SPACE-TIME MODELING OF TRAFFIC FLOW

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ABSTRACT

This paper discusses the application of space-time autoregressive integrated moving average (STARIMA) methodology for representing traffic flow patterns. Traffic flow data are in the form of spatial time series and are collected at specific locations at constant intervals of time. Important spatial characteristics of the space-time process are incorporated in the STARIMA model through the use of weighting matrices estimated on the basis of the distances among the various locations where data are collected. These matrices distinguish the space-time approach from the vector autoregressive moving average (VARMA) methodology and enable the model builders to control the number of the parameters that have to be estimated. The proposed models can be used for short-term forecasting of space-time stationary traffic-flow processes and for assessing the impact of traffic-flow changes on other parts of the network. The three-stage iterative space-time model building procedure is illustrated using 7.5 min. average traffic flow data for a set of 25 loop-detectors located at roads that direct to the centre of the city of Athens, Greece. Data for two months with different traffic-flow characteristics are modelled in order to determine the stability of the parameter estimation.

1. INTRODUCTION

The space-time autoregressive integrated moving average (STARIMA) model class was first presented in the literature in the early eighties. Since then it has been applied to spatial time series data from a wide variety of disciplines such as river flow (Pfeifer and Deutsch 1981a), spread of disease (Pfeifer and Deutsch 1980a), and spatial econometrics (Elhorst 2000, Giacomini and Granger 2001). The STARIMA methodology was illustrated in a series of papers by Pfeifer and Deutsch (1980a, 1980b, 1981a, 1981b). As they point out:

"*Processes amenable to modelling via this class are characterized by a single random variable observed at N fixed sites in space wherein the dependencies between the N time series are systematically related to the location of the sites. A hierarchical series of* $N \times N$ *weighting matrices specified by the model builder prior to analysing the data is the basic mechanism for incorporating the relevant physical characteristics of the system into the model form. Each of the N time series is simultaneously modelled as a linear combination of past observations and disturbances at neighbouring sites. Just as univariate ARIMA models reflect the basic idea that the recent past exerts more influence than the distant past, so STARIMA models reflect (through the specification of the weighting matrices) the idea that near sites exert more influence in each other than distant ones*."

To our knowledge it's the first time that a purely inductive model is proposed for the spatiotemporal behaviour of traffic flow. Till now the vast majority of inductive techniques were univariate in nature; that is only historical data from a given location were used for modelling and predicting its future behaviour. Specifications have ranged from Kalman filtering (Whittaker et al., 1997), non-parametric regression (Davis and Nihan, 1991), regression with time varying coefficients (Rice and van Zwet, 2001), neural networks (van Lint and Hoogendoorn, 2002) and ARIMA models (Williams et al., 1997, Lee and Fambro, 1999). Limited amount of work has been performed using multivariate modelling techniques (Ben-Akiva et al. 1996, Stathopoulos and Karlaftis 2002), all of them based on the state-space methodology and employed for short-term forecasting of traffic flow using a relatively small number of measurement locations.

This paper presents how the STARIMA methodology can be tailored to model the traffic flow of a road network; the approach is similar to the one adopted by Deutsch and Ramos (1987) for vector hydrologic sequences. In addition to its potential use for short-term forecasting, this model class contributes to the understanding of the spatiotemporal evolution of traffic flow since it can be used to estimate how changes in traffic-flow patterns in some specific locations are propagated to the rest of the network.

In the next session, an overview of the STARIMA model class and model building procedure is presented. Next, the experiment and the data are described followed by a report of the relevant model building details and an examination of the model coefficient estimates for different time periods. Finally, we discuss the results and their implications for the applicability of STARIMA modelling.

2. THE STARIMA MODEL CLASS AND MODEL BUILDING PROCEDURE

2.1 PHYSICAL BASIS

In traffic flow systems tree structures are the most common method for network representation. The direction of the vectors of the tree follows the permitted traffic direction, whereas traffic flow measurements are taken at specific points of the network (Figure 1). If we assume that the traffic flow process forms a “black-box” network, i.e. one that does not have access to any information other than past or present flows, then from Figure 1 it is clear that some measurement locations may not be connected through a path and therefore may act independently. If we also ignore any external effects and consider the distance between the measurement locations to be sufficiently long so as no congestion effects are introduced to disturb the flow pattern, no measurement location will be influenced by actions occurring downstream from it. Thus, downstream locations only depend on upstream locations but not vice versa. The question that has to be answered is how to exploit this structure in model identification and yet retain the statistical properties of the traffic flow process. The spatial topological relationships of a network as the one presented in Figure 1 can be introduced through a hierarchical ordering for the neighbors of each measurement site. This is the basis for system structuring using STARIMA model building. We shall call W_l a square $N \times N$ l^{th} order weight matrix with elements $w_{ij}^{(l)}$ that are nonzero only in the case that the measurement locations i and j are “l^{th} order neighbors”. First order neighbors are understood to be closer than second order ones, which are closer than third order neighbors and so on.

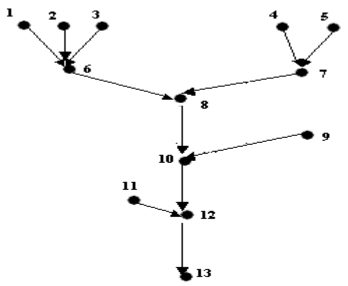

FIGURE 1. The typical road network tree structure for traffic flow. The dots represent measurement locations and the arrows the direction of flow.

The weights $w_{ij}^{(l)}$ are taken so that $\sum_{j=1}^{N} w_{ij}^{(l)} = 1$ and W_0 is the identity matrix since each site is its own zeroth order neighbour. Applying this rule to the network of Figure 1 and assigning equal weights to the l^{th} order neighbours of each site yields the following weight matrices for spatial lags 1 and 2:

$$
W_1 = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13 \end{array}
\begin{array}{c}
\begin{array}{ccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.33 & 0.33 & 0.33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
\qquad
W_2 = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13 \end{array}
\begin{array}{c}
\begin{array}{ccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0
\end{bmatrix}
\end{array}
$$

Additional features such as the distances of each neighbouring pair of sites are usually incorporated into the weighting matrices through an appropriate selection of weights**.**

2.2. THE STARIMA MODEL

The STARIMA model class expresses each observation at time t and location i as a weighted linear combination of previous observations and innovations lagged both in space and time. The basic mechanism for this representation is the hierarchical ordering of the neighbours of each site and a corresponding sequence of weighting matrices as presented in the previous section. The specification of the weighting matrices is a matter left to the model builder to capture the physical properties that are being considered endogenous to the particular spatial system being analysed.

If Z_t is the $N \times 1$ vector of observations at time *t* at the *N* locations within the road network then the seasonal STARIMA model family is expressed as,

$$\Phi_{\mathrm{P},\Lambda}\left(\mathrm{B}^S\right)\phi_{p,\lambda}(\mathrm{B})\nabla_S^D\nabla^d Z_t = \Theta_{Q,M}\left(\mathrm{B}^S\right)\theta_{q,m}(\mathrm{B})a_t \tag{1}$$

where

$$\Phi_{\mathrm{P},\Lambda}\left(\mathrm{B}^S\right) = \mathrm{I} - \sum_{k=1}^{p}\sum_{l=0}^{\Lambda_k}\Phi_{kl}W_l\mathrm{B}^{kS} \tag{1a}$$

$$\phi_{\mathrm{P},\lambda}(\mathrm{B}) = \mathrm{I} - \sum_{k=1}^{p}\sum_{l=0}^{\lambda_k}\phi_{kl}W_l\mathrm{B}^{k} \tag{1b}$$

$$\Theta_{Q,M}\left(\mathrm{B}^S\right) = \mathrm{I} - \sum_{k=1}^{p}\sum_{l=0}^{\lambda_k}\Theta_{k,l}W_l\mathrm{B}^{kS} \tag{1c}$$

$$\theta_{q,m}(\mathrm{B}) = \mathrm{I} - \sum_{k=1}^{q}\sum_{l=0}^{m_k}\theta_{kl}W_l\mathrm{B}^{k}\ . \tag{1d}$$

Φ_{kl} and ϕ_{kl} are respectively the seasonal and nonseasonal autoregressive parameters at temporal lag *k* and spatial lag *l*; similarly Θ_{kl} and θ_{kl} are the seasonal and nonseasonal moving average parameters at temporal lag *k* and spatial lag *l*; *P* and *p* are the seasonal and nonseasonal autoregressive orders; *Q* and *q* are the seasonal and nonseasonal moving average orders. Λ_k and λ_k are the seasonal and nonseasonal spatial orders for the k^{th} autoregressive term; M_k and m_k are the seasonal and nonseasonal spatial orders for the k^{th} moving average term; and *D* and *d* are, respectively, the number of seasonal and nonseasonal differences required, where ∇_S^D and ∇^d are the seasonal and nonseasonal difference operators, such that i.e., $\nabla_S^D = \left(\mathrm{I} - \mathrm{B}^S\right)^D$ and $\nabla^d = (\mathrm{I} - \mathrm{B})^d$ with seasonal lag *S*. Finally, a_t is the random, normally distributed, error vector at time *t* with statistics:

$$E\{a_t\} = 0 \tag{2a}$$

$$E\{a_t a'_{t+s}\} = \begin{cases} G \ \ if \ \ s = 0 \\ 0 \ \ if \ \ s \neq 0 \end{cases} \tag{2b}$$

$$E\{Z_t a'_{t+s}\} = 0 \ \text{for} \ s > 0\,. \tag{2c}$$

Equation (1) is referred to as a seasonal multiplicative STARIMA model of order $(p_{\lambda}, d, q_m) \times (P_{\Lambda}, D, Q_{\mathrm{M}})_S$.

When there is no seasonal component (quite unlikely in traffic flow) and d=0 the model collapses to the easier to interpret STARMA model which is of the form

$$Z_t = \sum_{k=1}^{p}\sum_{l=0}^{\lambda_k} \phi_{kl} W_l Z_{t-k} - \sum_{k=1}^{q}\sum_{l=0}^{m_k} \theta_{kl} W_l a_{t-k} + a_t \tag{3}$$

where p is the autoregressive order, q is the moving average order, λ_k is the spatial order of the k^{th} autoregressive term, m_k is the spatial order of the k^{th} moving average term, ϕ_{kl} and θ_{kl} are parameters to be estimated and W_l is the $N \times N$ matrix for spatial order l and a_t is the random normally distributed innovation or disturbance vector at time t.

STARMA models can be viewed as special cases of the Vector Autoregressive Moving Average (VARMA) models (Lutkerpohl 1987,1993). The VARMA models use general $N \times N$ autoregressive and moving-average parameter matrices to represent all autocorrelations and cross-correlations within and among the N time series. If the diagonal elements in these matrices are assumed to be equal (as in the case where the N series represent a single random process operating at different sites) and the off-diagonal elements are assumed to be a linear combination of the W_l weight matrices then the general VARMA family collapses to the STARMA model class. The VARMA model class on the other hand, can be viewed as a special case of the state-space model, which is the only multivariate technique presented in the literature of traffic-flow modelling so far. It's obvious from (1) and (3) that the STARIMA methodology provides a great reduction in the number of parameters that have to be estimated compared to the VARMA or the state-space model classes and thus facilitates the performance of applications of large spatial scale (large number of measurement locations).

2.3. MODEL IDENTIFICATION

Model identification is the first of the three stages of the iterative procedure commonly attributed to Box et al. (1994). The model form of the STARIMA class is tentatively chosen after an examination of the space-time autocorrelation and space-time partial autocorrelation functions that can be viewed as the 2-dimensional analogues of the usual autocorrelations and partials used to identify univariate ARMA models The sample space-time autocorrelation at spatial lag l and temporal lag s is calculated via

$$\rho_l(s) = \frac{T}{T-S}\frac{\sum_{t=1}^{T-s}\left[W_l Z_t\right]' Z_{t+s}}{\left(\sum_t \left[W_l Z_t\right]'\left[W_l Z_t\right]\sum_t Z_t' Z_t\right)^{1/2}} \tag{4}$$

For the space-time analogue of the Yule-Walker equations the space-time covariance function is needed

$$\gamma_{lk}(s) = E\left\{\frac{\left[W^{(l)} Z_t\right]'\left[W^{(k)} Z_{t+s}\right]}{N}\right\} \tag{5a}$$

which can be seen to be equivalent to

$$\gamma_{lk}(s) = tr\left\{\frac{W^{(k)\prime} W^{(l)} \Gamma(s)}{N}\right\} \tag{5b}$$

where $\Gamma(s) = E\left[Z_t Z_{t+s}'\right]$ and $tr[A]$ is the trace of A defined on square matrices as the sum of the diagonal elements. $\Gamma(s)$ is estimated by

$$\hat{\Gamma}(s) = \sum_{t=1}^{T-s} \frac{Z_t Z_{t+s}'}{T-s}. \tag{5c}$$

Premultiplying both sides of the general STAR model

$$Z_t = \sum_{j=1}^{k}\sum_{l=0}^{\lambda} \phi_{jl} W^{(l)} Z_{t-j} + a_t \tag{6}$$

by $\left[W^{(h)} Z_{t-s}\right]'$ gives

$$Z_{t-s}' W^{(h)\prime} Z_t = \sum_{j=1}^{k}\sum_{l=0}^{\lambda} \phi_{jl} Z_{t-s}' W^{(h)\prime} W^{(l)} Z_{t-j} + Z_{t-s}' W^{(h)\prime} a_t \tag{7}$$

Taking expected values and dividing both sides by N yields

$$\gamma_{h0}(s) = \sum_{j=1}^{k}\sum_{l=0}^{\lambda} \phi_{jl} \gamma_{hl}(s-j) \tag{8}$$

since $E\left[Z_{t-s}' a_t\right] = 0$ for s>0. This system is the space-time analogue of the Yule-Walker equations for univariate time series. The set of last coefficients ϕ_{kl}' obtained from solving the system of equations as l=0,1,..,λ for k=1,2,… forms the space-time partial correlation function of spatial order λ. Analogously to univariate time series STARMA processes are characterized by a distinct space-time partial and autocorrelation function. Purely autoregressive STAR$\left(p_\lambda\right)$ processes exhibit space-time autocorrelations that tail off both in space and time and partial autocorrelations that cut off after p lags in time and λ lags in space whereas STMA(q_m) processes exhibit autocorrelations that cut off after q lags and partials that decay over time and space. Mixed models exhibit partials and autocorrelations that tail off with both time and space. For a thorough discussion on these matters the reader should consult Pfeifer and Deutsch (1980a, 1980b).

2.4. ESTIMATION AND DIAGNOSTIC CHECKING

STARIMA (p,d,q) models with $q \neq 0$ are non-linear in form so parameter estimation is performed using any of a variety of non-linear optimisation techniques. As discussed in Pfeifer and Deutsch (1980a), gradient methods have found use, as has linearization, an iterative technique that at each stage "linearizes" the non-linear model using Taylor's expansion and solves approximate normal equations for the next guess at the optimum parameters. Normally, one has to minimize the expression

$$a_t = Z_t - \sum_{k=1}^{p}\sum_{l=0}^{\lambda_k}\phi_{kl}W^{(l)}Z_{t-k} + \sum_{k=1}^{q}\sum_{l=0}^{m_k}\theta_{kl}W^{(l)}a_{t-k} \tag{9}$$

where the first few alphas are functions of observations and errors at times before the initial epoch observed; this difficulty is sidestepped by substituting zero, the unconditional mean for all values of Z_t and α_t with $t<1$.

The first phase of diagnostic checking is the examination of the residuals from the fitted model; these should be distributed normally with zero mean, have a spherical variance–covariance matrix and autocovariances at nonzero lags equal to zero. Usually the sample space-time autocorrelations and partials of the residuals are computed and compared to their theoretically derived variance. If the residuals are approximately white noise, the sample space-time autocorrelation functions should all be perfectly zero; otherwise they may follow a pattern that can be represented by a STARMA model, which may be coupled with the one initially proposed and lead to a better updated model.

The second phase of the diagnostic checking involves checking the statistical significance of the estimated parameters based on the approximate confidence intervals proposed by Pfeifer and Deutsch (1980a). The insignificant parameters should be removed and the resulting simpler models should be again estimated and passed through the diagnostic checking stage until all parameters are statistically significant and the residuals meet the required constraints.

3. THE APPLICATION

3.1. THE STUDY AREA

The urban area of Athens, the capital of Greece, has an area of 60 km^2 and a population of approximately four million people. Total daily demand for travel is about 5.5 million trips with about 1 million occurring during the 2-hour peak period (Stathopoulos and Karlaftis, 2002). In the last ten years traffic flows have been increasing by about 3.5% annually. Travel times in such a congested network can be very long and the potential for travel time savings though Intelligent Transportation Systems Technologies are high.

A set of 88 loop detectors (Figure 2) has been installed by the Ministry of Environment and Public Works at major roads of the Athens network to measure traffic volume and road occupancy. The measurements take place every 90 seconds and are immediately transmitted to the Urban Traffic Control Center where they are used by the Siemens MIGRA traffic control system to adjust street lights timing, stored in databases for further analysis and displayed on a web site (http://test.AthensTraffic.gr) that shows real time traffic conditions in Athens (Kotzinos, 2002). An indicator of data quality ranging from 1 to 3 is transmitted as well since often electronic or system failures result in measurements that might not be accurate.

3.2. THE DATA ANALYSED

In this study, it's the dataset provided by 34 loop detectors located on major arterials leading to the center of the city that is being modelled. In 9 of them the measurements were of questionable quality during the time-period under investigation so the information they provided was discarded. The 25 loop detectors that remain are highlighted in Figure 2 and more formally presented in Table 1.

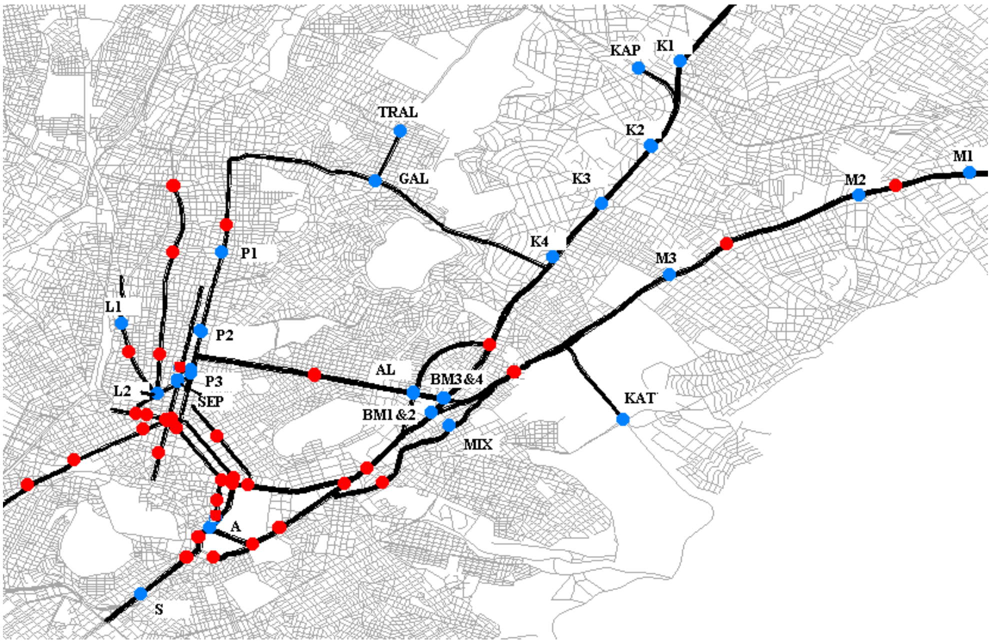

FIGURE 2. Loop detectors at the Athens road network. The ones used in this study are highlighted with different color and a label.

LABEL	ROAD	INTERSECT. STR.
M1	Mesogion	ERT
M2	Mesogion	Ipirou
M3	Mesogion	Paritsi
K1	Kifisias	Karella
K2	Kifisias	28 Oktovriou
K3	Kifisias	Ethn. Antistas.
K4	Kifisias	25 Martiou
KAP	Kapodistriou	El. Venizelou
TRAL	Veikou	Tralleon
GAL	Galatsiou	Veikou
P1	Patision	Kiprou
P2	Patision	Derigni
P3	Patision	Ipirou
SEP	Tr. Septemvriou	Marni
L1	Liosion	Sepolion
L2	Liosion	Pl. Vathis
AL	Alexandras	Panormou
BM1	Vas. Sofias	Mesogion
BM2	Vas. Sofias	Mesogion
BM3	Vas. Sofias	Alexandras
BM4	Vas. Sofias	Alexandras
KAT	Katehaki	Alimou
MIX	Mihalakopoulou	Sinopis
A	Amalias	Vas. Sofias
S	Sigrou	Frantzi

TABLE 1. Location of the loop detectors under study

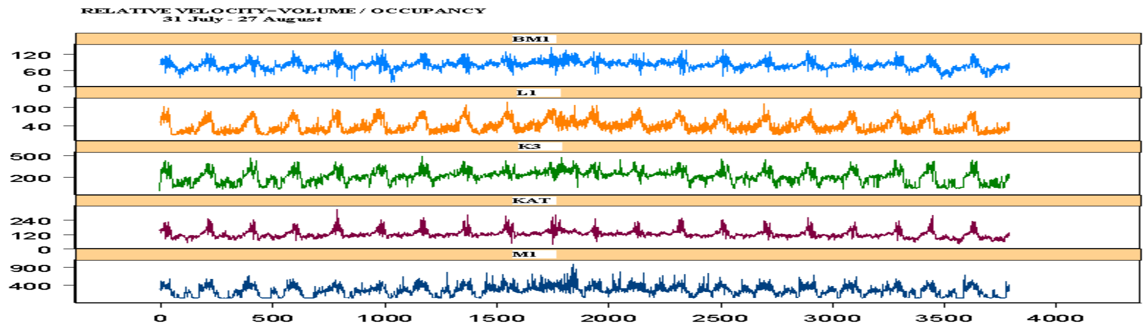

FIGURE 3. A subset from the dataset for the period 31 July-27 August.

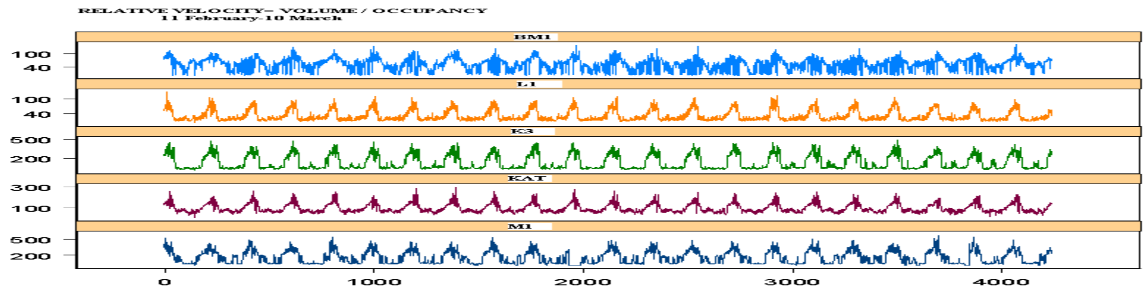

FIGURE 4. A subset from the dataset for the period 11 February-10 March.

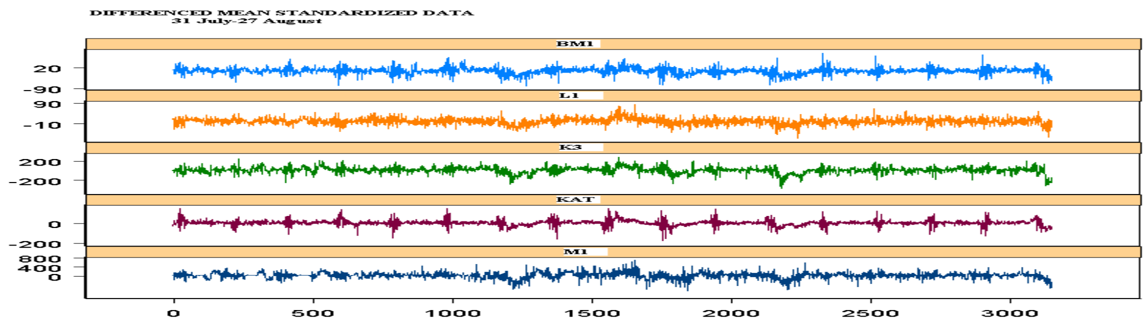

FIGURE 5. A subset from the differenced, mean-corrected data for the period 31 July-27 August.

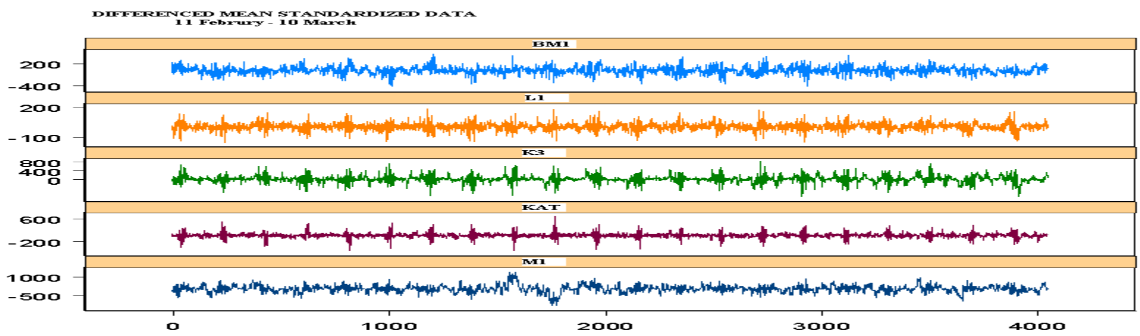

FIGURE 6. A subset from the differenced, mean-corrected data for the period 11 February-10 March.

The variable under study was the relative velocity, which was defined as the traffic volume divided by the road occupancy. This is a variable more volatile than the other two, but it reflects in a clear way the traffic condition. As indicated in Rice & van Zwet (2002), multiplied by a constant related to the average vehicle length it can provide a proxy for the exact speed. Averages over 5 consecutive time intervals were taken in order to ease the implementation and smooth out the noise, so each loop detector provided 192 measurements per day. Data measurements for weekends were discarded since traffic conditions during these two days differ significantly from the other weekdays. In order to check for the stability of the estimated model parameters, separate models were fitted for two time periods. The first one was from the 31st of July 2001 to the 27th of August 2001 and contained 3.727 observations (almost 20 days); the second one was from the 11th of February 2002 to the 10th of March 2002 and contained 4242 observations corresponding to 22 weekdays. August is a month of atypical traffic flow characteristics since most of the Athens' population takes their vacation at that time whereas the second time period is considered to be a typical one. As expected and is clearly depicted at Figures 3 and 4 that follow, the observed relative velocities for the August dataset are significantly higher. The variances of the spatial time series analysed for the two separate time periods (Table 3), also indicate a result pointed out quite often in the literature of traffic flow; the heavier the traffic the more volatile are the velocities (the variance of the relative velocities is clearly larger for all loop detectors the second period of our study). From simple observations of Figures 3 and 4 it is evident that there is a clear sinusoidal pattern with a daily period that should be accounted for in our models. The daily periodicity was removed by differencing and mean standardization took place so that the models presented in the previous sections can be applied (Figures 5 and 6). The time-sequence plots of the differenced, mean-corrected data, force us to check whether the spatial time series under investigation exhibit time dependent variation. For that purpose, the augmented Dickey Fuller test was performed to each differenced, mean-corrected spatial time-series and indicated no deviation from stationarity. The specifications related to the performance of the Dickey-Fuller test in the commercial statistical packages used for this purpose, were judged to be too restrictive for our purposes though, so another test, proposed by Bos-Fetherston (1992) was called for to ensure stationarity; fortunately, the constant variance hypothesis was not rejected (Figure 7). Table 3 shows the sample means, variances and measures of the relative skewness for the differenced mean standardized data. The skewness measures (none of which were statistically significant at the 0.01 level) and a visual inspection of the corresponding histograms were used to confirm the reasonableness of the normality assumption.

Before proceeding to the STARIMA model fitting, separate ARIMA models were fit to the 50 time series of the two datasets of the application. The patterns of the fitted models were quite similar for the series within the datasets that correspond to each of the two time periods examined; all the three stages of the ARIMA model fitting procedure indicated models that contain one autoregressive term (AR1) and two moving average ones, one at lag one (MA1) and one at lag 192 (MA192). The autoregressive term corresponds to the previous observation to the one being modeled, the first moving average term to the error from the previous prediction and the second moving average term to the prediction error one day before. The spatial time series from the first dataset that corresponds to August 2001 exhibit fairly stable behaviour concerning the parameters AR1, and MA192 (table 4). The AR1 term was proved to be of the greatest statistical significance with t-statistics greater than 100. The AR1 and MA192 terms proved to be stable for the second dataset also. The MA192 terms were of larger statistical significance in this dataset though (table 4). The standard errors tend to increase as the volatility of the time series increase; the proportion of variance explained from the models for the second dataset is not always smaller than the one explained from the models for the first though.

SITE	MEAN (AUGUST)	VARIANCE (AUGUST)	SKEWNESS (AUGUST)	MEAN (FEB-MAR)	VARIANCE (FEB-MAR)	SKEWNESS (FEB-MAR)
M1	-7.299171e-015	1.910245e+004	4.193507e-001	2.859715e-015	8.283703e+004	6.965089e-004
M2	1.716657e-015	1.261564e+003	2.419429e-001	5.087837e-016	1.367126e+004	7.938480e-002
M3	4.454296e-014	4.754314e+003	5.047244e-001	-1.235116e-015	2.834469e+004	3.501394e-002
KAT	9.286167e-015	7.986365e+002	1.684261e-001	3.684296e-016	6.592629e+003	1.491311e-002
K1	-4.031440e-014	8.874021e+003	4.570294e-001	-1.198273e-015	4.625433e+004	1.056192e-002
K2	1.245365e-014	8.171604e+003	4.382056e-001	3.059720e-015	3.915146e+004	3.471764e-002
K3	-4.149714e-014	4.702235e+003	6.957575e-001	-2.006187e-015	2.421069e+004	6.542651e-002
K4	-1.617983e-014	2.859344e+003	4.204714e-001	-1.256169e-015	1.409399e+004	8.135375e-002
KAP	-7.686658e-015	1.559097e+003	2.831333e-001	6.175581e-016	1.113532e+004	5.877742e-002
TRAL	6.042091e-015	1.837857e+003	3.887191e-001	7.228237e-016	2.581437e+004	8.899183e-003
GAL	5.303163e-015	2.442650e+003	2.028006e+000	-3.052702e-016	7.847806e+003	4.795165e-003
L2	-3.204651e-016	1.168043e+002	2.486567e-002	8.631778e-016	1.504720e+003	1.312467e-002
SEP	-2.297887e-016	4.170068e+002	1.417909e-001	-1.868464e-015	6.547218e+003	5.023091e-002
P1	-1.659210e-015	4.507253e+002	9.567459e-002	-8.298437e-016	5.394936e+003	3.062323e-002
P2	-2.127798e-015	6.417203e+002	4.058264e-002	-9.123018e-017	4.368673e+003	4.674180e-002
P3	1.397037e-015	1.949005e+002	3.043126e-002	1.228099e-016	1.842022e+003	3.901963e-002
S	-3.304902e-015	9.603511e+002	1.129470e-001	1.550913e-015	7.045201e+003	1.598506e-001
A	-4.573246e-015	1.665012e+003	5.719351e-002	-2.303562e-015	1.998369e+004	5.164928e-002
BM1	-3.288006e-015	2.239783e+002	1.115568e-001	-8.574760e-017	2.864741e+003	6.020362e-003
BM2	8.958380e-015	4.345446e+002	1.160451e-001	3.008841e-016	4.073353e+003	1.137899e-002

BM3	2.532181e-015	2.000895e+002	6.066701e-001	-3.223759e-016	9.563090e+002	1.190688e-001
BM4	1.242887e-014	6.284769e+002	1.481245e-001	1.614072e-016	5.065504e+003	4.021932e-002
MIX	-1.194451e-014	3.056875e+002	1.774820e-001	-5.614165e-017	9.923221e+003	3.288007e-002
L1	-4.618302e-016	2.463585e+002	7.130476e-002	-7.894919e-017	1.274060e+003	1.912110e-002
AL	-5.226567e-016	3.025340e+002	6.209760e-002	-5.438722e-017	2.985394e+003	5.475049e-002

TABLE 2. Sample moments of the seasonally differenced, mean corrected data.

	AUGUST	2001					FEBR.	MARCH	2002
LOOP DETECT.	**AR1 (T-STAT)**	**MA1 (T-STAT)**	**MA192 (T-STAT)**	**STAND. ERROR**		**AR1 (T-STAT)**	**MA1 (T-STAT)**	**MA192 (T-STAT)**	**STAND. ERROR**
M1	0.97 (162.4)	0.81 (57.75)	0.44 (26.25)	108.49		0.894 (100.54)	0.35 (18.86)	0.847 (99.56)	117.69
M2	0.977 (178.52)	0.87 (69.5)	0.49 (30.83)	28.89		0.818 (44)	0.55 (20.4)	0.826 (91.7)	71.56
M3	0.971 (176.47)	0.76 (51.46)	0.47 (28.67)	48		0.8347 (57.46)	0.474 (20.4)	0.84 (97.6)	93.7
KAT	0.967 (142.02)	0.84 (57.99)	0.43 (25.88)	23.5		0.46 (7.63)	0.257 (3.9)	0.849 (100)	51.37
K1	0.91 (78.42)	0.67 (31.52)	0.43 (24.94)	74.5		0.885 (77.11)	0.57 (28)	0.87 (109.22)	114.67
K2	0.95 (144.88)	0.6 (35.3)	0.47 (29)	54.87		0.88 (77.32)	0.546 (26.93)	0.86 (105.14)	104.4
K3	0.957 (138.76)	0.69 (41.66)	0.469 (28.52)	49.16		0.879 (68.63)	0.6 (28.15)	0.86 (104.4)	88.8
K4	0.97 (175.48)	0.73 (48.75)	0.44 (27)	36.83		0.859 49.65	0.653 (25.6)	0.852 (101.5)	71.76
KAP	0.981 (211.8)	0.84 (68.56)	0.45 (27.38)	30.56		0.925 (93)	0.73 (40.8)	0.858 (104.56)	61
TRA	0.98 (197.7)	0.858 (70.3)	0.44 (26)	34.73		0.876 (79.72)	0.465 (23.14)	0.84 (97.15)	81.23
GAL	0.8 (44.65)	0.4 (14.56)	0.32 (18.2)	40		0.782 (34.4)	0.529 (17.1)	0.87 (108.78)	52.27
L2	0.968 (117.53)	0.89 (61)	0.45 (27.36)	9.56		0.896 (75.39)	0.652 (32.11)	0.84 (97)	22.74
SEP	0.925 (83.2)	0.71 (35.2)	0.49 (30)	16.4		0.915 (112.4)	0.477 (26.9)	0.855 (102.45)	34.3
P1	0.974 (151.9)	0.884 (67.73)	0.453 (27.17)	18.4		0.83 (61.5)	0.386 (17.34)	0.857 (103.76)	37
P2	0.9 (39.86)	0.8 (26.19)	0.46 (28.5)	22.4		0.89 (48)	0.77 (29.56)	0.85 (100.73)	41.2
P3	0.9 (68.87)	0.7 (31.39)	0.48 (30)	11.55		0.869 (62.37)	0.605 (26.93)	0.816 (88.47)	25.58
S	0.82 (23.88)	0.69 (16)	0.447 (26.7)	28		0.933 (108.9)	0.7 (42)	0.832 (93.88)	46.61
A	0.93 (90.3)	0.72 (37.3)	0.5 (30.7)	32.8		0.76 (40.88)	0.36 (13.5)	0.867 (107.62)	77
BM1	0.9 (75.85)	0.65 (30.23)	0.48 (30)	11.96		0.8 (48.8)	0.414 (16.68)	0.85 (102)	29.17
BM2	0.979 (174.8)	0.89 (74)	0.47 (28.75)	17.8		0.82 (29.56)	0.68 (19)	0.862 (106.3)	39.5
BM3	0.967 (173.19)	0.677 (43)	0.462 (28.24)	8.98		0.9 (92.7)	0.54 (28.62)	0.837 (95.55)	15.9
BM4	0.885 (72.3)	0.513 (22.79)	0.462 (28.17)	18.28		0.752 (43.84)	0.271 (10.83)	0.87 (109.75)	35.92
MIX	0.98 (203.36)	0.88 (76.11)	0.476 (28.85)	14.17		0.57 (23.65)	0.05 (1.73)	0.87 (109.59)	53.44
L1	0.98 (209.5)	0.887 (78.6)	0.46 (27.9)	13.1		0.91 (61.42)	0.784 (35.35)	0.834 (94.81)	22.85
AL	0.97 (169.2)	0.825 (60.36)	0.51 (32.13	13.54		0.846 (55.61)	0.55 (23.12)	0.84 (96.66)	31.62

TABLE 4. Estimates of the ARIMA $(1,0,1)\times(0,1,1)_{192}$ models fitted to the datasets.

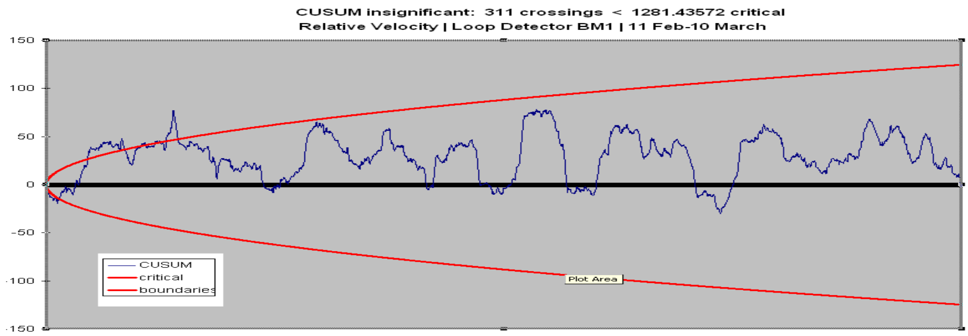

FIGURE 7. The Cusum test for zero mean, constant variance of the differenced, mean corrected series for loop BM1, second study period.

3.3. STARIMA MODEL BUILDING

Figure 2 is a map of the road network around the center of the city of Athens. For the illustrative purposes of this paper we defined a hierarchical system of neighbours (Table 4) that is comprised by two matrices where all l^{th} order neighbours of each measurement site are equally weighted. This specification is done a priori and allows the W_l matrices to be treated as exogenous constants rather than model parameters. There are considerable gains in simplicity and ease of model identification and estimation relative to multiple time series modelling that are achieved through this mechanism. The aforementioned definitions limit STARIMA model family to models with maximum spatial order of two.

Tables 5 and 6 contain the sample space-time autocorrelations and partials of the differenced mean standardized data. The sizes of the sample space-time autocorrelations at the temporal lag s are judged relative to the variance of the sample space-time autocorrelations of a pure white noise process, $[N(T-s)]^{-1}$. In the examined cases T=4050 for the first dataset and 3154 for the second one and N=25 so that the standard deviations of these space-time autocorrelations are approximately 0.003. For both datasets the space-time autocorrelations tail off and exhibit a significant increase of magnitude at spatial lag 0 and temporal lag 192, indicating the need for the presence of a moving average term of order 192 to the STARIMA models, analogously to the MA192 term of the ARIMA models. The partial autocorrelations appear to cut off at the third temporal lag at spatial lag zero and at the first temporal lag at spatial the first and second spatial lags so the candidate models for the two time periods under study are of the form

$$Z_t = \phi_{10} Z_{t-1} + \phi_{20} Z_{t-2} + \phi_{30} Z_{t-3} + \phi_{11} W_1 Z_{t-1} + \phi_{12} W_2 Z_{t-1} - \theta_{10} a_{t-192} + a_t \qquad (10)$$

Thus, each measurement taken at a specific site at time *t* is modelled as a linear combination of the three previous measurements at this site plus a weighted average of the measurements taken from its first order neighbours at time t-1 plus a weighted average of the measurements taken from its second order neighbours at time t-1 plus the prediction error that was made yesterday at the same time, plus a random error. The non-linear least squares estimates of the parameters are depicted in Table 7 The nonlinear least squares estimation of the parameters of model (10) was performed through a run of the SAS/ETS procedure PROC MODEL. The model formulation in the computer was quite similar to the one Pfeifer and Deutsch (1980a) propose for the STAR model. If spatially weighted moving average were present in (10), at least two recursive runs of PROC MODEL should have been performed.

Diagnostic checking of the model involves calculation of the space-time autocorrelation of the residuals; the results were fairly satisfactory except for the space-time autocorrelation for the 192nd temporal lag, zero spatial lag for both data sets. The variance-covariance matrix of the residuals was decidedly nonspherical; there were large differences among its diagonal elements. The hypothesis that G is of diagonal form was tested by using the results of Anderson (1958) and Pfeifer and Deutsch (1980c) and could not be rejected, so the models (10) were re-identified, following the procedure proposed in Pfeifer and Deutsch (1981c). This time the autocorrelations appear to cut-off at zero spatial lag first temporal lag (the large autocorrelation at lag 192 remains), so the models were re-formulated

$$Z_t = \phi_{10} Z_{t-1} + \phi_{20} Z_{t-2} + \phi_{30} Z_{t-3} + \phi_{11} W_1 Z_{t-1} + \phi_{12} W_2 Z_{t-1} - \theta_{20} a_{t-192} - \theta_{10} a_{t-1} + a_t \qquad (11)$$

and re-estimated (again one run of the PROC MODEL was sufficient). The updated estimations that lie at table 8 indicate that for both examined periods the parameters that correspond to decreasing temporal lags are also decreasing in statistical significance. A surprising result is that the parameters that correspond to the second order neighbours appear to be more significant than the ones that correspond to first order neighbours probably implying that the temporal intervals between observations were relatively long. Tables 9 and 10 contain the space-time autocorrelation functions and partials of the new residuals. The new residuals are of satisfactory form so the updated model can be used for forecasting and impulse control (i.e. quantification of the effect of a shock at one or more sites to their neighbours under the assumption of constant effects relative to time and scale. The total variance of each spatial time series, the standard errors of the ARIMA model fitting procedure and the root mean square errors of the STARIMA models indicate what amount of the total variability each model explains. The comparison of the average standard errors of the ARIMA models and the root mean square error of the STARIMA model shows that the aforementioned models are quite close as far as the explanation of the variation of the 25 time series that correspond to August is concerned; that happens even though the specification of the spatial weights was naïve and the total number of parameters for the STARIMA model was 7 whereas the ARIMA models used 75 different parameters in total.

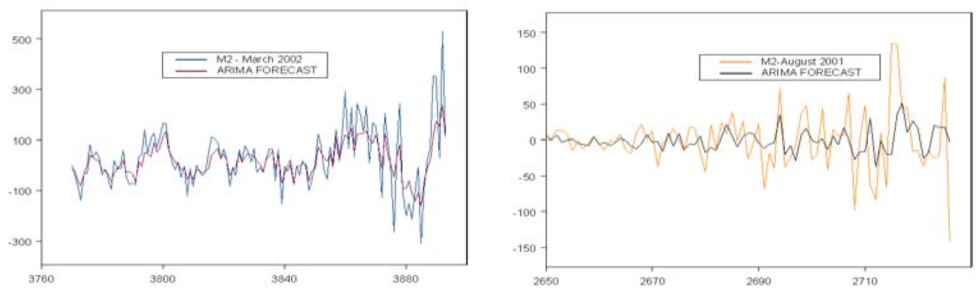

FIGURE 8. ARIMA model fit for loop M2

OP DETECTOR	FIRST ORDER NEIGHBORS	SECOND ORDER NEIGHBORS
M1	-	K1, K2, K3
M2	M1	K1, K2, K3, K4
M3	M1, M2	K1, K2, K3, K4
KAT	-	M2, M3
K1	-	M1, M2
K2	K1, KAP	M1, M2, M3
K3	K1, K2, KAP	M1, M2, M3
K4	K1, K2, K3	M1, M2, M3
KAP	-	K1
TRAL	-	KAP, K1, K2, K3
GAL	TRAL	K3, K4
L2	L1, SEP	P1, P2, P3
SEP	P1, P2, P3	L1
P1	-	GAL
P2	P1	GAL
P3	P1, P2	GAL
S	-	-
A	S	-
BM1	BM2, BM3	BM4, AL
BM2	BM1, BM3	BM4, AL
BM3	BM4	KAT, M3
BM4	BM3	KAT, M3
MIX	BM1, BM2	KAT, M3
L1	-	P1, P2
AL	BM4	K4

TABLE 4. First and second order neighbours of each measurement location.

S T	0	1	2		0	1	2
1	0.44	0.165	0.186		0.526	0.079	0.09
2	0.43	0.156	0.176		0.433	0.068	0.079
3	0.39	0.159	0.18		0.375	0.076	0.089
4	0.377	0.152	0.172		0.339	0.073	0.085
5	0.357	0.144	0.163		0.295	0.063	0.074
6	0.355	0.145	0.164		0.272	0.066	0.076
7	0.31	0.139	0.158		0.2459	0.063	0.073
8	0.309	0.137	0.155		0.239	0.05	0.067
9	0.3	0.128	0.145		0.212	0.045	0.055
10	0.26	0.129	0.146		0.206	0.035	0.042
…							
20	0.177	0.075	0.085		0.177	0.016	0.013
…	…	…	…		…	…	…
30	0.132	0.069	0.078		0.041	0.019	0.007
…	…	…	…		…	…	…
40	0.077	0.058	0.065		0.027	0.0158	0.004
…	…	…	…		…	…	…
50	0.049	0.033	0.037		0.049	-0.01	-0.011
….	…	…	…		…	…	…
192	-0.335	-0.026	-0.029		-0.727	-0.05	-0.02
…	…	…	…		…	…	…
384	-0.024	-0.027	0.042		0.258	-0.027	0.03
….	…	…	…		…	…	…
1000	0.066	0.044	0.05		0.014	0.01	0.04

Table 5. Space-time autocorrelations for the data that correspond to August (columns 2-4) and February-March (columns 6-9) .

S T	0	1	2		0	1	2
1	0.526	0.09	0.11		0.44	0.175	0.3
2	0.216	0.038	0.065		0.296	0.112	0.193
3	0.12	0.05	0.037		0.178	0.09	0.16
4	0.086	0.034	0.054		0.133	0.067	0.094
5	0.04	0.015	0.005		0.097	0.046	0.092
6	0.043	0.024	0.037		0.095	0.047	0.075
7	0.027	0.02	0.053		0.082	0.038	0.066
8	0.043	-0.003	0.048		0.084	0.039	0.044
9	0.026	-0.0014	0.037		0.075	0.027	0.047
10	0.038	-0.011	0.027		0.072	0.032	0.032

TABLE 6. Part of the space-time partial autocorrelations for the data that correspond to August (columns 2-4) and February-March (columns 6-9).

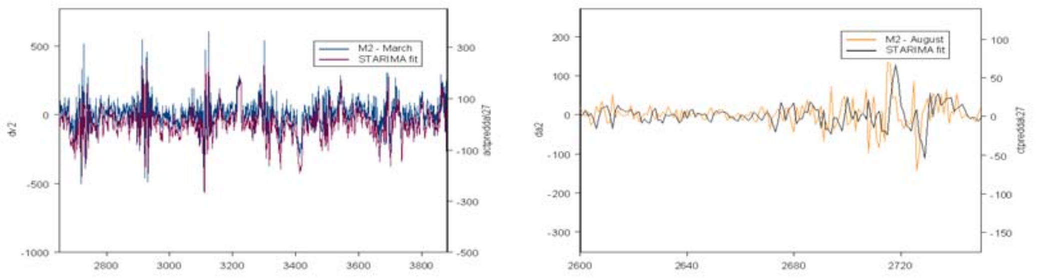

FIGURE 9. STARIMA fit for loop M2.

	ϕ_{10} (T-VALUE)	ϕ_{20} (T-VALUE)	ϕ_{30} (T- VALUE)	ϕ_{11} (T- VALUE)	ϕ_{12} (T- VALUE)	θ_{10} (T- VALUE)	**RMSE**
Model 1 (Aug.)	0.242 (68.73)	0.227 (64.36)	0.162 (46)	0.05 (15.27)	0.1 (27)	0.008 (2.13)	42.12
Model 2 (Feb.-Mar.)	0.3835 (122.8)	0.165 (49.95)	0.119 (38.06)	0.024 (7.93)	0.036 (10.43)	0.03 (9.57)	100.6

TABLE 7. Parameter estimation (before diagnostic checks) and root mean square error for the two STARIMA models.

	ϕ_{10} (T-VALUE)	ϕ_{20} (T-VALUE)	ϕ_{30} (T- VALUE)	ϕ_{11} (T- VALUE)	ϕ_{12} (T- VALUE)	θ_{20} (T- VALUE)	θ_{10} (T-VALUE)	**RMSE**
Model 1 (Aug.)	0.241 (68.75)	0.2264 (64.23)	0.1615 (45.89)	0.0572 (14.43)	0.1 (27.05)	0.0074 (2.11)	0.0133 (3.09)	42.1168
Model 2 (Feb.-Mar.)	0.3834 (122.79)	0.165 (49.92)	0.1188 (38.13)	0.0185 (5.29)	0.0354 (9.96)	0.03 (9.68)	-0.0113 (-3.13)	100.8

Table 8. Parameter estimation (after diagnostic checks) and root mean square error for the two STARIMA models.

S T	**0**	**1**	**2**		**0**	**1**	**2**
1	-0.0006	0.00078	<0.0001		0.0008	<0.0001	<0.0001
2	0.0002	-0.0009	<0.0001		<0.0001	0.0005	0<0.0001
3	<0.0001	0.0031	<0.0001		0.0001	<0.0001	0.0006
4	<0.0001	<0.0001	<0.0001		<0.0001	<0.0001	<0.0001
5	<0.0001	0.0005	<0.0001		-0.0004	0.0003	<0.0001
6	<0.0001	<0.0001	<0.0001		<0.0001	<0.0001	<0.0001

TABLE 9 Part of the space-time partial autocorrelations for the residuals that correspond to the second model fitted (August columns 2-4, February-March columns 6-9).

S T	0	1	2		0	1	2
1	-0.001	-0.0002	0.006		0.0028	0.0011	0.0007
2	0.003	0. 00016	0.0004		-0.0034	0.0 62	-0.009
3	0.009	0.0009	0.0009		0.0005	-0.0047	<0.0001
4	0.0047	0.0005	0.0002		-0.0009	<0.0001	0.0086
5	0.002	-0.0004	<0.0001		0.0025	0.00039	<0.0001
6	-0.0018	<0.0001	-0.0006		0.00062	0.007	-0.0007
7	0.0004	-0.00019	0.0058		-0.0009	<0.0001	0.0003
8	0.0007	0.00017	<0.0001		<0.0001	0.0004	<0.0001
9	-0.0003	<0.0001	-0.00145		<0.0001	<0.0001	0.00069

10	<0.0001	<0.0001	0.0001		<0.0001	<0.0001	0.00024
…	…	…	…		…	…	…
20	<0.0001	<0.0001	<0.0001		<0.0001	<0.0001	<0.0001
…	…	…	…		…	…	…
30	<0.0001	<0.0001	<0.0001		<0.0001	<0.0001	<0.0001
…	…	…	…		…	…	…
40	<0.0001	<0.0001	<0.0001		<0.0001	<0.0001	0.0061
…	…	…	…		…	…	…
50	<0.0001	<0.0001	<0.0001		<0.0001	<0.0001	<0.0001
….	…	…	…		…	…	…
192	-0.006	<0.0001	<0.0001		-0.007	<0.0001	<0.0001
…	…	…	…		…	…	…
384	-0.0004	<0.0001	<0.0001		0.0008	<0.0001	<0.0001
….	…	…	…		…	…	…
1000	<0.0001	<0.0001	<0.0001		<0.0001	<0.0001	<0.0001

TABLE 10. Space-time autocorrelations for the residuals that correspond to the second model fitted (August columns 2-4, February-March columns 6-9).

4. CONCLUSION

The STARIMA model class is a purely inductive method that can be used to statistically describe the spatiotemporal evolution of traffic flow in a road network when traffic conditions are stationary or can be made stationary by transformation. The impressive task that can be accomplished by using this strategy is that the traffic conditions of the complete network can be modelled and predicted by a single model. This is true irrespectively of the number of the traffic flow measurement locations. The definition of a hierarchical system of neighbours from the model builder, gives the opportunity for a limitation on the number of parameters which in the case of unconstrained multivariate models (VARMA, State-Space) are at least $N \times N$ where N is the number of measurement sites. As demonstrated in the example application, care in the definition of the weighting matrices results in significant predictive performance, while at the same time parsimony is not sacrificed.

The proposed strategy offers to practitioners the capability to produce traffic flow forecasts for a road network through a single model. Another significant contribution of the modeling strategy is related to the fact that it bridges the gap between traffic flow equilibrium theories and real world conjectures (as the relation between economic theories and econometrics). Traffic flow at any location of the network is related to the traffic in the nearby locations. This is equivalent in concept to Wardrop's user equilibrium concept, a fundamental law of traffic theory. Of course, its usage in assessing the effect of shocks occurring close to a measurement location to its neighbors (of first, or higher order) could be further improved. As formulated now, constant effects are assumed relative to time that is the effect of a shock at one or more sites site is the same to its neighbors irrespectively of the time it occurred. The stationarity and constant innovations' variance hypotheses are additional limitations of the model that may be unrealistic in real circumstances and should be relaxed through further research.

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