

# **An Analysis of Spatial and Economic Structure in a Region Comprising Different Kinds of Competition**

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## **Abstract**

A region comprises markets with different kinds of spatial competition such as free-entry competition and local monopoly. In this paper, a model is built to analyze spatial and economic structure established in each of the markets. This model shows as follows. Interactions between the markets with different kinds of competition influence a firm through the delivered price at the boundary of its market area and the net wage level at the edge of its commuting range. Under the interaction between these markets, a local monopoly firm first determines the size of its market, and then workers' commuting range to the firm is settled. As a local monopoly firm moves closer to a competitive market, the local monopoly's market area and commuting range become smaller. In addition, the firm, compared with a firm farther from the competitive one, is forced to sell consumers the goods at a lower price and to offer higher wages to workers. The nearer people reside to the competitive market, the higher their standard of living become because they are in a position to purchase the goods at a lower delivered price and to receive a higher net wage.

*Keywords:* Different kinds of competition, Market area, Commuting range, Delivered price, Net wage rates

*JEL Classification:* R10, R30

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## 1. Introduction

A region usually comprises different kinds of spatial competition such as free-entry competition, imperfective competition, and local monopoly in its market system: While many firms concentrate in one area and form a competitive market, firm of the same kind may exist in another, more remote area and enjoy local monopoly position on the market. When the markets with different types of competition are independent of one another economically as well as geographically, each market reveals the unique spatial and economic structure that is peculiar to the competition in question. Since these spatial and economic structures have the typical characteristics, the existing theory of spatial economics may elucidate adequately each of the structures. In general, however, the markets influence one another economically in spite of the geographical distances between them. In this case, the spatial and economic structure of the market area is different from that of the economically isolated market, and the structure may reveal complicated features due to the influences from other markets. In order to analyze the intricate structures successfully it is indispensable to build a model that is designed to describe essentially both the markets and the spatial situation surrounding them. The purposes of this paper are 1) to constitute a variant circumference model, 2) to clarify the mechanism of interaction between the markets with different kinds of competition, and 3) to analyze the spatial and economic structures of these markets in a region<sup>1</sup>.

Nakagome (1991) pointed out effects of the interrelation between different kinds of market on spatial economy<sup>2</sup>. He took up the labor markets in a country and investigated the effects on the labor markets of the relationship between competitive market and imperfectly competitive one. It is convinced that the analysis has a worth to be developed to include a commodity market

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<sup>1</sup>The circumference model is convenient for examining the operation of interactions among the economic agents such as manufacturer, retailers, and consumers in a spatial market. There are a number of studies of the spatial economy using the circumference model. The representative works are Evans (1972), Salop (1979), Novshek (1980), Heal (1980), Mathewson and Winter (1983), and Nakagome (1991).

<sup>2</sup> Wall (2001) provides another clue to investigation of the relationships between markets with different types of competition.

and to be sophisticated theoretically. This paper assumes the two kinds of competition, a kind of quasi-perfect competition and a local monopoly, and takes a goods market and a labor market simultaneously into the account in the analysis. Examining the sizes of markets and the commuting ranges of the firms and the firm's profit, price of goods, and wages of a worker, the paper shows a spatial and economic structure of a region with different kinds of competition.

The paper consists of four sections. Section II introduces assumptions of the analysis, and then builds a variant circumference model from two points of view, a spatial view and an economic view. It is also shown in the model building that the interaction operating between the markets with different kinds of competition plays an important role in generating a spatial and economic structure in a region. In section III the numerical analysis, which follows the analytical analyses, describes concretely the spatial and economic structure in a region: This analysis explicitly shows firms' market sizes and commuting ranges (spatial structure), and derives a firm's profit, the price of the goods, and the wages of workers (economic structure). Section IV summarizes the considerations of the spatial and economic structures generated in a region.

## **II. A model for the analysis of spatial and economic structure**

### **1. Spatial structure of the model and the types of competition**

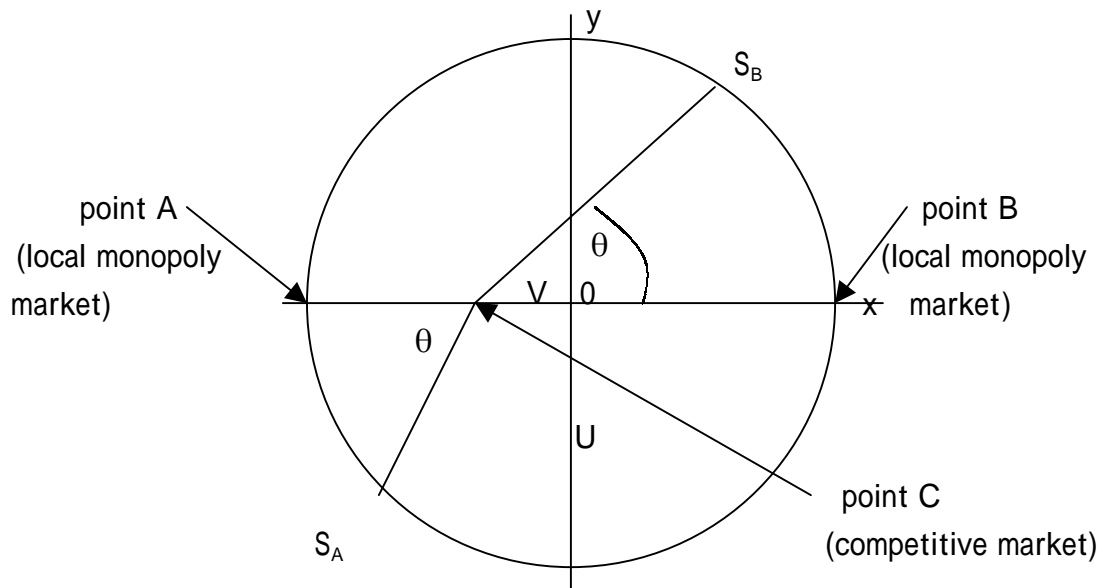
A circumference forming a circle with radius  $U$ , as illustrated in Figure 1, is assumed on the  $x$ - $y$  plane. Consumers are distributed evenly on the circumference with density  $d_S$ ; workers who wish to provide their labor to a industry in question are distributed evenly on the circumference with density  $d_L$ .

Firms A and B locate at the points A and B on the circumference, respectively, and they produce the same goods and sell them to consumers, employing workers who commute from varying inhabitant points to the firm. Firms A and B locate on the circumference market on the ground that they supply consumers with goods at the lower transportation cost. They are large and they enjoy a position of a local monopoly in their markets because there is no rival in the areas neighboring with the point A and B. Point A and B can

be, therefore, assumed to be a local monopoly market that is consist of a firm

There is a point C at the distance of  $V$  from the center  $O$  of the circumference. There is neither consumer nor worker at this point. It is assumed that this point is a kind of a port linked directly to the world market of the goods. All kinds of industry agglomerate at point C and this agglomeration provides firms with the so-called urban and localization economies. Firms concentrated on the point are small and they need agglomeration economies to manage their economic activities. They produce the same goods as firms A and B, employing workers commuting from varying inhabitant points on the circumference to the point, and they sell goods to the consumers on the circumference. The price  $p_C$  of the goods at the point is assumed to be given by the world market. In addition, it is assumed that the wage level  $w_C$  of an employee at the point is determined by the labor market involving all industries in the region. Based on the given price and wage level, each firm located at this point determines the number of employees to produce the optimal amount of production that maximizes profit. The economic behavior of these firms, thus, is similar to that of the firms in a perfectly competitive market. Point C is assumed a competitive market that consists of many firms

Figure 1 The Spatial Structure of the Circumference Model



## 2. Economic structure of the model and behavior of economic agents

### 1) The demand function of a consumer and net wage of an employee

The assumptions of behavior of a consumer and a worker are as follows. All consumers possess the same demand functions and each of the consumers purchases the goods at the firm supplying the consumer with the goods at the lowest delivered price. When a consumer purchases the goods at firm A or B, the consumer goes along the circumference to firm A or B, on the other hand, if a consumer goes to the firm at point C, the consumer goes straight to the place. The demand function, therefore, is to be described in two ways; when a consumer goes to the firm A or B, the function is shown by equation (1a); in the case of going to point C, it is given by equation (1b),

$$q_i = a - (p_i + t_s e_{si}) \quad (1a)$$

(i=A,B)

$$q_C = a - (p_C + t m_S) \quad (1b)$$

where  $q_i$  (i=A,B,) and  $q_C$  are quantities demanded for goods sold by each of the firms, respectively,  $a$  is the maximum reservation price of the goods,  $p_i$  (i=A,B) and  $p_C$  show the mill price of each of the firms, respectively,  $t_s$  is transport cost per mile when a consumer goes to firm A or B.  $e_{si}$  (i=A,B) shows the distance between a consumer in question and firm A or B along the circumference.  $t$  is transport cost per mile when a consumer goes to the point C.  $m_S$  is the distance between a consumer in question and the point. The distinction of the transportation cost reflects the difference of the transportation conditions between the transport on the circumference and that of distance between circumference and point C.

Workers commute and supply their labor to the firm that offers the highest net wage for the worker, less the transport cost to the firm; the commuting cost is borne by a employee. When a worker goes to firm A or B, the worker commutes to the firm along the circumference, and in the case of commuting to the firm at point C the worker goes straight to the place. Therefore, a net wage level for the worker need to be expressed in two ways; when a worker commutes to the firm A or B, the net wage  $W_i$  (i=A, B) is expressed by equation (2a); and in the case of working at point C, the net

wage  $W_C$  is given by equation (2b)

$$W_i = w_i - t_S e_{Li} \quad (2a)$$

(i=A,B)

$$W_C = w_C - t m_L \quad (2b)$$

where  $w_i$  (i=A,B) and  $w_C$  are wages offered by each of the firms, respectively.  $e_{Li}$  (i=A,B) is a distance from a worker in question to the firm A or B along the circumference, and  $m_L$  is a distance between a worker in question and the point C. Worker supplies his labor to the firm that offers the highest net wage and commutes to that firm. .

## 2) The production, revenue, cost, and profit functions of the firms

Firm A, firm B and the firms at point C produce the same goods using a certain number of workers and production facilities. All the firms, thus, have the same kind of production function. But the firms concentrated at point C, however, have slightly different values of parameters of the production function since they produce less than firms A and B. Equation (3a) shows the production function of the firm A and B, and that of the firms in point C are given by equation (3b).

$$Q_{Si} = S_{Gi} \text{Log}(S_{Li} n_i + 1) \quad (3a)$$

(i=A,B)

$$Q_{SC} = S_{GC} \text{Log}(S_{IC} n_C + 1) \quad (3b)$$

where  $Q_{Si}$  (i=A,B) and  $Q_{SC}$  are quantities produced by each of the firms, respectively.  $S_{Gi}$ ,  $S_{Li}$  and  $S_{GC}$ ,  $S_{IC}$  (i=A,B) are positive parameters; considering the sizes of these firms, the magnitudes of these parameters are assumed as follows:  $S_{Gi} < S_{GC}$  and  $S_{Li} > S_{IC}$ .  $n_i$  (i=A,B) and  $n_C$  are the number of workers employed by the firms, respectively.

Let us derive revenue of firms A and B. Since these firms are in position of a local monopoly in their market areas, the firms can set the mill price of the good they sell. After deriving mill price, the amounts of goods sold by the firms are obtained. Thus, the revenue  $R_i$  (i=A,B) of each firm is shown as

$$R_i = p_i Q_{Di} \quad (4)$$

(i=A,B)

where  $Q_{Di}$  (i=A,B) is the sales amount in each firm's market area.

Next, let us turn to the costs of the firms. The cost function of firms A and B is derived as follows. Since production quantity is determined by the firm to conform with the quantity demanded in its market, the number of workers employed by the firm is derived by using equation (3a) as a function of the quantity demanded in its market area. In consequence, the cost function  $TC_i$  (i=A, B) of the firm also is shown as a function of the quantity demanded  $Q_{Di}$ , given as equation (5):

$$TC_i = w_i \text{Exp}(Q_i / S_{Li}^{-1}) / S_{Gi} + F_i \quad (5)$$

(i=A,B)

where  $F_i$  (i=A,B) is the fixed costs. (Since  $Q_{Si}$  must be equal to  $Q_{Di}$ , hereafter,  $Q_i$  is used to express both production quantity and sales quantity of firms A and B).

Using the revenue and cost functions derived above, the profits  $Y_i$  (i=A,B) of firms A and B are obtained as a function of the sales amount  $Q_i$  of these firms:

$$Y_i = p_i Q_i - w_i \text{Exp}(Q_i / S_{Li}^{-1}) / S_{Gi} - F_i \quad (6)$$

(i=A,B)

Finally, let us derive the revenue, cost, and profit functions of the firms located at point C. Since the mill price  $p_C$  of the goods and the wage level  $w_C$  at point C are given exogenously, the revenue  $R_C$  and the cost function  $TC_C$  of the firms are simply shown by equations (7) and (8), respectively:

$$R_C = p_C S_{GC} \text{Log}(S_{IC}^{-nc+1}) \quad (7)$$

$$TC_C = w_C n_C + F_C \quad (8)$$

where  $F_C$  is fixed costs of the firm. Profit of a firm is, therefore, obtained as a function of the number of workers and equation (9) shows the profit of the firm.

$$Y_C = p_C S_{GC} \text{Log}(S_{IC} n_C + 1) - w_C n_C - F_C \quad (9)$$

### 3) Interaction between markets with different kinds of competition

Each of the consumers on the circumference is in position to purchase the goods at the delivered price of  $(p_C + t m_S)$  at point C. As firms A and B sell their goods to the consumers, these firms must set the mill price  $p_i$  ( $i=A,B$ ) for the delivered price  $(p_i + t s e_{Si})$  lower than  $(p_C + t m_S)$  for each of the consumers living inside their market areas. At any interior site of the market area, thus, inequality (10) must hold.

$$(p_i + t s e_{Si}) < (p_C + t m_S) \quad (10)$$

$(i=A,B)$

The boundary of the firm's market area is a place where the delivered price of the goods sold by the firm is the same as that of the firm at point C. At the boundary of the market, therefore, equation (11) must hold<sup>3</sup>,

$$(p_i + t s E_{Si}) = (p_C + t M_{Si}) \quad (11)$$

$(i=A,B)$

where  $E_{Si}$  ( $i=A,B$ ) is a distance from the firm to the boundary of its market along the circumference and  $M_{Si}$  is the distance from point C to the boundary of the market. From equation (11), equation (12) is derived:

$$p_i = p_C + t M_{Si} - t s E_{Si} \quad (12)$$

$(i=A,B)$

It is shown from equation (12) that the firms cannot freely price their goods; their prices are decisively affected by the relationship between the markets with different kinds of competition. They are restricted by the delivered price at the boundary of its market. And the delivered price is determined by, the mill price at point C, the distance from the firm to the

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<sup>3</sup> The consumer at the end-point of the market of the firm is indifferent between purchasing from each firm.

end-point of its market, the distance between point C and the end-point of the market, and the freight rates in these distances. Since the mill price set by the firm eventually becomes a function of the length of the firm's market, the firm's determination of the length of the market is a decisively important factor in its profit maximizing behavior <sup>4</sup>.

Another important effect on firms A and B of the relationship between the markets with different kinds of competition is shown by the determination of the wages offered by firms A and B. As mentioned above, since the wage level at point C is constantly given  $w_C$ , workers on the circumference are in position to commute to C and supply their labor to the firms at the net wage level  $(w_C - tm_L)$ . In order for firms A and B to attract workers, these firms offer the net wages  $(w_i - t_{seLi})$  higher than the net wages  $(w_C - tm_L)$ . Thus, at every interior site of the commuting range to the firms, inequality (13) must hold,

$$(w_i - t_{seLi}) > (w_C - tm_L) \quad (13)$$

(i=A,B)

The edge of the commuting range is a place where the net wage level presented by the firm A or B must be the same as that of the firms at point C. The worker at the edge-point is, then, indifferent in selecting the firm to supply its labor. At the edge of the commuting range, therefore, equation (14) must be held:

$$(w_i - t_{seLi}) = (w_C - tM_{Li}) \quad (14)$$

(i=A,B)

where  $E_{Li}$  (i=A,B) is a distance from the firm to the edge of its commuting range along the circumference.  $M_{Li}$  is a distance from point C to the edge of the range. From equation (14), equation (15) is obtained,

$$w_i = w_C - tM_{Li} + t_{seLi} \quad (15)$$

(i=A,B)

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<sup>4</sup> When the mill price at point C or the transportation cost is considerably high compared to the maximum reservation price, the firms do not determine the mill price according to equation (12). They can price their goods without effect from the point C.

Equation (15) shows that the firms cannot determine wages of its employee freely. The wage is dependent on the net wage level at the edge-point of firm's commuting range. This net wage is determined by many factors: the wage level at point C, the distance from the firm to the edge of the commuting range, the distance between point C and the edge of the range, and the freight rate in each distance. The wage determination of the firms explicitly shows the effect on the firm's decision making of the interaction between the markets.

The relationship between the markets with different kinds of competition influences the firms in point C. Once the lengths of markets and the commuting ranges of the firm A and B are determined, the length of the market area and the commuting range for all the firms in point C can be shown by equations (16a) and (16b), respectively.

$$2\pi U - 2(E_{SA} + E_{SB}) \quad (16a)$$

$$2\pi U - 2(E_{LA} + E_{LB}) \quad (16b)$$

Given the market size and the commuting range to the firms in point C by equations (16a, 16b), it is possible to obtain the total quantity demanded for the goods sold by these firms, from which the number of the firms can be derived. Then, it can be shown that the interaction between the markets influences the level of production and sales activity at point C.

### **3. Description of sales amount, commuting range, and wage in terms of length of the market and the angle indicating the length**

#### **1) Length of the market and sales amount and wages**

In this subsection, sales amount of firms A and B and the wage offered to an employee are expressed as a function of length of the firm's market.

Now, let  $\theta_i$  ( $i=A, B$ ) be the angle at point C, as illustrated at Figure 1, formed by the  $x$  axis and the line connecting point C and a consumer in question (for example, a consumer inhabited at  $S_A$  or  $S_B$  at Figure 1) on the market of the firm. Using the angle  $\theta$ , the quantity demanded  $Q_i$  ( $i=A, B$ ) on the firm's market is shown by equation (17a,b),

$$Q_A = 2d_S \int_{3.14}^{E_{SA}} (a - p_A - t_s \theta) d\theta \quad (17a)^5$$

$$Q_B = 2d_S \int_0^{E_{SB}} (a - p_B - t_s \theta) d\theta \quad (17b)$$

Integrating equation (17a, b) with respect to  $\theta$ , equation (18) is obtained,

$$Q_i = 2d_S E_{Si} (a - p_i - t_s E_{Si}/2) \quad (18) \\ (i=A,B)$$

As shown by equation (18), the sales amount of the firm is expressed as a function of the length of the firm's market. Since the mill price of the firm is, as shown by equation (12), a function of the length of the market, revenue of the firm can be shown as a function of the length of the market.

Using the production function of the firm, the number of workers  $n_i$  ( $i=A,B$ ) employed by the firm can be expressed by the length  $E_{Si}$  ( $i=A,B$ ) of its market,

$$n_i = (\text{Exp}(2d_S E_{Si} (a - p_i - t_s E_{Si}/2) / S_{Li}) - 1) / S_{Gi} \quad (19) \\ (i=A,B)$$

It follows that the number of workers can be shown in terms of the length of commuting range. The number is given as

$$n_i = 2d_L E_{Li} \quad (20) \\ (i=A,B)$$

Combining equations (19) and (20), it becomes possible to show the length  $E_{Li}$  ( $i=A, B$ ) of the commuting range to the firm as a function of the length  $E_{Si}$  of its market,

$$E_{Li} = (\text{Exp}(2d_S E_{Si} (a - p_B - t_s E_{Si}/2) / S_{Li}) - 1) / (2d_L S_{Gi}) \quad (21) \\ (i=A,B)$$

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<sup>5</sup> The upper bound  $E_{SA}$  of equation (17a) is a function of angle  $\theta$ .

The distances between point C and each of the edge-points of the commuting ranges of firms A and B are easily derived as a function of the lengths of the firms' markets. Equations (22a,b) show these distances,

$$M_{LA}=(U^2 + V^2 - \text{Cos} ((\text{Exp}(2 d_S E_{SA}(a - p_A - t_S E_{SA}/2)/ S_{Li}) - 1)/(2 d_L S_{Gi})))^{0.5} \quad (22a)$$

$$M_{LB}=(U^2 + V^2 + \text{Cos} ((\text{Exp}(2 d_S E_{SB}(a - p_B - t_S E_{SB}/2)/ S_{Li}) - 1)/(2 d_L S_{Gi})))^{0.5} \quad (22b)$$

Substituting  $E_{Li}$  and  $M_{Li}$  ( $i=A, B$ ) in equation (15) with (21) and (22a,b), respectively, the wage  $w_i$  ( $i=A, B$ ) offered by firms A and B can be described as a function of the lengths  $E_{Si}$  of the firms' markets. Equations (23a) and (23b) show the wages.

$$w_A = w_C - t(U^2 + V^2 - \text{Cos}((\text{Exp}(2 d_S E_{SA}(a - p_A - t_S E_{SA}/2)/ S_{Li}) - 1)/(2 d_L S_{Gi})))^{0.5} + t_S((\text{Exp}(2 d_S E_{SA}(a - p_A - t_S E_{SA}/2)/ S_{Li}) - 1)/(2 d_L S_{Gi})) \quad (23a)$$

$$w_B = w_C - t(U^2 + V^2 + \text{Cos}((\text{Exp}(2 d_S E_{SB}(a - p_B - t_S E_{SB}/2)/ S_{Li}) - 1)/(2 d_L S_{Gi})))^{0.5} + t_S((\text{Exp}(2 d_S E_{SB}(a - p_B - t_S E_{SB}/2)/ S_{Li}) - 1)/(2 d_L S_{Gi})) \quad (23b)$$

Since the number of workers employed by the firm is given as a function of the length of the market, using equations (19) and (23a, 23b), the cost of the firm is expressed by a function of the length of the firm's market. Eventually, revenue, costs, and the commuting ranges of the firms can be described by using the lengths of the firms' markets.

## 2) Expression of the length of the market of the firm in terms of angle

Market length, which is the only independent variable in the decision-making of the firm A and firm B, is to be depicted by a length of arc, a segment of circle. It may be less convenient using a length itself than an angle indicating the length to show the market size of the firm and to describe other economic items on a circumference. In this subsection, the angle indicating a length of the market of the firm is introduced in order to

more easily carry out the analysis. To this aim, first, denote by  $\theta_i^*$  (i=A, B) the angle formed at point C by the x axis and the line connecting point C and the end-point of the market of the firm. Secondly, let us express a distance  $M_{Si}$  between point C and the boundary of the market area of the firm in terms of using the angle  $\theta_i^*$  (i=A, B).

The distance  $M_{Si}$  is shown by equation (24),

$$M_{Si} = V \cos \theta_i^* + ((V \cos \theta_i^*)^2 - V^2 + U^2)^{0.5} \quad (24)$$

(i=A,B)

The length  $E_{Si}$  (i=A,B) of the market area is obtained by integrating equation (24) with respect to  $\theta$  from zero to  $\theta_B^*$  for the firm B, and from  $\pi$  to  $\theta_A^*$  for the firm A. The lengths of firms' market area are shown as equation (25a) and (25b), respectively.

$$E_{SA} = \int_{\pi}^{\theta_A^*} ((dM_{SA} / d\theta)^2 + M_{SA}^2)^{0.5} d\theta \quad (25a)$$

3.14

$$E_{SB} = \int_0^{\theta_B^*} ((dM_{SB} / d\theta)^2 + M_{SB}^2)^{0.5} d\theta \quad (25b)$$

The lengths  $E_{Si}$  (i=A,B) of the markets of firms A and B are obtained as a function of the angles  $\theta_i^*$  (i=A,B),

$$E_{SA} = U(\theta_A^* + \text{ATN}(k/(1 - k^2)^{0.5}) - \pi) \quad (26a)$$

$$E_{SB} = U(\theta_B^* + \text{ATN}(k/(1 - k^2)^{0.5})) \quad (26b)$$

where  $k$  is  $(V/U) \sin \theta_i^*$  (i=A,B).

The lengths of the markets of firms A and B can be now identified by the angles  $\theta_i^*$  (i=A,B). It becomes possible to derive the commuting ranges, wages of employees, profits and sales amounts of goods sold by the firms by using the angles  $\theta_i^*$ . For example, the profit of the firm B is expressed by equation (27) as a function of the angle  $\theta_B^*$ ,

$$\begin{aligned}
Y_B = & ((p_C + t - t_S) * 2d_S * (a - (p_C + t - t_S) - (t_S/2))) - \\
& ((1/s_{G,B})(\text{Exp}((1/s_{L,B}) * 2d_S * (a - (p_C + t - t_S) - (t_S/2)))) - 1))^* \\
& (w_C - (V^2 + U^2 + 2VU \cos((1/(2*d_L U))((1/s_{IB})(\text{Exp}((1/s_{GB}) * 2d_S * (a - (p_C + t - t_S) - \\
& (t_S/2)))) - 1)))^{0.5} + t_S(1/2d_L)((1/s_{IB})(\text{Exp}((1/s_{L,B}) * 2d_S * (a - (p_C + t - t_S) - \\
& (t_S/2)))) - 1))) - F_B \tag{27}
\end{aligned}$$

where  $\theta_B^* = V \cos \theta_B^* + ((V \cos \theta_B^*)^2 - V^2 + U^2)^{0.5}$  and

$$= U(\theta_B^* + \text{ATN}((V/U) \sin \theta_B^* / (1 - ((V/U) \sin \theta_B^*)^2)^{0.5})).$$

#### 4. The optimization condition of the firms and the equilibrium condition in the goods market

##### 1) The optimization condition of the firms

The firms have to determine the angles  $\theta_i^*$  ( $i=A, B$ ) that indicate the lengths of their markets to maximize profits. The angle is obtained by differentiating the profit function with respect to the angle  $\theta_i^*$  and setting it equal to zero: Solving equation (28) with respect to  $\theta_i^*$  gives the optimal angle to the firm. Equation (28) shows the optimization condition of firms A and B:

$$\begin{aligned}
dY_i / d\theta_i^* &= 0 \tag{28} \\
&(i=A, B)
\end{aligned}$$

The optimization condition of a firm located at point C is obtained as follows: Since the profit function of the firm is a function of the number of workers, as shown by equation (9), the firm determines the number of workers to maximize profit.. Solving equation (29) with respect to  $n_C$  gives the optimal number of workers for the firm. Thus, the optimization condition of the firm at point C is shown by equation (29).

$$dY_C / dn_C = 0 \tag{29}$$

## 2) Equilibrium condition of goods market in a region

Once the lengths of the markets of firms A and B are derived, the sizes of the market and commuting range to the firms at point C are obtained automatically: Total quantity  $TQ_{DC}$  demanded on the market left to the firms in the point is obtained by integrating the demand function (1b) of a consumer with respect to  $\theta$  from  $\theta_A^*$  to  $(2\pi - \theta_B^*)$ ,

$$TQ_{DC} = 2d_s \int_{\theta}^{2\pi-\theta} (a - p_C - t(V \cos \theta + ((V \cos \theta)^2 - V^2 + U^2)^{0.5})) d\theta \quad (30)^6$$

Since the quantity demanded for the goods of the firm A and B is equal to the quantity supplied by firms A and B, equilibrium in the goods market is established when the quantity remaining demanded coincides with the quantity supplied by firms at point C. Equating the total demand and supply function relevant to the firms at C, the equilibrium in the goods market is derived. Namely, solving equation (31) with respect to the number of firms  $N_C$  in the point, the number of the firms that equilibrate the goods market is derived<sup>7</sup>,

$$TQ_{DC} = N_C Q^*_{SC} \quad (31)$$

where  $Q^*_{SC}$  is quantity produced by the firm at point C where each firm employs the optimal number of workers that is derived by the equation (29).

### III. The spatial and economic structure generated in a region comprising the markets with different types of competition

#### 1. The angle indicating the optimal length of the market of a local monopoly firm

Based on the optimization condition of the firm A and B, as shown by

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<sup>6</sup> This integration is called the second kind ellipse integration.

<sup>7</sup> In the paper wage assumes to be determined exogenously, then wage does not vary to clear the labor market. There is a possibility for some workers not to work at the industry in question.

equation (28), it is possible to derive the angles that maximize the profits of the firms. It is, however, quite difficult to solve equation (28) by an analytical method. A numerical calculation method is needed to solve equation (28) with respect to the angle. In this section, giving concrete figures to the parameters used in the equations, the optimal length of the market and commuting range and profit of the firm are derived. Derivation of the values of these items shows concretely the spatial and economic structure in the region. Table 1 shows the figures assigned to the parameters in the equations.

Table 1 Figures assigned to parameters

U	V	$d_S$	$d_L$	$p_C$	$w_c$	t	$t_S$
5	2	1.75	1.25	5	10	1	1.2
$S_{GA,B}$	$S_{IA,B}$	$S_{GC}$	$S_{Ic}$	a	$F_{A,B}$	$F_C$	
3	30	3.5	3	15	5	1	

Given these concrete figures to the parameters on equations, it is possible to solve equation (28) with respect to the angles and derive the optimal angles indicating the optimal lengths of the markets of firms A and B. The numerical calculation gives the optimal angles for the firm A and B as follows:

$$\theta_A^* = 4.0790$$

$$\theta_B^* = 0.6252$$

Solving equation (31) with respect to  $N_C$  gives the equilibrium number of firms at point C:

$$N_C = 9.04$$

## 2. The spatial and economic structure established on the circumference

Now, let us derive the spatial and economic structure in the region using the obtained optimal angles in the previous subsection. Table 2 shows the

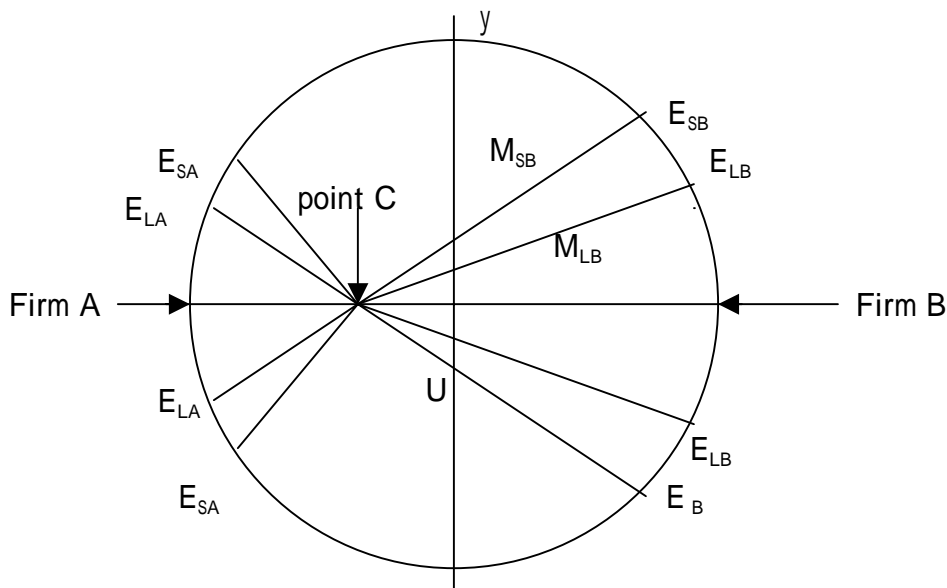
items and their values that describe the spatial and economic structure, the lengths of the markets and the commuting ranges of the firms, wages of employees, the firm's profit, the mill price and the sales amount of the goods.

Based on the values provided in Table 2, the following observations arise: Firstly, firm A, which is closer to the competitive market, possesses a smaller market than that of firm B, which is farther from the competitive market. The markets of the firm A and B are larger than their commuting ranges. Figure 2 depicts, using the values shown in Table 2, the spatial structure, that is, the firms' markets and their commuting ranges.

Table 2 A spatial and economic structure established in the region

Firm	A	B	C, the firm in point C
Length of market: $2E_S$	6.08	8.6	-
Commuting range: $2E_L$	4.78	5.44	-
Profit: $Y$	369.9	531.5	0.27
Sales amount: $Q$	88.2	91.9	3.08
Mill price: $p$	4.89	6.31	5.0
Wage: $w$	9.51	6.48	10.0
Number of worker: $n$	5.98	6.82	1.41
Number of firms: $N$	1	1	9.04
Average sales quantity: $Q/2E_S$	14.5	10.5	1.66

Figure 2 A spatial structure established in the region



Secondly, while the mill price of firm A is lower than the price of the competitive market, firm B's price is higher than the price in the competitive market. The wages paid by firms A and B are lower than that offered in the competitive market. The wage presented by firm A is higher than that of firm B.

Thirdly, people who reside closer to the competitive market obtain the goods at a lower delivered price and receive higher wage. They, as consumers and workers, can enjoy an advantageous position compared to the people who live farther from the competitive market.

In summary, the difference between the distances from each of firms A and B to the competitive market reflects the different performances of these firms, the differences in profits, prices, and wages. Since firm B, compared with firm A, is protected by a longer spatial distance from the competition at C, firm B can price its goods relatively high and offer lower wage to its workers. Firm B's profit, therefore, is higher than that of firm A. The firm located farther from the competitive market enjoys an advantageous position in the region. It is noteworthy, however, that firm A sells almost as much as firm B because firm A's delivered price is lower than that of firm B.

### **3. The effects on the spatial and economic structure of the exogenous environment change in the market**

This subsection observes the spatial and economic structure that results as the exogenous environment of the market system changes.

#### **1) The effects of an increase in consumer density on the spatial and economic structure**

Firstly, let us examine the effects on the spatial structure of an increase in consumer density ( $d_s$ ) by 0.75 to 2.5. Table 3(1) shows the values of the items describing the structure resulting from the density change. When the consumer density increases, the markets of firms A and B become smaller, and their commuting ranges become longer; it is especially interesting that the commuting range of firm A becomes larger than its market area.

The observation of the economic aspect of the structure shows the following. The increase in consumer density raises the mill prices and wage levels of firms A and B. The wage level of firm A becomes higher than that

of point C. But profit of the firm A rises due to the increase in number of consumers in its market area.

Examining the firms located at point C, the effects of the changes in external conditions are reflected by the number of the firms: the increase in consumer density raises the number of firms at point C because of the increasing demand for the goods sold by the firms.

**2) The effects of a decrease of the freight rate on the spatial and economic structure**

Secondly, let us investigate the effects on the structure of a decrease in the freight rate ( $t_s$ ) from 1.2 to 0.75. Table 3 (2) shows the values of the items describing the structure resulting from the decrease in the freight rate. The two changes of the structure attract attention: 1) the commuting range of the firm A is longer than that of firm B; 2) firm A has greater sales and employs more workers than firm B<sup>8</sup>.

Table 3 The changes in the external condition of the market system in the region and the spatial and economic structure

	(1) $d_s=2.5$			(2) $t_s=0.75$		
Firm	A	B	C	A	B	C
2E <sub>S</sub>	4.74	7.22	-	8.02	10.46	-
2E <sub>L</sub>	6.22	7.2	-	9.12	8.48	-
Y	442.7	655.4	0.27	516.7	690.7	0.27
Q	95.8	100.0	3.0	107.1	104.7	3.0
$\rho$	5.5	7.29	5.0	5.87	7.32	5.0
w	10.16	7.69	10.0	9.34	6.68	10.0
n	7.79	9.01	1.41	11.52	10.6	1.41
N	1	1	10.8	1	1	6.7
Q/2E <sub>S</sub>	20.2	13.8	1.6	13.3	10.0	1.5

Examining of the figures shown in Table 3 (1) and (2), the following insights are obtained: If the change in external conditions expands firms'

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<sup>8</sup> The above analysis suggests that the change in freight rate ( $t_s$ ) gives rise to the great change of the spatial structure and an improvement in the transportation system in the suburban area. These changes favor firms located close to the competitive city center over firms farther away from the center.

markets and commuting ranges, the change benefits the firm closer to the competitive market, compared to the firm farther from the competitive market. The explanation is that expanding the market and the commuting range of the firm close to point C, the competitive market, makes their end-points recede from point C. Thus, the delivered price at the end-point becomes higher and the net wage level at the end-point becomes lower. The firm, then, can price its goods relatively higher and offer relatively lower wage to workers. These changes increase the profits of the firm closer to the point C, the competitive market.

### **3) The effects on firms' locations of changes in mill price and freight rate of competitive market**

This subsection analyses the relationship between the mill price and wage level at point C and the locations of the firms in the region. When the mill price given exogenously at the point C varies upwards or downwards on large scale, profits of all firms in the region decrease considerably. It may become difficult for the firms to continue operating in the region. All firms, however, may not necessarily exit from the region simultaneously due to the aggravation of the managerial environment. For example, when the mill price at point C decreases to 0.1 and the wage level decreases to 5.85, profits of the firms at point C become zero. They are forced to exit from the region, while firms A and B can still obtain positive profit in each location: The locations apart from the competitive market survive in the region<sup>9</sup>. In this case, then, the consumers who are supplied with the goods from firms at point C begin to import the goods from other regions at the point.

If the wage level increases to 179.8, assuming that the mill price remains 0.1, the profit of firm A decreases to zero and firm A exits from the market. Only firm B can engage in business in the region. Many consumers on the circumference, except the consumers living near the firm B, purchase the imported goods at point C. If the wage level at the point C increases further to 400, no firm can make a positive profit, and the industry in question disappears from the region. All consumers on the circumference buy

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<sup>9</sup> It is safe to say that when the regional or national economy enters a slowdown, the firms in the competitive market disappear first from the region, while as an economic expansion starts, they appear last in the region.

the imported goods at point C.

#### 4. The spatial and economic structure in the closed economic system

This section carries out the analysis of the spatial and economic structure under the so-called closed economic system. To this aim, some of the assumptions are altered as follows: The mill price at point C is set to meet the supply of the goods with the quantity demanded on the circumference: New firms continue to enter at point C until profits of the firms located at point C become zero. All other assumptions remain unchanged and the analysis uses the figures shown in Table 1 to yield the parameters.

Let us derive the lengths of the markets of firms A and B, the number of firms located at point C, and the mill prices of the firms to describe the spatial and economic structure under this closed system. The procedure of derivation is as follows. The number of the firms  $N_C$  located at the point is handled as a parameter. Increasing the value of the  $N_C$  at intervals of 0.1 from a certain level (e.g., level 10), the following simultaneous equations of (32a), (32b), and (32c) continue to be solved with respect to  $\theta_A^*$ ,  $\theta_B^*$ , and  $\rho_C$  for each level of  $N_C$  until the profit of the firm at point C becomes zero (until  $Y_C = 0$ , is satisfied).

$$dY_A/d\theta_A^* = 0 \quad (32a)$$

$$dY_B/d\theta_B^* = 0 \quad (32b)$$

$$TQ_{DC} - N_C Q_{SC} = 0 \quad (32c)$$

The equilibrium values of  $N_C$ ,  $\theta_A^*$ ,  $\theta_B^*$ , and  $\rho_C$  are derived by the numerical calculation method as  $N_C = 28.3$ ,  $\theta_A^* = 3.72$ ,  $\theta_B^* = 0.43$ ,  $\rho_C = 1.78$ , respectively<sup>10</sup>. It is possible from these equilibrium values to describe the spatial and economic structure established under closed economic system in the region.

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<sup>10</sup> As the number  $N_C$  of the firms increases up to 25, the profit of firms at point C decreases to zero and the lengths of the markets of firm A and B reduce to the final equilibrium.

Table 4 A spatial and economic structure generated in the closed system

Firm	A	B	C
2E <sub>S</sub>	3.62	5.98	-
2E <sub>L</sub>	2.52	4.48	-
Y	166.9	389.7	0
Q	70.6	86.4	2.19
ρ	2.81	4.94	1.78
w	8.41	5.84	10.0
n	3.17	5.62	0.29
N	1	1	28.3
Q/2E <sub>S</sub>	19.5	14.4	2.85

The figures in Table 4 describe the spatial and economic structure in this situation. Compared to the structures in the open economic system assumed in the previous sections, the closed system reflects a greater role of the firms at point C: The number of firms at point C and their sales increase. The other characteristics relevant to firm A and B under this system are not so different from those of the open system.

#### IV. Summary and conclusions

Using the variant circumference model, this paper analyzes the spatial and economic structure of a region containing different kinds of spatial competition in its market system. The interaction between markets with different kinds of competition influences firm through the delivered price at the boundary of its market and the net wage level at the edge of its commuting range. Under this interaction, a local monopoly firm determines the optimal size of the market from which the optimal commuting range is derived. Based on the derived market size, the profit, the sales amount, and the number of employees of the firm are derived. From these derivations, it is possible to clarify a spatial and economic structure established in a region.

The market area and commuting range of the firm closer to the competitive market is smaller than those of firm located farther from the competitive market. Firms' markets are usually larger than their commuting ranges regardless of the distance from the firm to the competitive market. However, when consumer density is relatively high on the circumference

market, the commuting range of the firm closer to the competitive market becomes larger than its market area

Regarding the economic structure, the firm closer to the competitive market is forced to set a lower mill price and to offer a higher wage than firm farther from the competitive market. The managerial environment is harsher for the firm located closer to the competitive market.

It is said that since firms at the competitive market are subjected directly to the world goods market and they are not protected by geographical distance, their economic position is not robust. When the conditions of the world market start to deteriorate, these firms are first to disappear from the region.

From the viewpoint of the people, the area closer to the competitive market is favorable for consumers and workers; in this area, the delivered price of the goods is lower and the net wage level is higher. If people could move freely, they would come to reside in the area nearer the competitive market. Such a movement may improve the managerial environment of the firm closer to the competitive market since it increases its sales.

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