

University of Jyväskylä
Jyväskylä, Finland
27th-30th August 2003

Comparing home-grown fruits: Productivity convergence across industries and regions

Stein Ostbye* and Olle Westerlund**

**Department of Economics, University of Tromsø, N-9037 Tromsø, Norway*

***Department of Economics, University of Umeaa, S-90187 Umeaa, Sweden*

Version: 15 June 2003

Abstract

Output per worker is radically unevenly distributed across space. Several authors have asked why the differences are so large between countries and hypothesized that differences in social infrastructure provide an answer. However, differences in output per worker are also very different when comparing spatial units, at lower levels of resolution, without substantial variation in social infrastructure. The purpose of this paper is to discuss possible reasons why. We will do so by looking at regional data for the Scandinavian Peninsula at a spatial resolution equivalent to the European NUTS3. Since Norway and Sweden is considered particular egalitarian and homogeneous societies, differences in broad measures of social infrastructure can hardly be invoked as substantial important determinants of productive performance. Instead we suggest that differences in industrial structure and human capital are able to explain the differences we observe.

JEL classification: O57, R11, R12

Key words: Comparative study, Economic growth

* Tel: +47-776-46135; fax: +47-776-46021; e-mail: steino@fh.uit.no

1. Introduction

Yet another paper based on the neoclassical growth model, you may object. So what is the interest of the present one that makes it indispensable reading? We suggest five reasons why. First, we formally link the conventional growth equation, used in numerous empirical applications, to the parameters of a translog neoclassical growth model. With different production technology in different production sectors in an economy, represented by different Cobb-Douglas production functions, van Garderen et al. (2000) have shown that the translog is the only aggregate setup that does allow consistent aggregation. Second, we use regional data in order to look at the issue of convergence at the industry level during the 80s and the 90s. Previously, Bernard and Jones (1996a) has been looking at the same issue during the 70s and 80s based on international data, but as argued by Sørensen (2001), the analysis is not robust. Using international data, they need an international conversion factor in order to make the data comparable and their conclusions depend on this construction, which is basically arbitrary. Third, we look for evidence of regional convergence based on Swedish and Norwegian data and discuss the implied parameter values of the translog production functions based on benchmark values for exogenous parameters. The results are compared to the standard Cobb-Douglas case. Fifth, we extend the model to include human capital in the translog setup.

Comparative analysis of productive performance across regions has in particular been related to questions concerning economic growth. Are less productive regions catching up to more productive ones, eventually how quickly and for what reasons? The results have often been interpreted in terms of the one-sector neoclassical growth model, following the research by Barro and Sala-i-Martin (1991).

Bernard and Jones (1996a) argue that looking for why countries appear to converge based on the one-sector model is misleading since it is too narrowly focused on the role of diminishing returns to capital interpreted in either a narrow (physical) or broad sense (physical and human). Comparing countries, output per worker in some sectors may converge and in other diverge. The net effect depends on how these opposing effects balance and how the industry structure is changing over time. In this sense the insights made by Colin Clark (1940) and the role played by changing industry structure in the process of economic growth should be integrated into the analysis and not ignored (but see, e.g., Kongsamut et al, 2001, Caselli and Coleman, 2001). As pointed out by Sørensen (2001) comparative studies of sectoral productivity based on international data, like the one by Bernard and Jones (1996a), should not take place if robust conversion factors for making data internationally comparable are not available. A posteriori, it appears that manufacturing is sensitive and others not, including aggregate productivity. A priori, we do not know. This problem of comparability is avoided when we look at regions within one country. In particular, we may provide an answer to whether manufacturing productivity has converged or not.

The production function used in neoclassical growth theory was postulated in macro without any attempt to provide micro foundations. Robert Solow himself ‘never thought of the macroeconomic production function as a rigorously justifiable concept’ but ‘either an illuminating parable or else a mere device for handling data, to be used so long as it gives good empirical results, and to be abandoned as soon as it doesn’t, or as soon as something better comes along’ (Solow, 1966, pp. 1259-1260, cited by Sato, 1975, p. 3). However, attempts were later made in order to provide micro foundations, notably by Franklin Fisher and Kazuo Sato (see the collection of papers by Fisher, edited by Monz and with an interesting introduction by the author, Fisher, 1992, and the book by Sato, Sato, 1975).

Unfortunately, the aggregation problem as traditionally formulated is only resolved under very stringent conditions. The problem is therefore often ignored altogether in applied work.

Another, more promising, route to the issue is suggested by Garderen et al. (2000). Let us restrict attention to the Cobb-Douglas specification since it is often used for analytical convenience, even in empirical work. Based on sectoral Cobb-Douglas production functions, consistent aggregation into an aggregate Cobb-Douglas function is extremely restrictive. As is well known, the Cobb-Douglas is only permissible when all the sectoral factor weights are equal (see, e.g., *ibid.*). Less noticed however, under a generalized version of the Hicks aggregation condition, sectoral factor weights may be different provided that squares and cross-products of the explanatory variables in the log-linear specification of the model are included, in other words that the aggregate relationship takes the flexible form known as translog (*ibid.*).

In order to accommodate differences in industry structure between units of analysis in a consistent way, it is therefore tempting to suggest that growth empirics should be reinterpreted in terms of a more flexible representation than Cobb-Douglas. This issue is elaborated upon in Section 3. Estimation matters are considered in Section 4 and Section 5 concludes. First, however, we take a look at regional output per worker in different production sectors, based on Norwegian data for the two last decades of the 20th century.

2. Comparing home-grown fruits

The title of Bernard and Jones (1996a) is ‘Comparing apples to oranges’, implying that it would be wrong to lump different industries together in order to concentrate attention on the aggregate economy only if the productivity of some industries diverge and others converge. Very much so, but in the paper home-grown apples are compared to foreign ones and unfortunately this turns out to be just as bad (see Sørensen, 2001). To avoid this problem, here we only compare home-grown fruits from different parts of one country.

Bernard and Jones (1996a) suggest that manufacturing productivity has diverged, and that convergence in aggregate productivity therefore is driven by convergence in other sectors. Using power purchasing parities for different base years, Sørensen (2001) shows that their conclusion may be reversed. Let us therefore start out by looking at manufacturing productivity for the 19 NUTS 2 regions in our Norwegian sample. The productivity measure used is productivity per worker. All Norwegian data are provided by Statistics Norway.

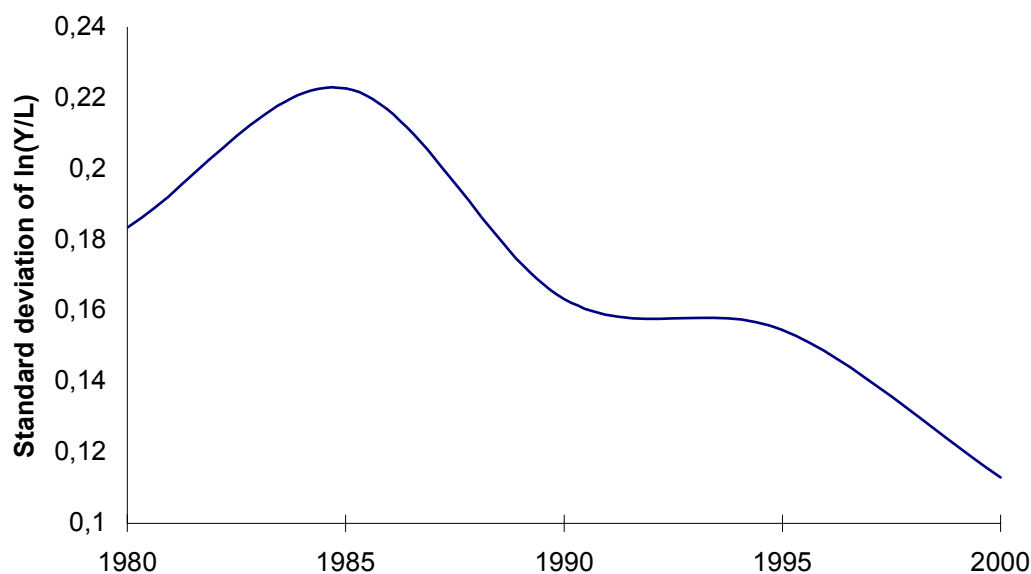


Figure 1. Regional labor productivity dispersion in Norwegian manufacturing

Referring to Figure 1, the dispersion rose in the first period and fell in all the following three periods, leaving the dispersion reduced by one third compared to the initial situation. The conclusion is therefore that, except for the first period, we have reduced dispersion in our data, contrary to what Bernard and Jones (1996a) believed to have found in the OECD data for the 70s and 80s. This corresponds to the concept ‘absolute convergence’ or ‘ σ -convergence’ in the convergence literature (see, e.g., Sala-i-Martin, 1996).

When we aggregate across industries, the almost perfect log-linear relationship in Figure 2 emerges. Now, σ -convergence appears to have taken place even in the first period.

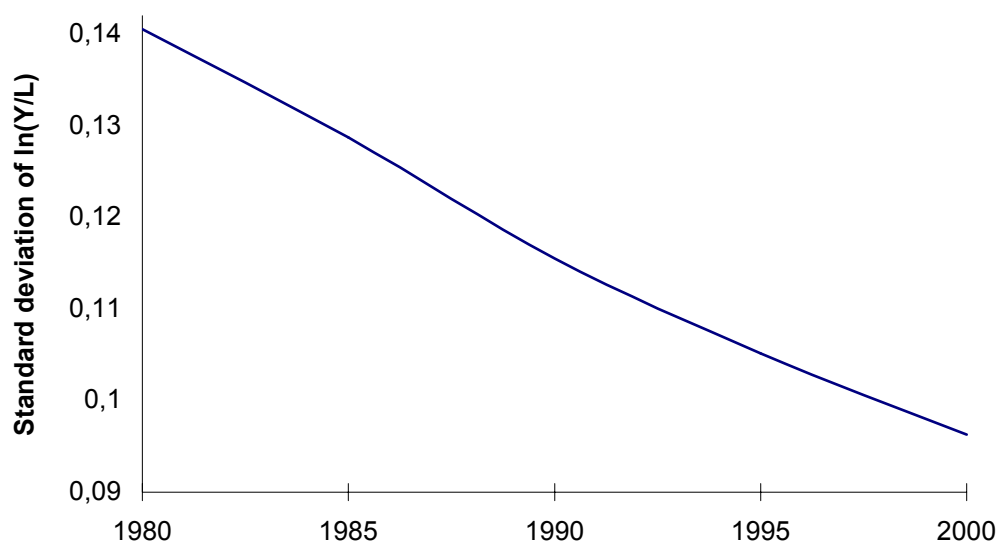


Figure 2. Regional labor productivity dispersion for total Norwegian production

The explanation is clear from Figure 3, showing the development of dispersion for services. A very substantial drop in the dispersion from 1980 to 1985 dominates the rise in dispersion for manufacturing. It is also clear that services do not contribute to the overall σ -convergence after 1985. Here manufacturing dominates services.

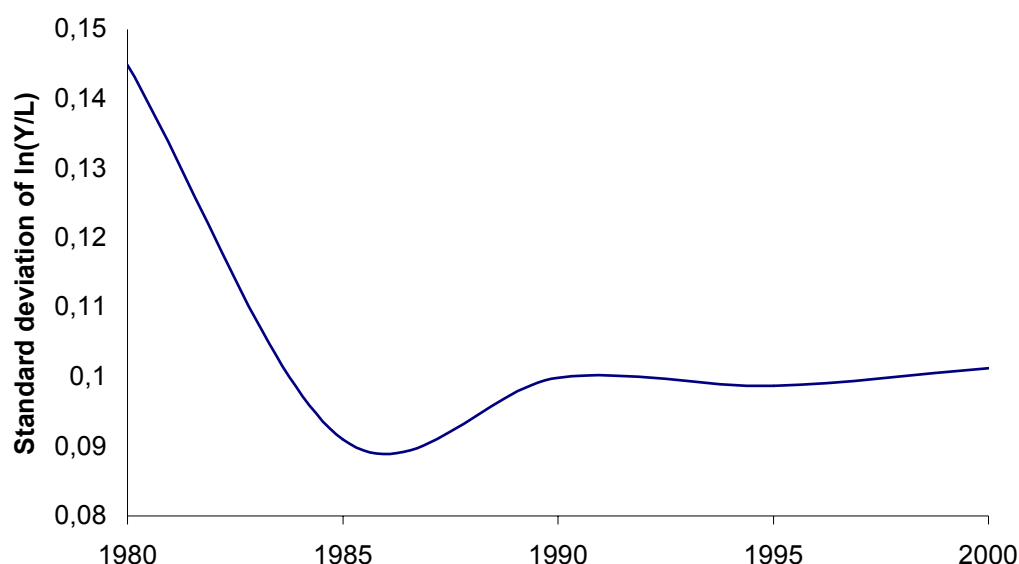


Figure 3. Regional labor productivity dispersion in Norwegian services

We do not, at least for the moment, have data on Swedish ‘apples’ and ‘oranges’. However, Figure 4 shows the regional dispersion in income per capita (aged 20-64) for the 24 Swedish counties in the same period from 1980 to 2000. The income data are based on gross income net of taxable government transfers, source: Statistics Sweden.

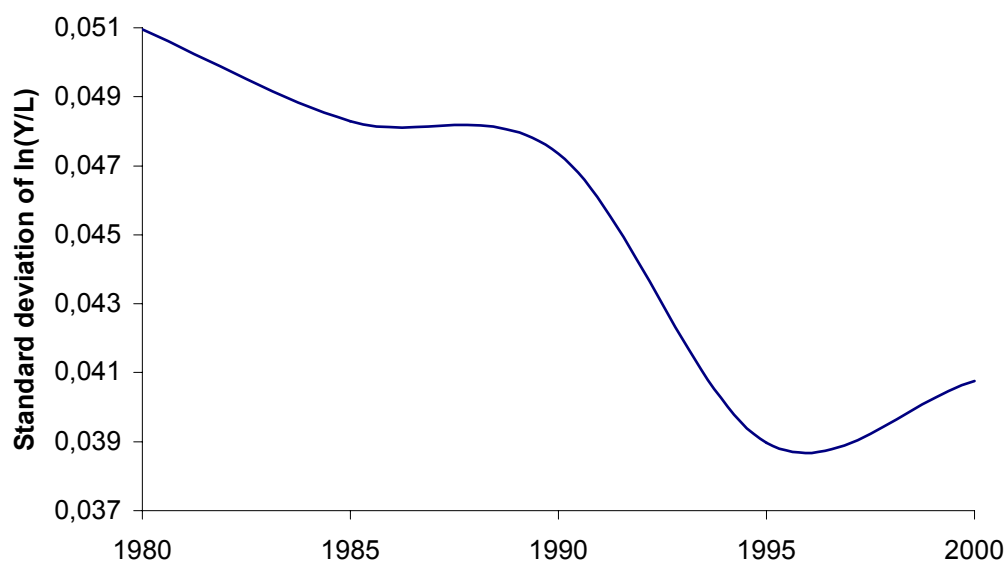


Figure 4. Regional income dispersion in Sweden

Compared to the Norwegian case, convergence appears less smooth and is partially reversed in the last period from 1995. Is this because the industry composition in Sweden and Norway differs or is it because the development in services and manufacturing has been different? We can only guess, but an understanding of what is behind the aggregate must surely be interesting from a policy-making perspective.

If we return to Norway, an interesting regularity is the counter cyclical movements of services and manufacturing. This may be more clearly seen from Figure 4, where also the substantial less important agricultural sector is included. When the productivity of services is diverging, the convergence of productivity elsewhere is sufficient for the aggregate to converge, and the other way around.

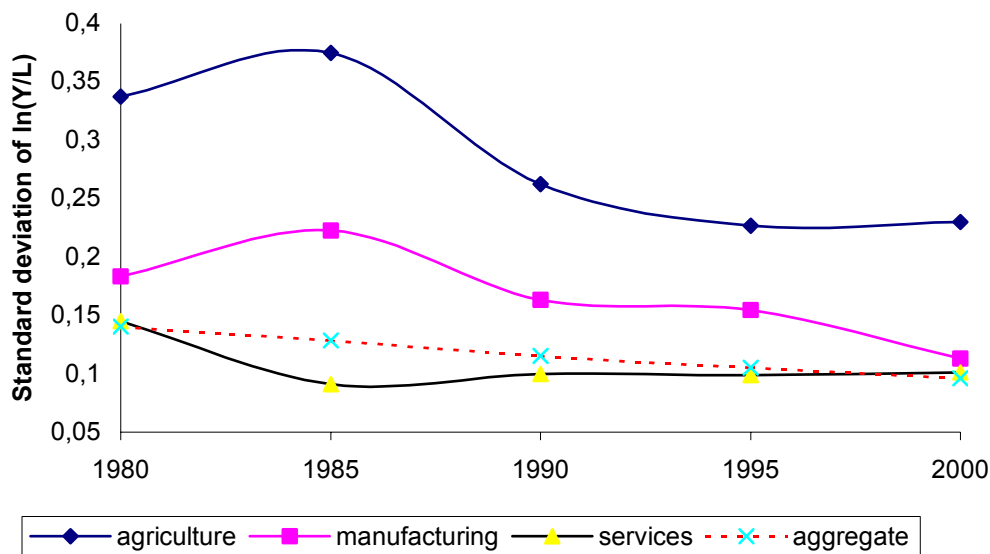


Figure 5. Regional labor productivity dispersion for Norwegian production sectors

The different development for different sectors suggests that industry composition plays a key role in maintaining stable convergence over time. To get an impression of the industry structure in 1980, we have computed location quotients for manufacturing and industries. We have used production rather than employment as basis for the computations. Location quotients for services are plotted against location quotients for manufacturing in Figure 6. The few observations in the upper left corner compared to the many in the lower right one suggest that services are more heavily concentrated in space than is manufacturing. The two observations to the far left represent Oslo and Akershus, the most important urban concentration. Hence, using agglomeration proxies in the growth equation, as is done in some studies, raise the question whether one is controlling for industry structure or agglomeration effects.

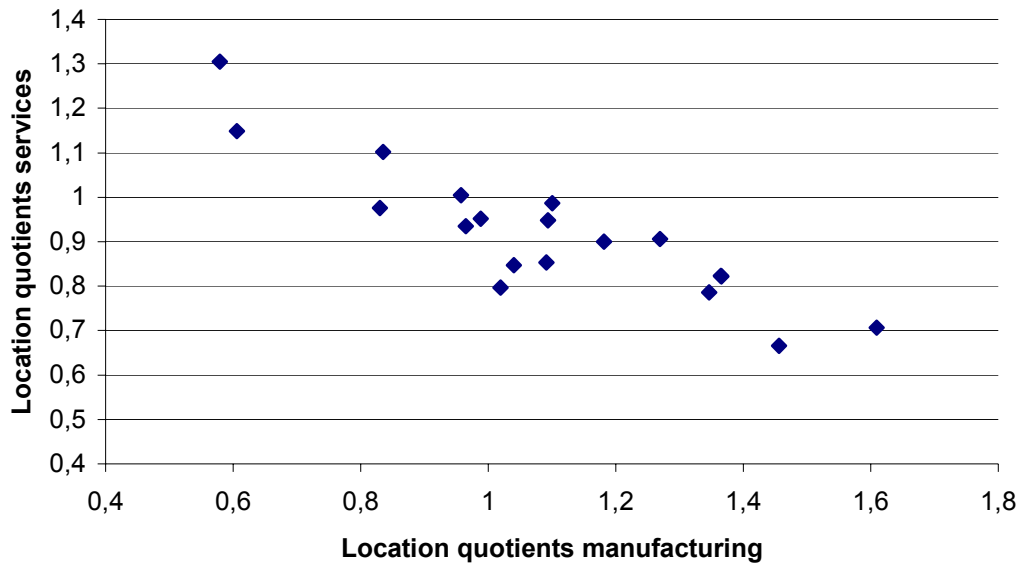


Figure 6. Industry structure 1980

How stable is the structure over time? We have plotted the location quotients for 2000 against 1980 for services (Figure 7) and manufacturing (Figure 8).

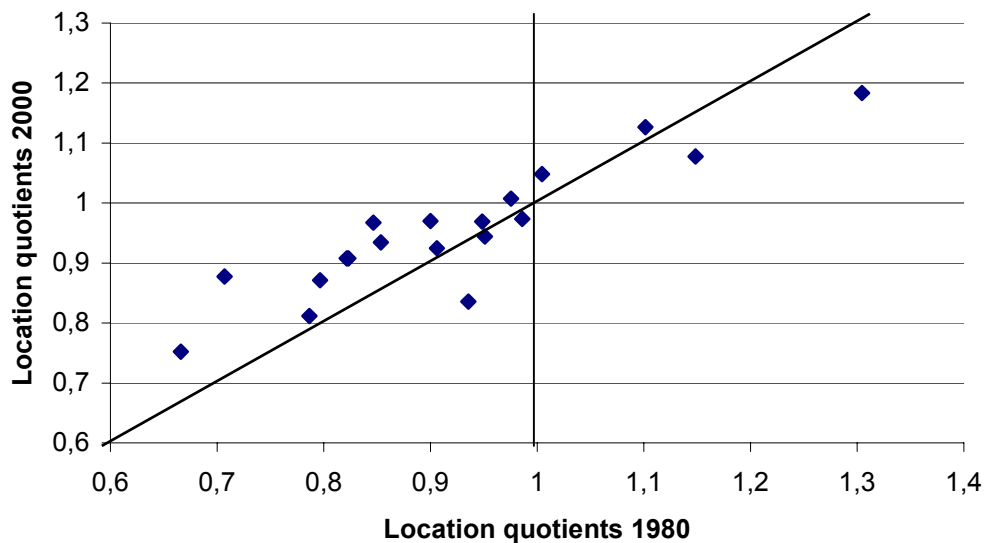


Figure 7. Persistence in industry structure 1980-2000: services

For services we note that the regional concentration has been decreasing. The regions to the left of the vertical line (location quotients less than one for 1980) are almost all above the diagonal, and the other way around for the regions to the right (location quotients exceeding one). Hence, there is a move towards a more equal spatial distribution. There is no such tendency for manufacturing as can be verified by inspecting the scatter plot in Figure 8.

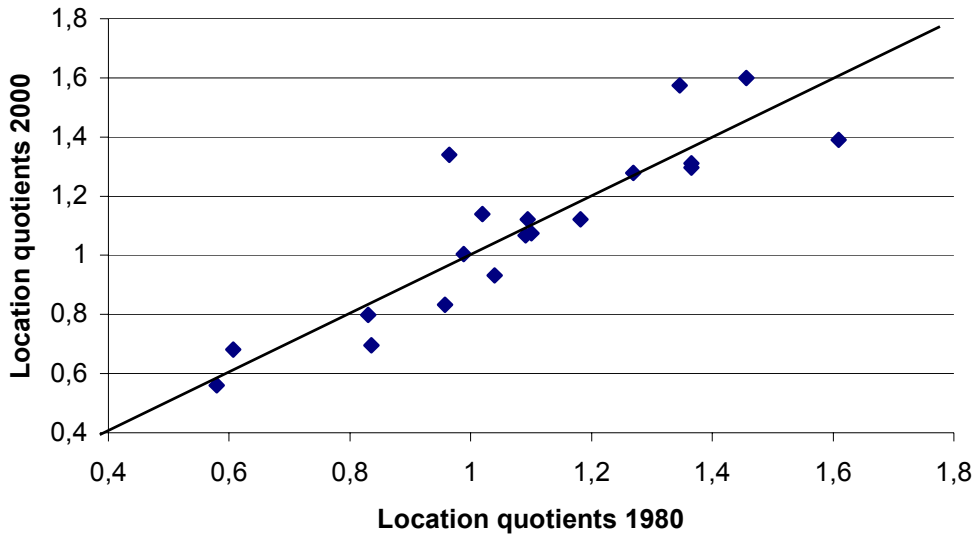


Figure 8. Persistence in industry structure 1980-2000: manufacturing

In view of the differences in industry composition, revealed in Figure 6, and the differences in convergence between industries, revealed in Figure 5, it is difficult to proceed based on the one-sector neoclassical growth model, as if industry composition did not matter. We will take this under consideration when we now turn to the issue of an appropriate theoretical setup for determining how fast the gap between productivity laggards and leaders, poor and rich, are closed.

3. Convergence and translog technology

In order to accommodate for differences arising from heterogeneity of goods produced within an economy, Bernard and Jones (1996b) suggest that the aggregate production function could be written in Cobb-Douglas form as

$$Y_i = K_i^{\alpha_i} (A_i L_i)^{1-\alpha_i} \quad (1)$$

where efficiency of labour, A_i , varies between countries because of, e.g., initial differences in efficiency due to different industrial structure (or climate, institutions and whatever, see Mankiw et al., 1992, and Islam, 1995), but here even the factor weights, α_i , are country specific. Again this can be attributed to different industry structure or as Bernard and Jones (1996b) put it: "Economically, we can interpret this difference as arising from the heterogeneity of goods that are produced within an economy. Such heterogeneity may be particular important if we examine the convergence of productivity at the sectoral rather than the aggregate level, for example." (p. 1039) From an econometrical point of view, the BJ suggestion amounts to potential different regressions for every spatial units and may be translated into a log-linear model with different intercepts as well as slope parameters. Available data are often insufficient in order to identify all these parameters.

A feasible approach could be to impose a common slope parameter, α_s , for spatial units with similar industry structure, based on, e.g., information on location quotients as used in Figure 6 in the preceding section.

$$Y_i = K_i^{\alpha_s} (A_i L_i)^{1-\alpha_s} \quad (2)$$

Employing lower cases for variables per effective labour unit, we may write (2) more conveniently in intensive form as

$$y_i = k_i^{\alpha_s} \quad (3)$$

The Cobb-Douglas form is unfortunately at odds with consistent aggregation, as mentioned in the introduction. Based on for example sectoral Cobb-Douglas production functions, consistent aggregation into an aggregate Cobb-Douglas function is extremely restrictive. It is only permissible when all the sectoral factor weights are equal (see Garderen et al., 2000). In this sense, using Cobb-Douglas in the one-sector model is less a problem, from a theoretical point of view, than in a multisector model. What should we do then? Perhaps ignore the aggregation problem and proceed as if it did not exist? We think not. Interestingly, under a generalized version of the Hicks aggregation condition, sectoral factor weights may be different provided that squares and cross-products of the explanatory variables in the log-linear specification of the model are included, in other words that the aggregate relationship takes the flexible form known as translog (ibid.). In order to accommodate differences in industry structure between units of analysis in a consistent way, it is therefore tempting to suggest that growth empirics should be reinterpreted in terms of a more flexible representation than Cobb-Douglas. It is simply wrong when Bernard and Jones (1996b, p.1232) claim: “Using either the CES or translog production setup does not address the fundamental problem that the parameters of the production function may vary across countries. For this reason and with an appeal to simplicity, we maintain the assumption of Cobb-Douglas functional form...”

By Young’s theorem and imposing linear homogeneity (see, e.g., Chambers, 1986, p. 181), the equivalent to (3) in translog form may be written

$$\ln y_i = \alpha_s \ln k_i + \frac{1}{2} \gamma_s \ln^2 k_i \quad (4)$$

Differentiating (4) logarithmically, we obtain the marginal product,

$$dy_i / dk_i = y_i (\alpha_s + \gamma_s \ln k_i) / k_i \quad (5)$$

Subtracting the logarithm of capital per effective labour unit from both sides of equation (4), and again differentiating logarithmically, we obtain

$$d(y_i / k_i) / dk_i = y_i (\alpha_s + \gamma_s \ln k_i - 1) / k_i^2 \quad (6)$$

Hence, the production function exhibits positive and diminishing marginal product when

$$0 < \alpha_s + \gamma_s \ln k_i < 1 \quad (7)$$

The Inada (1963) conditions are not globally satisfied, since (7) will only hold if k_i is constrained. For γ_s positive

$$-\alpha_s / \gamma_s < \ln k_i < (1 - \alpha_s) / \gamma_s \quad (8)$$

and reversing the inequalities for γ_s negative. As long as (7) is valid, however, the translog is consistent with the Inada conditions in the sense that the marginal product approaches a value arbitrarily close to zero and infinity by an appropriate choice of parameter values, when k_i goes to infinity and zero, respectively. Consider, i.e., the case when γ_s is positive: In the limit, when capital is approaching the upper bound, we have

$$\ln k_i \rightarrow (1 - \alpha_s) / \gamma_s \Rightarrow dy_i / dk_i \rightarrow e^{-(1-\alpha_s)^2 / \gamma_s} \quad (9)$$

If we expand the upper bound, by letting γ_s approach zero, we see that the marginal product approaches zero, consistent with the Inada conditions. When considering the lower bound, it is convenient to set $\ln k_i$ equal to $-\alpha_s / \gamma_s + \varepsilon$, where ε is a small number arbitrarily close to zero. The marginal product then becomes

$$dy_i / dk_i = \frac{\gamma_s \varepsilon}{e^{(\varepsilon - \alpha_s / \gamma_s)(1 - \alpha_s - 1/2\gamma_s(\varepsilon - \alpha_s / \gamma_s))}} \quad (10)$$

Both the numerator and the denominator approach zero when γ_s approach zero, however at a very different pace and clearly the exponential function in the denominator dominates. Hence, the marginal product approaches infinity, again consistent with the Inada conditions. Hence, we may regard the translog as an approximation to a neoclassical production function.

The dynamics of the neoclassical growth model with the translog setup is given by

$$\dot{k}_i = s_i k_i^{\alpha_s + 1/2\gamma_s \ln k_i} - (n_i + g + \delta)k_i \quad (11)$$

where the rates of saving, labour force growth, technological progress and depreciation are s_i , n_i , g and δ , respectively. A dot means the time derivative. Following Mankiw et al. (1992), we allow specific rates of saving and labour force growth for each spatial unit, whereas the rate of technological progress and depreciation is the same. In steady state, capital per effective worker is constant. The steady state level, k_i^* , is therefore given by

$$s_i k_i^{*\alpha_s + 1/2\gamma_s \ln k_i^* - 1} = n_i + g + \delta \quad (12)$$

or, for later reference (Figure 5), solving for the steady state level of capital per unit output,

$$k_i^{*1 - \alpha_s - 1/2\gamma_s \ln k_i^*} = s_i / (n_i + g + \delta) \quad (13)$$

Taking logs on both sides of (12) and rearranging, we obtain

$$\ln^2 k_i^* + \frac{2(\alpha_s - 1)}{\gamma_s} \ln k_i^* = \frac{2}{\gamma_s} \ln \left(\frac{n_i + g + \delta}{s_i} \right) \quad (14)$$

Completing the square and solving, we have

$$\ln k_i^* = \frac{1 - \alpha_s}{\gamma_s} \pm \frac{1}{\gamma_s} \sqrt{(\alpha_s - 1)^2 + 2\gamma_s \ln \left(\frac{n_i + g + \delta}{s_i} \right)} \quad (15)$$

However, for (7) to hold in steady state, only the negative root is feasible.

$$\ln k_i^* = \theta_i, \quad \theta_i \equiv \frac{1 - \alpha_s}{\gamma_s} - \frac{1}{\gamma_s} \sqrt{(\alpha_s - 1)^2 + 2\gamma_s \ln \left(\frac{n_i + g + \delta}{s_i} \right)} \quad (16)$$

Substituting $\ln k_i^*$ for $\ln k_i$ in (4), we obtain the steady state level of output per effective worker,

$$\ln y_i^* = \theta_i (\alpha_s + 1/2\gamma_s \theta_i) \quad (17)$$

The steady state solution is illustrated in Figure 5.

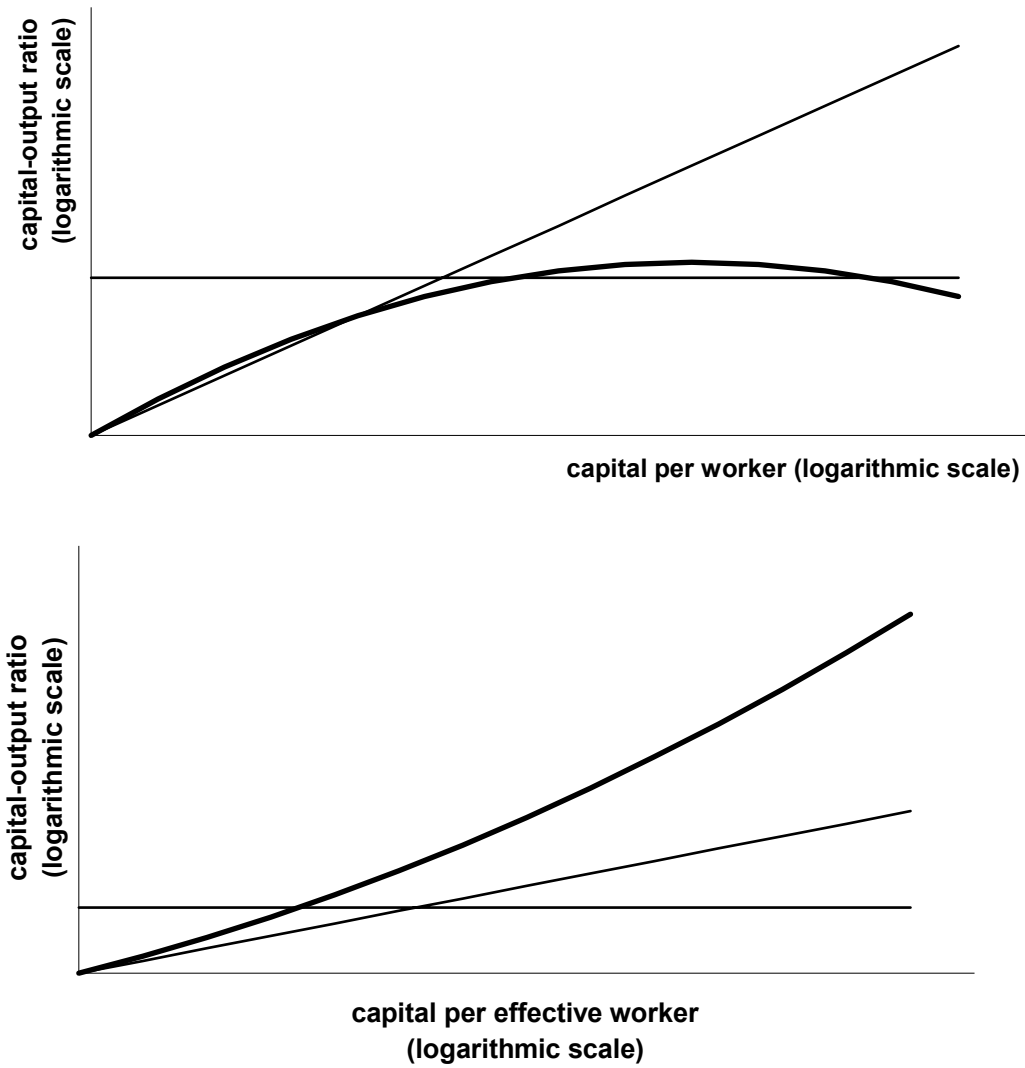


Figure 5. Steady-state

The horizontal line represents the capital-output ratio in steady-state. The intersection with the ray, representing Cobb-Douglas, gives the steady-state level of capital per worker in the Cobb-Douglas case. The intersection with the curve, representing translog, gives the equivalent in the translog case. For positive γ_s , the curve is concave (the upper panel) and for negative γ_s it is convex (the lower panel). The intersection with the concave curve to the far right in the upper panel represents the solution that violates the restriction imposed by (7). If we had extended the convex curve to the left in the lower panel, we would have seen the infeasible solution to the far left.

Let us now look at the dynamics outside steady state and return to equation (11). Instead of working with a specified form, it is now convenient to write $y_i = f(k_i)$. Around steady state,

$$\dot{y}_i = f'(k_i^*) \dot{k}_i \quad (18)$$

Approximating the true functional form around steady state by a first order Taylor expansion, we have approximately,

$$f(k) = f(k_i^*) + f'(k_i^*)(k_i - k_i^*) \quad (19)$$

or

$$y_i^* - y_i = f'(k_i^*)(k_i^* - k_i) \quad (20)$$

The steady state level of capital is given by

$$s_i f(k_i^*) = (n_i + g + \delta) k_i^* \quad (21)$$

and the dynamics may be written

$$\dot{k}_i = s_i f(k_i) - (n_i + g + \delta) k_i \quad (22)$$

Substituting for $f(k)$ from (19) and for s_i from (21),

$$\dot{k}_i = \left[\frac{f'(k_i^*) k_i^*}{f(k_i^*)} - 1 \right] (n_i + g + \delta) (k_i^* - k_i) \quad (23)$$

Substituting for $k_i - k_i^*$ from (20) in (23), and then substituting for \dot{k}_i from (23) in (18),

$$\dot{y}_i = \left[\frac{f'(k_i^*) k_i^*}{f(k_i^*)} - 1 \right] (n_i + g + \delta) (y_i^* - y_i) \quad (24)$$

Under Cobb-Douglas, $f'(k_i^*) k_i^* / f(k_i^*)$ is equal to α_s , under translog, $\alpha_s + \gamma_s \ln k_i^*$. Switching back to translog, we may therefore approximately write

$$\dot{y}_i = [1 - \alpha_s - \gamma_s \ln k_i^*] (n_i + g + \delta) (y_i^* - y_i) \quad (25)$$

This is an ordinary first-order linear differential equation with constant coefficient and constant term that is easily solved. The solution may be written,

$$\frac{y_{i,t} - y_{i,t-T}}{y_{i,t-T}} = [1 - e^{-\beta_i T}] \left(\frac{y_i^* - y_{i,t-T}}{y_{i,t-T}} \right), \quad \beta_i \equiv (1 - \alpha_s - \gamma_s \ln k_i^*) (n_i + g + \delta) \quad (26)$$

It is convenient to approximate the growth rates, using logarithms,

$$\ln(y_{i,t} / y_{i,t-T}) = [1 - e^{-\beta_i T}] \ln(y_i^* / y_{i,t-T}) \quad (27)$$

In empirical applications we like to have the variables expressed in terms of labour units, not effective labor units. Define output per labour unit by, $\bar{y}_{i,t} \equiv Y_{i,t} / L_{i,t} = A_{i,t} y_{i,t}$. Since efficiency by assumption grows at the constant rate, g , $\ln A_{i,t} = \ln A_{i,0} + g$. Hence,

$\ln y_{i,t} = \ln \bar{y}_{i,t} - \ln A_{i,0} - g$ and $\ln y_{i,t-T} = \ln \bar{y}_{i,t-T} - \ln A_{i,0} - g$ ($t - T$). Substituting in (27), and dividing by the length of the time period, we get the average growth rate of output per labour unit,

$$\frac{1}{T} \ln(\bar{y}_{i,t} / \bar{y}_{i,t-T}) = g + \frac{t - e^{-\beta_i T} (t - T)}{T} + \frac{1 - e^{-\beta_i T}}{T} \ln(y_i^* A_{i,0} / \bar{y}_{i,t-T}) \quad (28)$$

Here $A_{i,0}$ constitutes a small problem since it is not observable. Following Mankiw et al.

(1992), we simply assume that it is stochastic and uncorrelated with the rates of saving and labour force growth and that the expected value is equal to the constant A . Introducing dummy variables to allow the α and the γ parameters to vary between the different groups of regions, and appending a stochastic error term, we may pool data across regions and time in order to estimate the parameters: A , and the α and the γ for each group. In order to impose all restrictions between parameters, the structural form should be estimated.

Equation (28) may also be used as basis for a simple cross section regression. Then t is equal to T , and (28) is more compactly written as

$$\frac{1}{T} \ln(\bar{y}_{i,T} / \bar{y}_{i,0}) = g + \frac{1 - e^{-\beta_i T}}{T} \ln(y_i^* A_{i,0} / \bar{y}_{i,0}) \quad (29)$$

The model can be extended to allow for human capital effects, similar to the extension of the Cobb-Douglas version of the neoclassical growth model by Mankiw et al. (1992). Introducing human capital in addition to physical capital and labour input, the equivalent to equation (4) is

$$\ln y_i = \alpha_{ks} \ln k_i + \alpha_{hs} \ln h_i + \frac{1}{2} (\gamma_{ks} \ln^2 k_i + \gamma_{hs} \ln^2 h_i + \gamma_s \ln^2 (k_i / h_i)) \quad (30)$$

with $\alpha_{ks} + \alpha_{hs} < 1$. There are decreasing returns to all capital, since constant returns have been imposed on the underlying production function. Human capital per effective worker is denoted by h_i .

The dynamics of the model is now governed by two equations of motion, one for each type of capital. We adopt the same system as used by Mankiw et al. (1992),

$$\begin{aligned} \dot{k}_i &= s_{ki} y_i - (n_i + g + \delta) k_i \\ \dot{h}_i &= s_{hi} y_i - (n_i + g + \delta) h_i \end{aligned} \quad (31)$$

where s_{ki} and s_{hi} are the fractions of income invested in physical and human capital. This means that we assume that both types of capital depreciate at the same rate.

In steady state, capital per effective worker is constant. The steady state level of physical capital, given by (15) when we had one type of capital, is now given by

$$\begin{aligned} \ln k_i^* &= \frac{1 - \alpha_{ks} - \alpha_{hs} + \gamma_{hs} \ln(s_{hi} / s_{ki})}{\gamma_{ks} + \gamma_{hs}} \pm \frac{1}{\gamma_{ks} + \gamma_{hs}} \text{Sqrt}[(\alpha_{ks} + \alpha_{hs} - 1 - \gamma_{hs} \ln(s_{hi} / s_{ki}))^2 \\ &\quad + 2(\gamma_{ks} + \gamma_{hs})(\ln(n_i + g + \delta) - \ln s_{ki} + \alpha_{hs} \ln(s_{hi} / s_{ki}) - (\gamma_{hs} + \gamma_s) \ln^2(s_{hi} / s_{ki}) / 2)] \end{aligned} \quad (32)$$

Imposing a zero-restriction on the human capital variable, (32) is reduced to (15). The positive root can therefore be ruled out for the same reason as before. Once we have obtained the steady state level of physical capital, the steady state level of human capital is simply given by

$$\ln h_i^* = \ln k_i^* - \ln(s_{hi} / s_{ki}) \quad (33)$$

Substituting for steady state levels from (32) and (33) in the production function, (30), we obtain the steady state level of output per effective worker as well. The steady state solution is illustrated in the three-dimensional Figure 6.

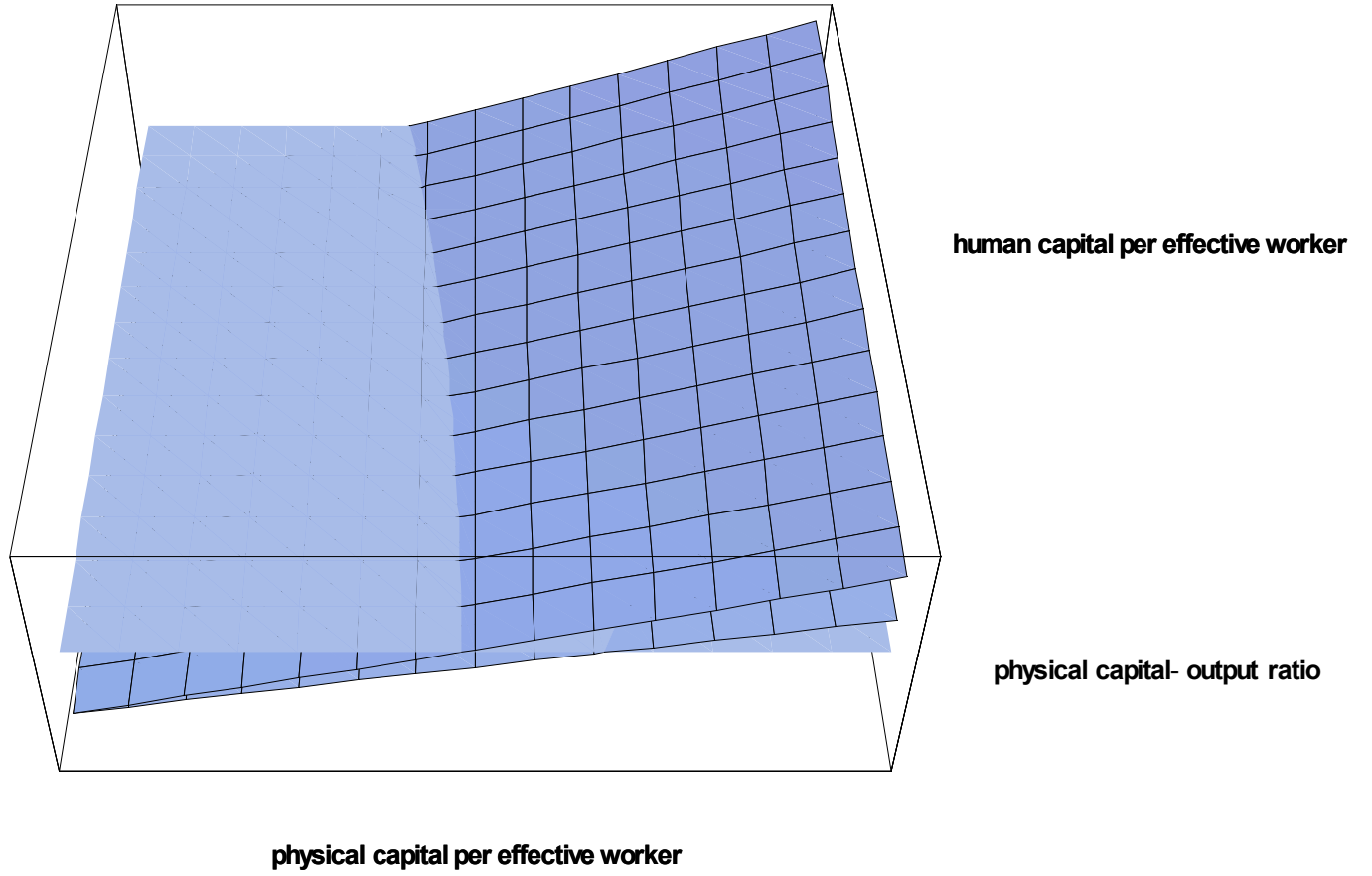


Figure 6. Steady-state with two types of capital (logarithmic scale)

The horizontal plane represents the physical capital-output ratio in steady-state. The intersection with the lower plane, rising from the left corner, gives the steady-state level of capital per worker in the Cobb-Douglas case. The intersection with the convex surface above the Cobb-Douglas plane, gives the equivalent in the translog case. You should recognize the image in the front plane from the lower panel of Figure 5.

Let us look at the dynamics outside steady state and return to equation (30). Applying the same approach as we used in case of one type of capital, instead of working with a specified form we choose to write $y_i = f(k_i, h_i)$. Around steady state,

$$\dot{y}_i = f_k(k_i^*)\dot{k}_i + f_h(h_i^*)\dot{h}_i \quad (34)$$

Approximating the true functional form around steady state by a first order Taylor expansion, we have approximately,

$$f(k_i, h_i) = f(k_i^*, h_i^*) + f_k(k_i^*, h_i^*)(k_i - k_i^*) + f_h(k_i^*, h_i^*)(h_i - h_i^*) \quad (35)$$

or

$$y_i^* - y_i = f_k(k_i^*, h_i^*)(k_i^* - k_i) + f_h(k_i^*, h_i^*)(h_i^* - h_i) \quad (36)$$

The fractions spent on either type of capital are constant and therefore always the same as in steady state,

$$s_{ki} = \frac{(n_i + g + \delta)k_i^*}{y_i^*} \quad (37)$$

$$s_{hi} = \frac{(n_i + g + \delta)h_i^*}{y_i^*}$$

Substituting for $f(k_i, h_i)$ from (35) and for s_{ki} and s_{hi} from (37) in (31),

$$\dot{k}_i = \left[\left(\frac{f_k(k_i^*, h_i^*)k_i^*}{y_i^*} - 1 \right) (k_i - k_i^*) + \frac{f_h(k_i^*, h_i^*)k_i^*}{y_i^*} (h_i - h_i^*) \right] (n_i + g + \delta) \quad (38)$$

$$\dot{h}_i = \left[\left(\frac{f_h(k_i^*, h_i^*)h_i^*}{y_i^*} - 1 \right) (h_i - h_i^*) + \frac{f_k(k_i^*, h_i^*)h_i^*}{y_i^*} (k_i - k_i^*) \right] (n_i + g + \delta)$$

Substituting from (38) in (34) and making use of (36), we obtain, after some manipulations,

$$\dot{y}_i = \left[\frac{f_k(k_i^*, h_i^*)k_i^*}{y_i^*} + \frac{f_h(k_i^*, h_i^*)h_i^*}{y_i^*} - 1 \right] (n_i + g + \delta)(y_i - y_i^*) \quad (39)$$

Under Cobb-Douglas, $f_k(k_i^*, h_i^*)k_i^* / y_i^* + f_h(k_i^*, h_i^*)h_i^* / y_i^*$ is equal to $\alpha_{ks} + \alpha_{hs}$, under translog,

$$\alpha_{ks} + \alpha_{hs} + \gamma_{ks} \ln k_i^* + \gamma_{hs} \ln h_i^*. \text{ In the translog case, we may therefore approximately write} \quad (40)$$

$$\dot{y}_i = [1 - \alpha_{ks} - \alpha_{hs} - \gamma_{ks} \ln k_i^* - \gamma_{hs} \ln h_i^*] (n_i + g + \delta)(y_i - y_i^*)$$

This is a differential equation of the same kind as with one type of capital. Indeed, the solution is the same, given by (26), provided that we redefine β_i :

$$\frac{y_{i,t} - y_{i,t-T}}{y_{i,t-T}} = [1 - e^{-\beta_i T}] \left(\frac{y_i^* - y_{i,t-T}}{y_{i,t-T}} \right), \quad (41)$$

$$\beta_i \equiv (1 - \alpha_{ks} - \alpha_{hs} - \gamma_{ks} \ln k_i^* - \gamma_{hs} \ln h_i^*)(n_i + g + \delta)$$

With this redefinition, (27), (28) and (29) remain valid, as well.

4. β - convergence

There has been done a lot of research on the measurement of human capital since Kendrick (1976) suggested that the stock of human capital was at least as large as the stock of physical capital in the U.S. in 1969, see, e.g., Mulligan and Sala-i-Martin (2000). However, it is probably fair to say that at this state of knowledge, an equal split between the two types of capital in steady state is as good a working hypothesis as any, and at least acceptable as a first approximation. This greatly simplifies the model outlined in Section 3. On this assumption we do not need data on human capital and the deterministic part of the model is almost as if there were only physical capital, the only difference being the interpretation of the parameters used to define β_i for the one-type capital case (equation (26)). Provided that $\alpha_s \equiv \alpha_{ks} + \alpha_{hs}$ and that $\gamma_s \equiv \gamma_{ks} + \gamma_{hs}$, equation (41) is reduced to (26). With this reinterpretation in mind, we may use (29) as the setup for simple cross-section regressions.

We know that there will be unconditional β -convergence in the Norwegian sample since this is a necessary condition for the σ -convergence shown in Figure 2 (see Sala-i-Martin, 1996). However, we do not know the speed of convergence and we do not know if the data are consistent with a well-behaved underlying aggregate production technology. For the Swedish sample we do not even know if there is unconditional convergence. In order to shed some light on these questions, we have presented some results in Table 1-3. Please observe that in this version of the paper prepared for the 43rd annual conference of ERS, we have not had time to investigate all possibilities before the deadline for submission. Notably, we have not allowed for different technology in different regions as we should and have argued for and we have not considered panel data possibilities. However, we will return to these issues in later versions of the paper.

Table 1. Unconditional convergence

Dependent variable log difference GDP per worker (Norway) and log difference income per capita aged 20-64 (Sweden) 1980-2000.

Sample	Norwegian counties	Swedish counties
Observations	19	24
Constant	.097 (.020)	-.005 (.035)
$\ln \bar{y}_{i,1980}$	-.021 (.005)	.011 (.012)
\bar{R}^2	.43	.01
Implied speed of convergence	.028 (.010)	.012 (.016)

Note: estimation method OLS

The results for the Norwegian sample is in line with results found in the literature based on other datasets. The difference in initial productivity accounts for a large fraction of the variation in growth rates, the adjusted R^2 being 0.43. There has been unconditional convergence in the sense that the laggards have narrowed the gap to the leaders, but we already knew this from the fact that the data revealed absolute convergence, or σ -convergence (see Section 2). However, the implied speed of convergence obtained means that the time to close half the gap is about 25 years.

The results for the Swedish sample are less encouraging. Essentially, none of the variation in growth rates is explained by the model. However, this is perhaps as could be expected from previous studies. Persson (1997) using similar data, report the same finding for the period 1980-1993.

Although there is no evidence of neither absolute, nor unconditional, convergence in the Swedish sample, there still may be conditional convergence in the sense that the Swedish counties converge when we control for variables that may cause different steady states. The neoclassical model suggests that differences in investment (the ‘saving rates’), labour force growth and depreciation, result in different steady states. Unfortunately, we have only data that can be used to proxy for labour force growth. The results when we condition on this variable are presented in Table 2.

Table 2. Conditional convergence

Dependent variable log difference GDP per worker (Norway) and log difference income per capita aged 20-64 (Sweden) 1980-2000.

Sample	Norwegian counties	Swedish counties
Observations	19	24
Constant	.157 (.032)	-.075 (.028)
$\ln \bar{y}_{i,1980}$	-.030 (.009)	-.002 (.009)
$\ln s$	-.008 (.011)	n.a.
$\ln (n+g+\delta)$.015 (.007)	.032 (.006)
\bar{R}^2	.53	.54
Implied speed of convergence	.055 (.022)	.002 (.009)

Note: estimation method NLS

We do see that this model performs much better for the Swedish sample. The adjusted R^2 is 0.54. The coefficient on the initial level of income per capita remains small and is not statistically different from zero, but the coefficient on the conditioning variable is highly significant. What is somewhat surprising is the sign. Population growth appears to be positively correlated with income growth. Although the inclusion of the conditioning variable improves the fit of the equation considerable, there is still no sign of convergence.

The inclusion of conditioning variables for the Norwegian sample also improves the fit of the equation. We see, though, that variation in investment in physical capital does not explain anything. However, the growth rate of the normalizing variable (the growth rate of workers) is significant and takes the same sign as for the Swedish sample. The point estimate of the speed of convergence is substantially increased, but so is the standard error. Interval estimates on the 95-percent significance level would clearly be overlapping: 0.055 ± 0.047 in the conditional model and 0.028 ± 0.021 in the unconditional one. However, if we put faith in the point estimate, the suggested time necessary to close half the gap between current productivity and productivity in steady state, which is different from region to region, is about 12 years.

We have seen that the sign of the coefficient for the labour force growth is positive in both samples. Neoclassical growth theory without human capital suggests it should be negative. However, when we consider that human capital is embodied in labour, the predicted sign is

ambiguous. On the one hand, *the quantity effect*, increasing the amount of labour input, should reduce the GDP per worker growth rate. On the other hand, *the composition effect* when new workers possess above average human capital, should increase the stock of total capital and possibly more than offset the quantity effect (see Shioji, 2001). This may be the explanation behind the results.

The results reported in Table 1 and 2 are based on reduced form equations where the restrictions suggested by the model outlined in Section 3 are not imposed. In Table 4 we report the results from estimation of the structural model, using the Norwegian sample. Both the model based on the Cobb-Douglas specification and the translog have been estimated by nonlinear least squares (software: TSP 4.3). More work should be done to verify how robust the results are since non-linear estimation may be sensitive to many aspects not important for linear estimation: starting values, level of tolerance, derivative calculations, software package etc. However, as a preliminary statement, the data appear not to be consistent with the constant returns to scale Cobb-Douglas technology since the estimated capital share (the share on rental income on capital (physical and human) in total income) is equal to α which is negative. On the other hand, the data seems to be consistent with the translog technology, although the parameters are not as sharply identified as we could hope for. This is encouraging in view of the efforts undertaken to spell out the implications of the translog technology in Section 3.

Table 4. Structural parameters

Dependent variable log difference GDP per worker 1980-2000.

Functional form	Cobb-Douglas	Translog
Sample	Norwegian counties	Norwegian counties
Observations	19	19
$\ln A_{1980}$	4.196	3.986
	(.137)	(.132)
α	-.663	.439
	(.275)	(.222)
γ		-.202
		(.178)
Log of likelihood function	86.787	87.704
\bar{R}^2	.53	.54

Note: estimation method NLLS

5. Concluding discussion

Bernard and Jones (1996a) were basically right – there does not seem to be a uniform pattern of convergence across industries: the pattern is different for ‘apples’ and ‘oranges’, as confirmed by our regional data. They were possibly right in concluding that overall convergence was driven by convergence in services while manufacturing productivity diverged when looking at a specific time span although this conclusion cannot be inferred from their international data. The regional data suggest a more general regularity. Productivity

dispersion in manufacturing and services appear to move out of phase following a downward trend. For 1980-1985 overall convergence was driven by services whereas manufacturing and agriculture diverged. For 1985-1995, overall convergence was driven by manufacturing and agriculture while services diverged. For 1995-2000, overall convergence was driven by manufacturing alone.

The industrial composition could be critical to a smooth pace of convergence over time and suggests that if we use a production function approach to growth, we should use a functional form that permits aggregation over industries with different technology. Assuming that Cobb-Douglas is permissible for separate industries, the aggregate production function cannot be Cobb-Douglas. In fact, the only aggregate form that is permissible is translog. It is interesting to note that our data seems to be inconsistent with Cobb-Douglas but consistent with translog.

There appears to have been absolute convergence between Norwegian regions over the whole time span from 1980 to 2000 based on 5 years intervals. There is a 95 percent probability that the time it takes to close half the gap is in the interval between 14 and 99 years, with a point estimate of 25. Not very accurate, in other words. For the Swedish regions there appear to be no convergence at all.

A somewhat different question concerns convergence to the region specific long run dynamic equilibrium, the steady state. Again, there is no support in the data for convergence in the Swedish case. The speed of conditional convergence for the Norwegian regions suggest that half the gap to own steady state is achieved after 12 years, but the 95 percent interval referred to above remains wide – between 7 and 87 years.

Although we have not used explicit data to proxy for human capital, the setup is consistent with a model where the stocks of human and physical capital are equal in steady state, as well as a model without human capital. However, only the first interpretation appears to be consistent with the data since the growth rate of the labour force is positively correlated with the productivity or income growth rate in both the Norwegian and the Swedish sample. In the terms of Shioji (2001), the composition effect due to embodied human capital dominates the quantity effect, leading to higher growth when the labor force grows faster because of improved quality, contrary to what the model with homogeneous labour predicts.

In order to corroborate the results obtained, it is urgent to replicate this study based on other regional datasets. We very much welcome initiatives in this respect and will contribute ourselves to the extent that data are made available.

REFERENCES

- Barro, R.J. and X. Sala-i-Martin 1991, Convergence across states and regions. *Brookings Papers on Economic Activity* 1: 107-82
- Barro, R.J. and X. Sala-i-Martin 1995, *Economic growth*. New York: McGraw-Hill.
- Bernard, A.B. and C.I. Jones 1996a, Comparing apples to oranges: Productivity convergence and measurement across industries and countries. *American Economic Review* 86: 1216-38
- Bernard, A.B. and C.I. Jones 1996b, Technology and convergence. *The Economic Journal* 106: 1037-44.
- Caselli, F. and W.J. Coleman 2001, The U.S. structural transformation and regional convergence: A reinterpretation. *Journal of Political Economy* 109: 584-616
- Chambers, R.G. 1989, *Applied Production Analysis: A Dual Approach*. New York: Cambridge University Press.
- Clark, C. 1940, *The Conditions of Economic Progress*. London: Macmillan.
- Fisher, F.M. 1992, *Aggregation*. New York: Harvester Wheatsheaf.
- Garderen, K.J. van, K. Lee and M.H. Pesaran 2000, Cross-sectional aggregation of non-linear models. *Journal of Econometrics* 95: 285-331
- Inada, K.-I. 1963, On a two-sector model of economic growth: Comments and a generalization. *Review of Economic Studies* 30: 119-27.
- Islam, N. 1995, Growth empirics: A panel data approach. *Quarterly Journal of Economics* CX: 1127-70
- Kendrick, J. 1976, *The Formation and Stock of Total Capital*. New York: Columbia University Press.
- Kongsamut, P. S. Rebelo and D. Xie 2001, Beyond balanced growth. *Review of Economic Studies* 68: 869-882
- Mankiw, N.G., D. Romer and D.N. Weil 1992, A contribution to the empirics of economic growth. *The Quarterly Journal of Economics* 107: 407-38
- Mulligan, C.B. and X. Sala-i-Martin 2000, Measuring aggregate human capital. *Journal of Economic Growth* 5: 212-52
- Persson, J. 1997, Convergence across the Swedish counties, 1911-1993. *European Economic Review* 41: 1835-52
- Sala-i-Martin, X. 1996, The classical approach to convergence analysis. *The Economic Journal* 106: 1019-36
- Sato, K. 1975, *Production Functions and Aggregation*. Amsterdam: North-Holland.
- Shioji, E. 2001, Composition effect of migration and regional growth in Japan. *Journal of the Japanese and International Economies* 15: 29-49
- Sørensen, A. 2001, Comparing apples to oranges: Productivity convergence and measurement across industries and countries: Comment. *American Economic Review* 91: 1160-67