A Random Utility Model of Demand for Variety under Spatial Differentiation

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Abstract

This paper studies a linear random utility model of demand for variety under spatial product differentiation. A motivation is given by previous work establishing a connection between the demand generated by linear random utility and CES demand. Two interpretations of the random utility model are considered. First, the utility of a representative consumer from a given variety is assumed to be random e.g. due to a random linear trade cost or unobservable quality. In the second interpretation the uncertainty is due to the incomplete information of the modeller regarding the utility parameters. The demand system implied by stochastic utility maximization has some resemblance with a CES demand system. An application of the demand for variety model to new economic geography is discussed. The approach to stochastic utility is based on maximizing linear random utility functions via probabilistic constraints. The main focus is on the case where the random utility/cost parameters are i.i.d. normal random variables. In economic geography linear random utility implies aggregate demand for variety and can explain intra-industry trade between symmetric regions.

1 Introduction

This paper studies a linear random utility (RU) model of the demand for variety under spatial product differentiation. A stochastic programming model of linear random utility model is presented. The main focus is on the case where the random taste parameters of the population of consumers, or of individual consumers, associated with the different varieties, are independent and normally distributed. The stochastic programming approach to dealing with random utility is based on maximizing random utility via probabilistic constraints. This framework provides a simple approach to optimizing a linear random objective function. The goal is to extend previous work on random linear utility models based on assuming discrete choice of a single variety [Anderson et al.1996] to a stochastic optimization framework allowing for demand for variety.

A motivation for the random utility approach to the study of demand for variety under spatial differentiation is given by previous work, demonstrating a connection between the CES utility model of demand for variety and a linear random utility model [Andersson et al.1988, Anderson et al.1996]. It has been shown that the CES model can be constructed as being representative of a population of consumers making discrete choices. The main objective of this paper is to study demand for variety from the point of view of stochastic linear utility optimization. The demand system implied by stochastic utility maximization is shown to have some resemblance to a CES demand system. The random utility model endogenizes demand for variety, provided the number of varieties available meets a certain threshold. Under this condition, and in the special case of symmetric prices, both the random utility model and the CES model imply an equal allocation of income across varieties.

The consumer may face uncertainty regarding the utility parameters, e.g. due to unobservable characteristics. In this case optimization is performed before the realization of the random variable, like in models of state dependent utility, and unlike in linear random utility models where the modeller faces uncertainty [Anderson et al.1996]. In this context the consumers are not restricted to buy a single variety. An application of the model is to consider a network economy such as an electronic marketplace where the consumers choose between different varieties characterized by attributes that can be unobservable and/or a trade cost that can be random. The random utility/cost parameters reflect the lack of complete information regarding the characteristics of the alternatives. When the uncertainty is interpreted to be due to the lack of information of the modeller, the discrete choice assumption can be endogenized in the stochastic programming framework similarly as in random utility models (ibid.). If the utility parameters are i.i.d., the aggregate consumer is then predicted to demand all varieties like in random utility models based on discrete choice.

The key features in the new economic geography NEG models include general equilibrium, transportation costs and Dixit-Stiglitz utility functions giving rise to demand for variety [Krugman1991]. As an application of the random utility framework this paper discusses an NEG model based on a linear random utility approach. Most of the previous literature on spatial economics with trade is based on deterministic models [Dean et al.1970, Fujita et al.1999]. In general the preferences, technologies and/or endowments can be random, leading to a particular form of a random trade model [Pomery1984]. In an NEG application random utility formalizes intra-industry trade between symmetric regions [Dixit and Norman1993]. The focus is on spatial economy with linear random utility and a deterministic trade cost.

The main results from this paper can be summarized as follows:

1. A stochastic programming framework for the optimization of linear random utility is introduced. The uncertainty in utility can be due to the lack of complete information available either to the modeller or the consumer. The random utility parameter associated with a given variety can be interpreted as a utility parameter reflecting a random trade cost between the representative consumer and the producer of the variety (cf. the Hotelling model of product differentiation). The outcome of individual stochastic utility optimization coincides with the predicted solution to the allocation

of the income of an aggregate consumer constructed from a population with each consumer buying a given fixed number of varieties (or none). Increasing the product diversity in the model both increases the (estimated) utility of the aggregate consumer and, due to diversification, the utility that can be obtained by consumers, like in [Dixit and Stiglitz1977].

2. In the stochastic utility interpretation, a random linear utility parameter makes the goods imperfect substitutes. The demand system implied by the stochastic programming model of random utility under independent normally distributed utility parameters resembles a modified CES demand system. In an NEG application of the model with an aggregate consumer, trade takes place in spatially different CES composites. It is optimal for the consumer to demand either all varieties or use all income as money, unless the consumer is restricted to buy at most a single variety.

3. Assuming normally distributed utility parameters is sufficient for the existence of an equilibrium in a price game between the produces of the different varieties, provided the suppliers ignore the impact of the own price on the price index. In [Caplin and Nalebuff1991] a joint logconcave distribution of the random utility parameters has been shown to be a sufficient condition for the existence of a price equilibrium. A market-based solution to product differentiation under uncertainty does not imply too much differentiation but it may imply too little.

The structure of the paper is as follows. Section 2 introduces the stochastic programming approach to random utility in the context of spatial differentiation. Section 3 discusses the demand system implied by utility maximization under normally distributed utility coefficients. Section 4 applies the random utility model to study demand for variety under spatial differentiation. Optimal product diversity is studied when assuming the normally distributed random utility coefficients are i.i.d. random variables. Section 5 discusses applications of the random utility model, first to an economic geography model and second to network formation. Like in previous work on random utility [Anderson et al.1996], the random utility model is the same irrespective of whether the uncertainty is due to the modeller's incomplete information or stochastic utility.

2 Spatial Differentiation under Uncertainty

In what follows spatial differentiation is studied from a random utility point of view. Spatial differentiation may refer to either distance in a geographical space or the difference in qualities of different products in a characteristics space [Anderson et al.1996].

2.1 A Random Utility Model of Spatial Differentiation

Consider an economy with n consumers and m varieties. Let x_{ij} denote the consumption of consumer i of variety j, i.e. the jth element in $\mathbf{x}_i^T = (x_{i1}, ..., x_{im})$. Let $\mathbf{x} = (\mathbf{x}_1, ..., \mathbf{x}_n)$ denote the mn-vector of consumptions. The utility function $u_i(\mathbf{x}_i)$ of consumer i is assumed to be linear in the random utility parameters g_{ij} associated with spatially different varieties $j = 1, ..., m$ (cf. related work in [Caplin and Nalebuff1991]):

$$u_i(\mathbf{x}_i) = b_i \sum_{j=1}^{m} g_{ij} x_{ij} + h(z_i),\ i = 1, ..., n, \tag{1}$$

where $b_i \geq 0$ is a deterministic utility parameter, z_i denotes consumer i's consumption of the numeraire and h is nondecreasing concave function. In what follows, two cases of h are considered: net utility maximization with $h(z_i) = z_i$ and gross utility maximization with $h(z_i) = 0$. Let P_j, the jth component in $\mathbf{P}$ denote the unit price of variety j. Let $z_i = E_i - \mathbf{P}^T \mathbf{x}_i = z(E_i, \mathbf{P}) \geq 0$ in (1) where E_i denotes the income of agent i. Each variety j is associated with a one-dimensional utility parameter g_{ij}, which is the jth element in $\mathbf{g}_i^T = (g_{i1}, ..., g_{im})$. In what follows it is assumed that these parameters are normally distributed. The extended version of this paper [Heikkinen2003] presents a model based on a lognormal distribution with a nonnegative support, applicable to spatial differentiation when the parameters g_{ij} measure decay due to distance in geographical space.

The utility model (1) implies an ex post unit trade cost between i and j, $E(g_{ij}) - g_{ij}$, measured as the difference between the expected marginal utility and realized marginal utility. Let $g_{il} = L$, the utility value of the ideal variety l of i, denote a large value of the parameter g_{ij} after which further increases in g_{ij} bring no additional benefit. Another trade cost measure can be defined as $L - g_{ij}$, where L can be chosen so that the probability of a negative cost (reward) is small. This definition of a trade cost applies also to normally distributed coefficients and can be rationalized as follows. Let $z_i = L \in R$ and $z_j = g_{ij} \in R$ denote the address of consumer i's ideal variety and the address of variety j, respectively. Redefine the distance to j as $d_{ij} = |z_i - z_j|$. Consumer maximizing (1) is indifferent between getting g_{ij} from one unit of j and obtaining L from one unit of l by incurring the trade cost $L - g_{ij}$ (cf. address models [Anderson et al.1996]). I.e. for $z_i \geq z_j$ the trade cost of j is equal to the distance to j, $L - g_{ij} = d_{ij}$. (For $z_i \leq z_j$ the trade cost is negative as $L - g_{ij} = -d_{ij} \leq 0$.)

The model implies an ex post unit trade cost between i and j, $E(g_{ij}) - g_{ij}$, measured as the difference between the expected marginal utility and realized marginal utility. Let $g_{il} = L$, the utility value of the ideal variety l of i, denote a large value of the parameter g_{ij} after which further increases in g_{ij} bring no additional benefit. Another trade cost measure can be defined in terms of the value of the ideal variety l: $L - g_{ij}$. The number L can be defined so that the probability of a negative cost (reward) is small. This definition of a trade cost applies also to normally distributed coefficients and can be rationalized as follows. Let $z_i = 0$ denote the address of consumer i, located at the origin. Let z_j denote the address of variety j, both on the real line. The distance from i to j is redefined as $d_{ij} = |z_j - z_i|$. When $z_j \leq z_i$ let $g_{ij} = -d_{ij} \leq 0$ denote the disutility value of the distance from i to j. When $z_j > z_i$, let $g_{ij} = d_{ij}$ denote the utility from j to i. Thus, $L = d_{il} = g_{il}$. Note that if $z_i = z_l$, the utility from j depends negatively on the distance to the ideal variety, $g_{ij} = -d_{lj}$ for all j with $z_j \leq z_l$, like in address models [Anderson et al.1996]. The trade cost of i between j and l, $L - g_{ij}$ is then equal to the distance from z_j to z_l. Consumer i is indifferent between obtaining g_{ij} from j and incurring the trade cost $L - g_{ij}$ to get L at z_l. The coefficient g_{ij} in (1) can thus be seen as the difference between the utility from l and the trade cost between j and l. For example, consider an economy with n spatially distinct demand markets. In each market a representative consumer chooses between $m \leq n$ varieties, produced at spatially distinct markets. Variety j is associated with a random coefficient g_{ij} at demand market i. In this context x_{ij} can be interpreted as the volume of trade from j to i.

Two Interpretations of Random Utility

Two interpretations of stochastic utility are discussed in [Anderson et al.1996]. In the first original approach the random utility model is the assignment of a random variable measuring utility to each alternative. In the second approach, the uncertainty in utility can be interpreted to be due to the lack of information of the modeler (ibid.). The modeler is assumed to have imperfect knowledge about the utility functions. This case is discussed in 2.4. In the first approach the utility parameters $\{g_{ij}\}$ of each agent i are random. In the presence of random utility parameters two models are considered. First, in 2.2 utility maximization is considered assuming each variety is associated with a random utility coefficient. In 2.3 deterministic iceberg transportation cost parameters are introduced, and x_{ij} refers to a CES composite from region j demanded by representative consumer i in the presence of random utility parameters regarding the composites.

2.2 Random Utility Maximization

When $h(z_i) = z_i$ the ith consumer's random objective function can be written as

$$b_i \mathbf{g}_i^T \mathbf{x}_i + E - \sum_{j=1}^{m} P_j x_{ij}. \tag{2}$$

Letting U_i denote the new objective, the corresponding optimization problem can be stated as:

$$\max U_i = U_i^* \; s.t. \; Pr(b_i \sum_{j=1}^{m} g_{ij} x_{ij} + E_i - \sum_{j=1}^{m} P_j x_{ij} \geq U_i) \geq \bar{p}_i, \; s.t. \; \sum_{j=1}^{m} P_j x_{ij} \leq E_i. \tag{3}$$

Random utility in (2) is derived by agent i only if the probability that the ex ante value of this term is at least E_i meets a given reliability threshold $\bar{p}_i$. Otherwise the utility of i is $h(E_i) = E_i$ by setting the demand equal to zero $\mathbf{x}_i = \mathbf{0}$. When $h = 0$, the objective is written as:

$$\max u_i = u_i^* \; s.t. \; Pr(b_i \sum_{j=1}^{m} g_{ij} x_{ij} \geq u_i) \geq \bar{p}_i, \; s.t. \; \sum_{j=1}^{m} P_j x_{ij} \leq E_i. \tag{4}$$

The theory of random utility has essentially been developed under the assumption that the utility coefficients associated with different varieties are independent and identically distributed [Anderson et al.1996]. In what follows for simplicity i.i.d. coefficients are assumed.

2.3 Aggregate Consumer's Problem: Income Allocation between Composites of Goods

This section summarizes the [Dixit and Stiglitz1977] utility model from the point of view of a spatial economy. Consider an economy with m regions each producing V distinct varieties and $n \leq m$ regions represented by an aggregate consumer. Let d_{jk} denote the consumption of variety k in region j. Let $\theta > 1$ denote the constant elasticity of substitution CES parameter and let $\tilde{g}_j$ denote the deterministic decay cost between the representative consumer and region j. The CES composite utility from varieties from region j is given by the [Dixit and Stiglitz1977] utility:

$$x_j = \tilde{g}_j \left(\sum_{k=1}^{V} d_{kj}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}. \tag{5}$$

The deterministic allocation problem, discussed first, is for the (aggregate) consumer to maximize the utility function

$$\max \sum_{j=1}^{m}(x_j + z_j) \tag{6}$$

subject to the aggregate income constraint

$$\sum_{j=1}^{m}(z_j + \pi_j x_j) = E, \tag{7}$$

where π_j denotes the price index of composite j. Letting p_{jk} denote the price of variety k in region j, the CES price index in region j is defined as (ibid.)

$$\pi_j = \frac{1}{\tilde{g}_j}(\sum_{k=1}^{V} p_{jk}^{1-\theta})^{\frac{1}{1-\theta}}. \tag{8}$$

The income allocated to varieties from region j is denoted by E_j. Problem (6)-(7) can be solved in two stages: first, treating E_j as a given parameter, the optimal allocation of E_j within region j is determined. Second, substituting the solutions $d^*_{jk}(E_j)$ from the first stage in (6) gives the new objective that is then solved with respect to the allocations E_j subject to $\sum_{j=1}^{m}(E_j + z_j) = E$. The first problem for each region $j = 1, ..., m$ is to

$$\max_{(d_{j1},\dots,d_{jV})}(\sum_{k=1}^{V}(\tilde{g}_j d_{jk})^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}} \quad s.t. \sum_{j=1}^{m} p_{jk} d_{jk} = E_j, \tag{9}$$

The kth solution to (9) is the CES demand in region j for variety k [Dixit and Stiglitz1977]:

$$d^*_{jk} = \frac{E_j p_{jk}^{-\theta}}{\sum_{k=1}^{V} p_{jk}^{1-\theta}}, \tag{10}$$

If the trade cost increases, the decay parameter $\tilde{g}_j$ decreases, decreasing the utility from the composite from region j as can be seen from (5), keeping the expenditure constant at E_j. Substituting (10) in (5),

$$x_j(E_j) = \frac{E_j \tilde{g}_j}{(\sum_{k=1}^{V} p_{jk}^{1-\theta})^{\frac{1}{1-\theta}}} = \frac{E_j}{\pi_j}. \tag{11}$$

The remaining problem then is to max $\sum_{j=1}^{m}(x_j(E_j))$ subject to $\sum_{j=1}^{m}(z_j + E_j) = E$.

This model can be extended to random utility as follows. Let g_j (omitting the consumer index) denote the random utility parameter associated with a composite good from region j. When the parameters g_j are random, the aggregate consumer is not able to verify whether the composites from each region are exactly equivalent from utility point of view. The aggregate consumer's problem is to find the optimal income allocation $\mathbf{E}^* = (E_1^*, .., E_m^*)^T$ solving (cf. (1)):

$$\max_{\mathbf{E},\mathbf{z}} \sum_{j=1}^{m}(g_j x_j(E_j) + h(z_j))$$

subject to $\sum_{j=1}^{m} z_j + \sum_{j=1}^{m} E_j = E$, where $x_j(E_j)$ is defined in (11). When $h(z_j) = 0$ for all $z_j \geq 0$ the stochastic gross utility maximization problem can be written as

$$\max_{\mathbf{x}} \ u \ s.t. \ Pr(\sum_{j=1}^{m} g_j x_j \geq u) \geq \bar{p} \tag{12}$$

subject to the aggregate income constraint:

$$\sum_{j=1}^{m}(\pi_j x_j + z_j) = E, \tag{13}$$

where x_j is defined in (11). The solution $\{x_j^*\}_{j=1}^{m}$ to (12)-(13) implies the optimal income allocation between different regions $j = 1, ..., m$. Section 5.1 returns to the above model.

2.4 Econometric Model with Discrete Choice

The econometrician's model is based on assuming that the utility functions are deterministic but the modeller has incomplete information about the utility function [Anderson et al.1996]. Assume that this uncertainty is captured by a distribution of the taste parameter g across the population of individuals that are statistically identical: the choices are governed by the same probability distribution. In random utility models with discrete choice it is assumed that each consumer buys a single variety. The motivation is given by the assumption that the consumer knows his linear utility function and is able to determine the best choice (ibid.). Consider the aggregate consumer's problem:

$$\max\ U\ s.t.\ Pr(\sum_{j=1}^{m} g_j x_j - \sum_{j=1}^{m} P_j x_j \geq U) \geq \bar{p},\ \sum_{j=1}^{m} P_j x_j \leq E, \tag{14}$$

where g_j is the random taste parameter of a representative consumer regarding variety j. It can be shown that under price symmetry, if the modeller imposes the discrete choice hypothesis, the choice probability of variety j is:

$$Pr(j) = \frac{x_j^* P_j}{E}, \tag{15}$$

where E denotes the aggregate income and x_j^* is the jth solution to the aggregate income allocation problem (14) (derived in section 3). In the special case of price symmetry the modeller's income allocation problem subject to the discrete choice hypothesis coincides with an aggregate consumer's unrestricted problem. E.g. the model in 2.3. can be interpreted in this way, as a model of an aggregate consumer maximizing random utility subject to an aggregate income constraint, as well as as a model of individual optimization).

3 The Demand System with Random Utility

Consider the stochastic utility optimization problem of a representative consumer to maximize the gross utility:

$$\max\ u \tag{16}$$

subject to

$$Pr(\sum_{j=1}^{m} g_j x_j \geq u) \geq \bar{p} \tag{17}$$

and subject to the income constraint

$$\sum_{j=1}^{m} P_j x_j = E. \tag{18}$$

Gross utility is denoted by u to distinguish from the net utility U in (3). It is argued in the extended version of this paper ([Heikkinen2003]) that this can be rewritten as the constrained optimization problem:

$$\max \sum_{j=1}^{m} \bar{g}x_j + \phi{-1}(1-\bar{p})\sigma\sqrt{\sum_{j=1}^{m} x_j^2} + \lambda(E - \sum_{j=1}^{m} P_j x_j), \tag{19}$$

where λ denotes the Lagrangian multiplier, $\bar{g} = E(g)$, $\sigma^2 = Var(g)$ and ϕ denotes the standard normal distribution function. Letting $\bar{p} \geq 0$, $\phi^{-1}(1-\bar{p}) \leq 0$. Let $v = (-\phi^{-1}(1-\bar{p})\sigma)^2$. Solving for λ implies:

$$\lambda = \frac{\bar{g}\Pi}{\hat{\Pi}} - \frac{\sqrt{\bar{g}^2\Pi^2 - \hat{\Pi}(m\bar{g}^2 - v)}}{\hat{\Pi}}, \tag{20}$$

where $\Pi = \sum_{j=1}^{m} P_j$ and $\hat{\Pi} = \sum_{j=1}^{m} P_j^2$ denote price indices. Assume that the marginal utility from income $\lambda \geq 0$. Letting

$$D_j = \frac{\bar{g} - \lambda P_j}{\sqrt{v}}. \tag{21}$$

the demand function x_j under i.i.d. utility parameters can be stated as (ibid.)

$$x_j = \frac{ED_j}{\sum_{j=1}^{m} D_j P_j}. \tag{22}$$

Relation to the CES Demand System

Note the formal similarity between the demand due to random utility (22) and the CES demand (10). In the special case with price symmetry $P_j = P,\ j = 1, .., m$, the *RU* demand and CES demand coincide: $x_j = E/(mP),\ j = 1, ..., m$, assuming $\lambda \geq 0$ in (20).

It follows from (22) that for any two varieties j and $k \neq j$:

$$r = \frac{x_j}{x_k} = \frac{D_j}{D_k} = \frac{\bar{g} - \lambda P_j}{\bar{g} - \lambda P_k}. \tag{23}$$

Under CES demands (10), referred to as x_j^{ces}, x_k^{ces},

$$r = \frac{x_j^{ces}}{x_k^{ces}} = \frac{P_j^{-\theta}}{P_k^{-\theta}}. \tag{24}$$

It can be observed that the ratio of the demands for any two varieties only depends on the prices of these varieties in the CES model. In the case of the *RU* model it also depends on the price index Π via λ. Let $t = P_j/P_k$. In the CES model, for any two varieties the elasticity of substitution is constant, $e^{ces} = \theta$, where θ is the utility parameter in (5). In the RU model, when evaluated at a fixed P_k, the elasticity of substitution depends on λ, t and P_k:

$$e^{ru} = \frac{\lambda t}{\frac{\bar{g}}{P_k} - \lambda t}. \tag{25}$$

It can be seen from (25) that e^{ru} approaches ∞, indicating perfect substitutability between the different varieties, only in the special case when $\bar{g} = \lambda P_j = P,\ j = 1, ..., m$, corresponding to the case without uncertainty, i.e. with $\sigma = 0$ or $\bar{p} = 0.5$. Thus, a linear random utility model under i.i.d. utility parameters does not imply that the varieties are perfect substitutes as in general $e^{ru} \neq \infty$.

Proposition 1. *Let the demand x_j for variety j be defined by (22) and let c denote the unit production cost. When ignoring the impact of own price on the price index, the profit function of the producer of variety j in region j, $I_j = P_j x_j - c x_j$ is quasiconcave in P_j.*

Proposition (1) shows that the random utility model satisfies the sufficient condition for the existence of an equilibrium price P^* in the game between the distinct producers of $j = 1, ..., m$.

4 Optimal Variety under Stochastic Utility

The following proposition is proved in [Heikkinen2003]:

Proposition 2. *Consider problem (3) of a representative consumer i. Let $P_j = P,\ j = 1, ..., m$. Assume that the parameters g_{ij} are i.i.d. random variables with a normal distribution. Let $E(g_{ij}) = \bar{g}$ and let $Var(g_{ij}) = \sigma^2$. Denote the objective function by: U_i. Suppose there is a feasible solution to this problem. Let ϕ denote the standard normal probability distribution function. Then, an optimal allocation is to let $x^*_{ij} = x^* = \frac{E}{Pm},\ i = 1, ..., n,\ j = 1, ..., m,$ if $m \geq \lceil m^*_i \rceil$ and $x^*_{ij} = 0,\ i = 1, ..., n,\ j = 1, ..., m$ otherwise, where the threshold product variety is $m^*_i = (\frac{-\phi^{-1}(1-\bar{p})\sigma b_i}{b_i \bar{g} - P})^2$.*

The threshold number m^* is the minimum sufficient variety that yields non-negative utility U. Proposition (2) implies for gross utility maximization as defined in section 3 the modified threshold:

$$\bar{m} = (\frac{-\phi^{-1}(1-\bar{p})\sigma}{b\bar{g}})^2. \quad (26)$$

Likewise, the following can be shown (ibid.).

Proposition 3. *Consider problem (16)-(17). If $m \geq \bar{m}$, the optimal demand of a representative consumer for variety j, x_j^*, is given in (22). If $m < \bar{m}$, it is optimal to set $\mathbf{x}^* = \mathbf{0}$.*

The required threshold variety makes the RU model of demand for variety different from the CES model (where the marginal utility from income always is nonnegative). The demand for variety that results from the stochastic programming model is also different from the outcome of expected utility maximization e.g. in portfolio theory where under linear Bernoulli utility functions and i.i.d. coefficients there is no gain in expected utility from demanding all varieties.

Proposition (2) implies the following:

Remark 1. *Assume that a representative consumer-producer at region i has income equal to the competitive profit from production at i, and that the labour (resource) is equally distributed across the locations. A decentralized directed network economy as modelled by independent normally distributed network coefficients is either with no trade or with symmetric trade along all links.*

Remark 2. *Under symmetric prices, the maximum utility is obtained when the number of available varieties m approaches infinity: $U(m) = \frac{E}{P}[(b\bar{g} - P) + \sigma\phi^{-1}(1-\bar{p})\sqrt{\frac{1}{m}}] \rightarrow \frac{E}{P}[(b\bar{g} - P)]$ as $m \rightarrow \infty$.*

Consider an economy with n consumers and m varieties. The equilibrium price P and variety m are jointly determined from

$$\sum_{i=1}^{n} x_{ij}(P) = R/m,$$

where the total amount of resources is R, and one unit of each variety requires one unit of resource. The maximum price is determined from:

$$P^{max}(m) = b\bar{g} + \sigma\phi^{-1}(1 - \bar{p})\frac{1}{\sqrt{m}}. \tag{27}$$

Any P such that $0 \leq P \leq P_{max}$ yields the same total utility $nU + nE$. A market based equilibrium may produce too little variety since any $m \geq m^*$ can be supported as a price equilibrium, assuming $P(m) \leq P^{max}$ (cf. [Dixit and Stiglitz1977]).

Assuming $\bar{p} \geq \frac{1}{2}$, the following observations can be made differentiating welfare U^* with respect to $\bar{g}, \sigma$ and m: U^* is increasing in both $\bar{g} = E(g)$ and m and U^* is decreasing in uncertainty as measured by σ. The discrete choice model with $x_{ij} \in \{0, 1\}$ for $i = 1, ..., n$ and at most one $j = 1, ..., m$ is an optimal solution only if $\bar{p} = \frac{1}{2}$ or $\sigma = 0$.

5 Applications to Network Economics

In what follows, An application to NEG based on the model of an aggregate consumer in 2.3 is summarized in 5.1. An application to network formation is discussed in 5.2.

5.1 A Random Utility Model of a Spatial Economy

The NEG approach to trade [Fujita et al.1999] studies trade assuming 1) price equilibrium 2) iceberg transportation costs and 3) product differentiation. In the new trade theory space is often modelled by iceberg transportation costs [Krugman and Venables1996]. In what follows consider the demand for variety model of an aggregate consumer in 2.3 where random utility is independent of a deterministic iceberg trade cost $\tilde{g}_{ij}$ between regions i and j. Consider a spatial setting with $n = m$ distinct demand markets, as modelled by the decay trade cost parameters $\tilde{g}_{ij} \in [0, 1]$. Each variety $j = 1, ..., m$ is produced at a distinct location, and each location is associated with a representative consumer with income E/m, where E denotes the aggregate income. Assume that the price of each variety k in each region j is competitively determined. The price of a composite from region j to region i modifies (8) to:

$$\pi_{ij} = \frac{V^{\frac{1}{1-\theta}} p_j}{\tilde{g}_{ij}}.$$

Assume that each region j has the amount of x^j of a productive immobile resource. Production of one unit of variety j takes one unit of resource. The competitive equilibrium price π_j of composite j determined from the resource market equilibrium condition:

$$\sum_{i=1}^{m} x_{ij}(\pi_j) = x^j, \ j = 1, ..., m,$$

where x_{ij} denotes the aggregate demand in a representative region i for the composite from region j and is the jth solution to (12)-(13).

The CES-demand for variety k (10) in a representative region is the solution to maximizing the constant elasticity of substitution CES utility subject to a budget constraint as formalized in problem (9). The function in the probabilistic constraint in the aggregate consumer's problem (12)-(13) is linear in the decision variables x_j. The random utility parameter can be interpreted as being independent of the deterministic parameter $\tilde{g}_{ij}$ measuring a linear decay cost between regions i and j. The allocation problem is then formally equivalent to the problem studied in Proposition (3). This implies the following:

Remark 3. *Suppose that the number of demand markets n is equal to the number of supply markets m. Assume that the number of regions m meets the threshold in Proposition (3). Then the aggregate consumer's demand defined at demand market i on the CES composite from each region $j = 1, ..., m$ is given by (cf. (22))*

$$x_{ij} = \frac{ED_{ij}}{\sum_{j=1}^{m} D_{ij}\pi_{ij}} \tag{28}$$

where now

$$D_{ij} = \frac{\bar{g} - \lambda\pi_{ij}}{\sqrt{v}}. \tag{29}$$

All inter-regional trade is intra-industry trade IIT. By the Grubel-Lloyd definition [Greenaway and Milner1986]) in terms of production for imports and *exports the volume of IIT is* $\sum_{i=1}^{m}\sum_{j\neq i} x_{ij}(\pi_{ij})$ *where* $x_{ij}(\pi_{ij})$ *is the demand in region* i *for the composite from* j *at price level* π_{ij}*.*

Aggregate Consumer vs. Individual Consumer

The demand for variety that the *RU* model predicts under discrete choice can be interpreted to follow from the incomplete knowledge of the econometrician estimating demand by specifying the distribution of the utility parameter, as argued in 2.5. Both the *CES* model and the random utility model can be interpreted as models of a representative consumer. The stochastic utility model of an individual consumer gives another motivation for trade in goods that are substitutes but can not be considered as perfect substitutes due to the random utility parameter (as shown in section 3). Like in the Hotelling type models, the utility parameter measures the decay cost to the different varieties. The consumer is unable to specify exactly how far the different alternatives are from his address (his ideal variety [Anderson et al.1996]).

5.2 Network Formation

As an application of the simple model in section 4 with price symmetry, consider [Bala and Goyal2000] where network formation is studied when the deterministic utility of i is given by

$$u_i = b_i(1 + \sum_{j=1}^{m} \tilde{g}_{ij} x_{ij}) - P \sum_{j=1}^{m} x_{ij} \tag{30}$$

where the level of communication is fixed $\mathbf{x} \in \{0,1\}^m$. By symmetry the deterministic parameter $\tilde{g}_{ij} = \tilde{g},\ i = 1, ..., n,\ j = 1, ..., m$ measures the decay in a direct link between node i and node j. It is assumed that by paying the link formation cost P i obtains from j $\tilde{g}_{ij} = \tilde{g} \in (0, 1]$. Furthermore, if there is a link between j and $k \neq j, i$, the amount node i obtains from k through the indirect link via j is $\tilde{g}^2$. Node i then prefers a direct link to k to an indirect link via j if

$$\tilde{g} - P \geq \tilde{g}^2. \tag{31}$$

If (31) is satisfied, the outcome is referred to as the complete network. In general, if the shortest path between i and j is via m links, i obtains $\tilde{g}^k$. A connected network need not be complete as some links may be indirect if the condition (31) does not hold.

In a strict Nash network each agent gets a strictly higher payoff with his strategy than he would with any other strategy [Bala and Goyal2000]. A strict Nash network with utility functions (30) is either empty (with no links) or connected via direct or indirect links between all nodes (ibid). Consider first the stochastic decision problem of a representative node i as written in (3). The parameters g_{ij} are assumed to be normally distributed. As argued in 2.1, the coefficient g_{ij} is negatively related to the distance between nodes i and j, $d_{ij} = L - g_{ij}$ for some large L. The distance can be defined in characteristics space or geographical space. This formulation simplifies the network formation game in [Bala and Goyal2000] by assuming each node makes its link formation decision without knowledge of existing links independently of each other. Like under (31), there is no incentive to consider indirect links. Letting $E = m$ captures the conditions $\mathbf{x}_i \in \{0,1\}^m,\ i = 1, ..., n$ since by Proposition (2) under price symmetry and the i.i.d. assumption only symmetric allocations emerge. In the deterministic model the empty network results whenever the cost of a single link exceeds its benefit $P > \tilde{g}$ (ibid.). In a stochastic network, there can be an incentive for network formation even if there is no incentive to form a single link. Proposition (2) implies that there is an incentive to form $m \geq m^*$ links or none under the i.i.d. assumption. Then, in the absence of knowledge of the decisions of other nodes, the symmetric random network is either empty or connected, via direct links only.

Assuming the nodes know the network (decisions of others) may change the outcome of Proposition (2) in two ways. First, some of the direct links may be replaced by indirect links (star network). Second, if the outcome of the simultaneous move game is an empty network, a connected network can still emerge if link formation can be supported via indirect links (this is referred to as a wheel network, ibid.). In both cases (star or wheel), the network is connected. To rule out indifference between forming and not forming links, consider only strict Nash networks. In summary, the outcome of network formation with i.i.d. decay parameters is analogous to that observed in previous work on deterministic network formation: a strict Nash network is either empty or connected.

6 Conclusion

This paper has revisited the modelling of demand for variety, from the point of view of linear random utility optimization via probabilistic constraints. In an economy with a single factor and constant returns to scale, the random utility model under the i.i.d. assumption implies demand for the spatially different varieties. This is analogous to the demands generated by CES-utility. In empirical work on intra-industry trade "industrial organization" features can explain part of this type of trade and the current work adds horizontal differentiation due to uncertainty in utility to the set of such variables. Uncertainty endogenizes demand for diversity. An application of the new model of demand for diversity to new economic geography was discussed.

Two interpretations of the linear random utility model are considered. First, it can be assumed that the utilities are random, e.g. due to unobservable characteristics. The demand system due to stochastic utility resembles the CES demand system. Under the second interpretation, made in most linear *RU* models, the modeller has incomplete information about the utility functions. The random parameters associated with the varieties, or composites, capture this uncertainty.

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