l} q_i^L - \sum_{j=1}^2 q_j^H] \cdot q_m \quad m \in \forall H, L,$$

and the F.O.C. is:

$$(***) \quad \frac{\partial \pi_m}{\partial q_m} = \bar{k}[1 - Q] + \bar{k}[q_m^H] * [-1] = 0 = \underbrace{\bar{k}[1 - Q - q_m]}_{\substack{\neq 0 \\ = 0}},$$

and the output level per firm is  $q = 1/(3 + n_L) = 1/8$ , and the total market output is  $Q = (2 + n_L)/(3 + n_L) = 7/8$ .

The marginal consumer is located at the distance  $\theta_0 = 1 - Q = 1/(3 + n_L) = 1/8$ . Since consumer have utility of consuming low quality products,  $k_l \neq 0$ , the market-clearing price is somewhat different from the base model:

$$(***) \quad \theta_0 = \frac{p^e}{\bar{k}} \Rightarrow p^e = \theta_0 \cdot \bar{k} \rightarrow p^e = \frac{(2 \cdot k_H + n_L \cdot k_L)}{(2 + n_L)(3 + n_L)} \rightarrow p^e = \frac{5}{56} \cdot k_L + \frac{1}{28} \cdot k_H.$$

Profit for each firm is:

$$(***) \quad \pi_m^* = (2 * k_h + n_l * k_l) / ((2 + n_l)(3 + n_l)^2).$$

The per-producer profit is equal to:

$$(***) \quad \pi_m^* = \frac{5}{448} \cdot k_L + \frac{1}{224} \cdot k_H .$$

The consumer surplus is:

$$(***) \quad CS = \int_{\theta_0}^1 (\bar{\theta} k - p^e) d\theta = \frac{1}{2} \cdot \frac{(2+n_L)(2 \cdot k_H + n_L \cdot k_L)}{(3+n_L)^2} = \frac{35}{128} \cdot k_L + \frac{7}{64} \cdot k_H .$$

The total welfare is:

$$(***) \quad W = CS + \sum_i \pi_i^{H*} + \sum_j \pi_j^{L*} = \frac{[(2 \cdot k_H + n_L \cdot k_L)(2+n_L)]}{(3+n_L)^2} + \frac{(2 \cdot k_H + n_L \cdot k_L)}{(3+n_L)^2} = \frac{45}{128} \cdot k_L + \frac{9}{64} \cdot k_H .$$

Thus, in the scenario with no certification, but where the consumers have preferences for low quality products yields a higher welfare, than when  $k_L=0$ , ceteris paribus:

$$(***) \quad W(k_H \geq k_L \geq 0) = \frac{45}{128} \cdot k_L + \frac{9}{64} \cdot k_H \geq W(k_H \geq k_L = 0) = \frac{9}{64} \cdot k_H$$

### XXX. Modified demand: Certification

This particular model has two types of marginal consumers: those indifferent between buying either product, and those indifferent between buying nothing and the low quality product. First, the marginal consumer that is indifferent between consuming either product is described by the taste parameter  $\theta_0$ :

$$(***) \quad \theta_0 k_H - p_H = \theta_0 k_L - p_L ,$$

or, alternatively:

$$(***) \quad \theta_0 = \frac{p_H - p_L}{k_H - k_L} .$$

and the demand for high quality products is:

$$(\text{***}) \quad Q_H = 1 - \frac{p_H - p_L}{k_H - k_L}$$

The marginal consumer at  $\theta_{00}$  is indifferent between consuming nothing and the low quality product. Solving for  $\theta_{00}$ :

$$(\text{***}) \quad \theta_{0,0} = \frac{p_L}{k_L}$$

The demand for the low quality product is:

$$(\text{***}) \quad Q_L = \frac{p_H - p_L}{k_H - k_L} - \frac{p_L}{k_L}$$

and the inverse demands are:

$$(\text{***}) \quad p_H = k_H - Q_H k_H - Q_L k_L,$$

and

$$(\text{***}) \quad p_L = (1 - Q_H - Q_L) k_L.$$

Since  $k_L \neq 0$ , the demands are inversely dependent on the degree of substitutability.

Hence, this case is similar to the product differentiation case.

The profit for the high quality producer is:

$$(\text{***}) \quad \pi_1^H = p^* q_1^H = (k_H - Q_H k_H - Q_L k_L) q_1^H - C/2,$$

with F.O.C.:

$$(\text{***}) \quad \frac{\partial \pi_1^H}{\partial q_1^H} = (k_H - Q_H k_H - Q_L k_L) - k_H q_1^H = 0.$$

Solving for first high quality firm's best response function:

$$(***) \quad q_1^H(q_2^H, q_i^L, i=1..5) = \frac{(k_H - Q_L \cdot k_L - q_2^H \cdot k_H)}{2 \cdot k_H}.$$

By analogy, the profit for the (first) low quality producer is:

$$(***) \quad \pi_1^L = p \cdot q_1^L = (1 - Q_H - Q_L) \cdot k_L \cdot q_1^L,$$

and the first order condition is:

$$(***) \quad \frac{\partial \pi_1^L}{\partial q_1^L} = (1 - Q_H k_H - Q_L k_L - q_1^L) \cdot k_L = 0,$$

so the best response function for the (first) low quality producer is:

$$(***) \quad q_1^L(Q^H, q_{-i}^L) = \frac{(1 - \sum_{j \neq i} q_j^L - Q^H)}{2}.$$

The optimal output levels for each firm is found by solving the systems of best response functions, or by solving the respective first order conditions. Note that  $q_i^{L*} = q_j^{L*} \forall i, j = 1..n$ , so:

$$(***) \quad q_i^{L*} = \frac{k_H}{18 \cdot k_H - 10 \cdot k_L} \forall i \in [1..n]$$

by analogy, the optimal output level for each of the two high-quality firms is:

$$(***) \quad q_i^{H*} = \frac{6 \cdot k_H - 5 \cdot k_L}{18 \cdot k_H - 10 \cdot k_L} \forall i \in [1..2],$$

so the market output levels are:

$$(***) \quad Q^H = \frac{6 \cdot k_H - 5 \cdot k_L}{9 \cdot k_H - 5 \cdot k_L},$$

and

$$(***) \quad Q^L = \frac{5 \cdot k_H}{18 \cdot k_H - 10 \cdot k_L},$$

and the corresponding market clearing price for high quality products with certification is:

$$(***) \quad p_h = \frac{(6 \cdot k_h - 5 \cdot k_l)k_h}{18 \cdot k_H - 10 \cdot k_L},$$

and for the low quality products:

$$(***) \quad p_l = \frac{k_h \cdot k_L}{18 \cdot k_H - 10 \cdot k_L},$$

and the marginal consumer between buying the high or low quality product is defined as:

$$(***) \quad \theta_0 = \frac{3 \cdot k_H}{9 \cdot k_H - 5 \cdot k_L},$$

and the marginal consumer between buying the low quality product and nothing is defined as:

$$(***) \quad \theta_{00} = \frac{k_H}{18 \cdot k_H - 10 \cdot k_L},$$

and the profit for the high quality producers are:

$$(***) \quad \pi_1^H = \pi_2^H = \frac{1}{4} \frac{(6 \cdot k_h - 5 \cdot k_l)^2 k_h}{(9 \cdot k_H - 5 \cdot k_L)^2} - \frac{C}{2},$$

and for the low quality producer:

$$(***) \quad \pi_i^L = \frac{1}{4} \frac{k_H^2 \cdot k_l}{(9 \cdot k_H - 5 \cdot k_L)^2}, i \in [1..5],$$

The consumers' surplus from consuming the high quality products are:

$$(***) \quad CS_H = \int_{\theta_0}^1 (\theta k_H - p_H) d\theta = \left[ \frac{\theta^2 k_H}{2} - \theta p_H \right]_{\theta_0}^1 = \frac{3 \cdot (k_H)^2 (6 \cdot k_H - 5 \cdot k_L)}{(9 \cdot k_H - 5 \cdot k_L)^2},$$

and from consuming the low quality products:

$$(***) \quad CS_L = \int_{\theta_{0,0}}^{\theta_0} (\theta k_L - p_L) d\theta = \left[ \frac{\theta^2 k_L}{2} - \theta p_L \right]_{\theta_{0,0}}^{\theta_0} = \frac{25}{8} \cdot \frac{(k_H)^2 \cdot k_L}{(9 \cdot k_H - 5 \cdot k_L)^2},$$

so the total consumer surplus is:

$$(***) \quad CS_{TOT} = CS_H + CS_L = \frac{1}{8} \cdot \frac{(k_H)^2 (144 \cdot k_H - 95 \cdot k_L)}{(9 \cdot k_H - 5 \cdot k_L)^2}.$$

The analytical expression for total welfare is:

$$(***) \quad W_{Certification} = CS_{TOT} + \sum_i \pi_i^{H*} + \sum_j \pi_j^{L*} = \frac{1}{8} \cdot \frac{k_H (288 \cdot k_H^2 - 325 \cdot k_L \cdot k_H + 100 \cdot k_L^2)}{(9 \cdot k_H - 5 \cdot k_L)^2} - C.$$

### XXXI. Modified demand: summary

Numerical estimates for the no-certification and certification scenario are summarized in the table below. Collusive action is omitted, since the producer always has incentive to deviate from collusive quantity due to the existence of positive marginal profit.

**Table 6.** Numerical estimates with the extended Marette model.

| Strategy                        | Numerical estimates*   |                              |  |                                  |               |
|---------------------------------|--|------------------------------|--|----------------------------------|---------------|
|                                 | Output level   | Prices                       | Consumers' surplus   | Producers' profit (per producer) | Total welfare |
| No certification                | $q_L = q_H = .125$<br>$Q = .875$                             | $p^e = .161$                 | $CS = .492$  | $\pi = .020$                     | $W = .633$    |
| Certification, and Nash Cournot | $q_H = .269$<br>$q_L = .077$<br>$Q_H = .538$<br>$Q_L = .385$ | $p_H = .538$<br>$p_L = .077$ | $CS_{Low} = .089$<br>$CS_{High} = .393$<br>$CS_{TOT} = .482$ | $\pi_H = .145$<br>$\pi_L = .006$ | $W = .741$    |

\* Base model programmed in GAMS with the following parameter values:  $n_h=2$ ;  $n_l=5$ ;  $k_l=1$ ;  $k_h=2$ ;  $C=0.15$   
The result from collusive behavior is not presented here, as it is not a Nash equilibrium