draft

The utility of travelling when destinations are heterogeneous:

How much better is the next destination as one travels further?

2002

Piet Rietveld,
Stefan van Woudenberg
Vrije Universiteit
Amsterdam
prietveld@econ.vu.nl

Abstract

In many studies travel behaviour (for example, commuting) is analysed on the basis of a utility function with the distance travelled (d) as one of the arguments. An example is

$$U=U(d,Y-cd,T-td),$$

This standard approach is not without problems, however, since it ignores the fundamental fact that most transport has a derived character: travelling kilometres is not an activity that gives utility per se, but only because these kilometres bring people to certain places they want to visit. In this paper we develop a method that provides a justification for utility functions such as shown here by showing that these can be made consistent with theories that take into account the derived character of transport.

The core element of our approach is that individuals compare potential destinations of their trips that are heterogeneous in terms of distance travelled and quality offered. Given the spatial distribution of destinations and the distribution of the quality of the jobs, one can derive the set of non-dominated alternatives that may serve as a destination. This non-dominated set is essentially a monotone relationship between distance travelled and utility of the trip. Examples of this curve are given under various assumptions concerning the spatial density of destinations and urban form.

Implications of this approach are discussed for commuting distances of different types of jobs. Our approach gives an explanation for the paradox that highly educated workers tend to have long commuting distances. Given their high value of time one would expect short commuting distances, but the low spatial density of their jobs appears to dominate the outcome.

1. Introduction

It is common wisdom that travel demand has mainly a derived character: people usually do not travel for the fun of it[1]. In stead, people travel in order to reach certain destinations where they want to carry out certain productive or consumptive activities. This common fact is, however not reflected in the way travel demand is modelled in many studies of travel behaviour. Consider for example the following formulation of a utility function that is often used as a basis for the analysis of travel demand. In such studies (see for example Golob et al., 1981, De Jong 1989, McCarthy, 2001), travel behaviour is analysed on the basis of a utility function with the distance travelled (d) as one of the arguments. An example is

$$U=U(d,Y-cd,T-td), \qquad (1)$$

where Y and T are money and time budgets, and c and t are the money and time costs per unit distance. The first argument of the utility function relates to the benefits of a trip of a certain distance d, the second concerns the benefits of consuming other goods than transport (Y-cd equals the amount of money available for this after transport expenditures have been subtracted). The third term of the utility function concerns the total time available after time for travelling has been susbtracted. The partial derivatives of U with respect to its three arguments are assumed to be positive. As indicated by Small (1992, p 12) the theoretical foundation of this formulation is not entirely clear, however. One of the problems is that it ignores the derived character of transport: travelling kms is not an activity that gives utility per se, but only because these kms bring people to certain places they want to visit. The utility function as specified in (1) can be used to derive a demand function for transport where the distance travelled per time period d is explained by factors such as the price of transport, the travel time involved and income (see for example McCarthy, 2001):

$$d = d\,(c,t,Y)$$

Given the derived nature of travel demand one would have expected a utility formulation such as:

$$U=U(v,Y-cd,T-td), \qquad (2)$$

as a basis for the analysis. In this utility formulation v represents the gross[2] utility of a visit to a certain destination at distance d.

In this paper we give a justification for utility functions such as (1) by showing that these can be derived from formulations of type (2). We also indicate some pros and cons of both formulations and discuss implications for the differences in commuting distances of workers. In addition@@@

2. Individual consumer.

Consider a consumer who can visit a number of destinations at various distances from his residence (point of origin). We assume that he makes one visit per time unit. The total number of potential destinations within a certain maximum distance D is M. The alternatives are

[1] For a counter view refer to Mokhtarian and Salomon(1999).
[2] The gross utility is the utility of the outdoor activity without taking into account the money and time outlays related to the trip.

ranked in increasing distance from the origin to the destination: $d_1,...,d_M$. The corresponding gross utilities are $v_1,...,v_M$. Consider a certain distance d^+. Let $n < M$ be the destination with the longest distance d_n being shorter than d^+. Then the maximum utility derived from a trip with distance d^+ equals

$$v(d^+) = \max \{v_1,...,v_n\}$$

When we compare two distances d^+ and $d^+ + \varepsilon$
with $\varepsilon > 0$, we find that

$$v(d^+ + \varepsilon) \geq v(d^+).$$

Thus we arrive at a monotone non-decreasing function. The case of $v(d^+ + \varepsilon) = v(d^+)$ occurs when one of the two following conditions holds:
1. There is no potential destination with a distance between d^+ and $d^+ + \varepsilon$.
2. There are one or more potential destinations with a distance between d^+ and $d^+ + \varepsilon$, but these additional destinations have a lower utility than $v(d^+)$.
The form of the function $v(d^+)$ is as presented in Figure 1.

Figure 1. Maximum utility of visiting a destination within a certain distance d^+.

The points at the upper corners of $v(d^+)$ form together the set of non-dominated alternatives. The destinations below the line in figure 1 are dominated by destinations on the line. The consumer will never choose an alternative below this line. By deleting the dominated alternatives we arrive at a one-to-one relationship between utility levels and distance travelled. The function $v=f(d)$ as presented in Figure 1 can be translated into $d=f^{-1}(v)$ where the function f^{-1} is defined in the points v for which an observation exists. Consider an individual at i considering a destination j.
Then the basic utility function already introduced above

$$U_{ij}=U(v_i, Y-cd_{ij}, T-td_{ij}), \qquad (2')$$

can be reformulated as

$$U_{ij}=U[f^{-1}(d_{ij}), Y-cd_{ij}, T-td_{ij}] \qquad (3)$$

Equation (1) is a rewritten version of equation (3) with one difference: (3) is only defined in particular points (i.e., the distances corresponding to the non-dominated points in Fig. 1), whereas for (1) such an explicit limitation has not been introduced. In the next section we will demonstrate how the gap between these two can be bridged.

3. Utility of visits for an average consumer in a uniform space of infinite size.

Figure 1 gives a possible result of the relationship between distance travelled and utility for the particular spatial setting for one specific consumer. One may wonder how the transformation between utility and distance would look like for the average consumer. We will give a derivation based on the assumption of uniform density of locations in two dimensional space and uniform density of the utility of visiting a destination. Consider a set of

M destinations in two dimensional space that are uniformly distributed. Also consumers are assumed to be distributed uniformly across space. The two distributions are assumed to be independent. Consider a randomly drawn consumer. Then the distances to the M destinations are a random sample of the following uniform distribution[3]:

$$f(d) = d/[2D^2] \qquad \text{where } 0 \le d \le D$$

The cumulative distribution F(d) is:

$$F(d) = d^2/D^2 \qquad \text{where } 0 \le d \le D \qquad (4)$$

Then, when the consumer makes a trip with distance d, the expected number of potential destinations equals $M.d^2/D^2$. Therefore, the expected number of potential destinations *increases quadratically* with distance travelled. Thus, the *elasticity of the number of potential destinations with respect to distance travelled equals 2.* The quadratic form obviously follows from the assumption of a two-dimensional space. When the consumer can only make visits in a one-dimensional space (all destinations are located along one road), the number of potential destinations would be *proportional* to the distance traveled.

These results make clear that a longer trip yields potential benefits because it leads to a larger choice set, and hence to adding an especially attractive alternative to the choice set. Assume that the utility v of a visit to a particular destination (apart from transport costs) has a uniform distribution g(v) with values between a and 1: the values of a and 1 are the lower limit and upper limit of the utility level. Thus,

$$g(v) = 1/[1-a] \qquad \text{where } a \le v \le 1$$

The corresponding distribution function G(v) equals:

$$G(v) = [v-a]/[1-a] \qquad \text{where } a \le v \le 1 \qquad (5)$$

Suppose that the consumer can choose out of n potential destinations ($n \le M$). Let the destinations be ranked in increasing order: $v_1,...,v_n$. Then it follows from the theory of order statistics (Mood and Graybill, 1963) that the distribution of the utility of the best alternative v_n, $h(v_n)$ equals:

$$h(v_n) = [n!/(n-1)!]\,[G(v_n)]^{n-1}\,g(v_n) \qquad \text{where } 0 \le v_n \le 1$$

$$= n\,[1-a]^{-n}\,[v_n - a]^{n-1} \qquad \text{where } 0 \le v_n \le 1 \qquad (6)$$

Then the expected utility value of the best alternative among these n is equal to:

$$E(v_n) = \int_0^1 n\,[1-a]^{-n}\,[v_n - a]^{n-1}\,v_n\,dv_n = [n+a]/[n+1], \quad n=1,2,3,...,M \qquad (7)$$

---

[3] For the ease of presentation we ignore the problem that consumers near the 'border of the space' will have less potential destinations since the space is empty at the other side of the border. Thus, the space is assumed to be of infinite size.

From (7) it follows that *the elasticity of the expected maximum utility of a trip with respect to the number of alternatives n from which can be chosen equals [n(1-a)]/[(n+a)(n+1)]*. This elasticity strongly decreases with the number of alternatives. For example, when a=0 and n=1, the elasticity equals ½ but for larger values of n it gets close to 0. When we confront this result with the constant elasticity of value 2 of the number of potential alternatives with respect to distance, it is clear that with longer distances the relative gains of searching at even longer distances get very small[4].

For a more detailed analysis of the impact of distance travelled on utility, consider a consumer who wants to make a trip within distance d. Given equation (4) the expected number of destinations within this distance equals $M.d^2/D^2$. The probability that a particular destination out of the total set of potential destinations is within distance d equals d^2/D^2. Then the number of destinations n lying within this distance has the following binomial distribution:

$$k(M,d,n)= M!/[n!(M-n)!] \,.\, [d^2/D^2]^n[1-d^2/D^2]^{M-n} \qquad n=0,1,2,...,M \qquad (8)$$

where k(M,d,n) is the probability that a sample of M destinations leads to n alternatives within a distance d. If we only consider situations where at least one destination is found the result n=0 must be excluded, so that the probability k(M,d,n) has to be redefined as:

$$k'(M,d,n)= M!/[n!(M-n)!] \,.\, [d^2/D^2]^n[1-d^2/D^2]^{M-n} \;/\; [1-(1-d^2/D^2)^M] \qquad n=1,2,...,M \quad (8')$$

Assume that the distribution of distances f(d) and the distribution of utilities g(v) are independent. Then the expected value of a trip with distance d (denoted as E[v(d)]) is:

$$E[v(d)]= \Sigma_{n=1,..M}\, M!/[n!(M-n)!].[(n+a)/(n+1)].[d^2/D^2]^n[1-d^2/D^2]^{M-n}/[1-(1-d^2/D^2)^M] \quad (9)$$

In the special case that M goes to infinity, E[v(d)] equals 1 for all positive d. This is a plausible result: when there is a very high spatial density of potential destinations one will easily find a very good destination nearby without the need to travel long distances. In the extreme case that there is only one potential destination (M=1), E[v(d)] equals [1+a]/2. This can easily be understood: in this case we just find that the expected utility of the trip equals the expected utility of any destination, being the mean value between a and 1.

The expected value for a trip with distance d=0 equals [1+a]/2. For d=D we find that the expected utility of a trip is (M+a)/(M+1) which is close to 1 for larger values of M. This is again a credible result: when a large number of potential destinations exists, the expected utility value of the best alternative is close to the maximum possible value of 1.

Figure 2. Utility of a trip as a function of distance and the number of potential destinations; based on uniform distribution of utility (0.2 to 1) and based on uniform density in space with circular distance function.

[4] Given a search distance d and number of destinations n, an increase in the search distance with 1% leads to 2% extra alternatives. But these lead to an increase of only 2[n(1-a)]/[(n+a)(n+1)]% in utility.

In Figure 2 some results are presented for the expected utility of the trip as a function of the number of alternatives M (values are given for 1, 2, 5, 10, 50, 100) and distance d (d ranges from 0 to D=100). The value of the utility parameter a has been set equal to 0.2. As indicated above the curve's shape is influenced by two countervailing forces. First, an increase in distance d leads to a more than proportional increase in the number of alternatives (the area of a circle is a quadratic function of its radius). This would lead to *convex* curves. Second, an increase in the number of alternatives leads to higher expected utility values, but the increase fades away as the number of alternatives gets higher. The second effect would lead to a *concave* curve. From figure 2 it appears that only in the case that M=2 or slightly higher the final curve has a pure convex form[5]. In all other cases the curves are characterised by an inflection point separating a segment with increasing slopes from a segment with decreasing slopes. Pure concave shapes are never found.

4. Utility of visits for an average consumer in a uniform space, rectangular city.

The above analysis is based on the assumption that the space has infinite size so that the issue of consumers being located at the fringe of an urban area have less destinations than consumers in the centre does not arise. As a consequence of this assumption an increase in the distance travelled always leads to a more than proportional increase in the number of potential destinations. In the real world with its distinct cities, this is not realistic. Therefore we repeat our analysis for some specific urban forms. We start with the assumption of a rectangular city. This city consists of a grid of say 1000 x 1000 points. Each point represents a consumer. Thus there are one million consumers. A set of M destinations is randomly distributed among the grids according to the uniform distribution.

For the above described spatial setting it's difficult to give a theoretical derivation, therefore we adopt a simulation approach. We draw a certain consumer in a random way. His location appears to be grid i,j. This grid is labelled k_0. Then we draw M other grids that are randomly distributed among the grids. These grids are labelled $k_1,\ldots,k_M$. The distances of these grids to k_0 are $d_1,\ldots,d_M$. The distances are computed as city block distances:

$$d[(x_1,y_1),(x_2,y_2)] = |x_1-x_2|+|y_1-y_2|$$

The next step is that we rank the M distances in increasing order, which results in $d_1^*,\ldots,d_M^*$. Then we draw M utility values from the distribution g(v), which results in $v_1,\ldots,v_M$. We are now able to compute the maximum utility w of a certain trip as a function of the distance d travelled. This maximum utility is not defined for distances between 0 and d_1^*. For distances that are larger than d_1^* we compute this maximum utility as follows:

$$w(d) = \max \{v_1,\ldots,v_n\} \qquad \text{where } d_n^* \leq d \leq d_{n+1}^*$$

We repeat this procedure 10000 times in order to get results for an aggregate traveller similar to the approach followed in section 4. Now we are able to compute the average value of w(d) for all 10000 iterations in the points d = 0,1,2,…,1998 (1998 is the maximum possible distance according to the city block distance).

[5] We observed already earlier that when M=1 the expected utility of a trip is just equal to the constant (1+a)/2.

Figure 3. Expected maximum utility level of a destination, assuming there are one or more destinations, as a function of the distance and the number of potential destinations; based on uniform distribution of utility (0.2 to 1) and based on uniform density in space, in a 1000 x 1000 rectangular city.

In Figure 3 some results are presented for the expected maximum utility of a destination as a function of the number of alternatives M (values are given for M = 1, 2, 5, 10, 50, 100) and distance d (d ranges from 0 to 1998). The value of the utility parameter a has again been set equal to 0.2. We see that for distances between 0 and 100 the curve pattern is rather unstable. This is due to the fact that the curve is based only on 10000 iterations and that the probability of a distance lower than 100 between the randomly drawn consumer and a randomly drawn destination is very small. All curves have a sigmoid shape: the expected marginal utility of distance of a trip starts at a low level, as distances it gets higher, but finally it declines again. A regular pattern of inflection points appears: as there are more destinations the transition from increasing to decreasing marginal utilities of distance take place at shorter distances.

When we compare Figures 3 and 2, we note that for M=2, Figure 2 yields an inflection point, whereas Figure 3 does not. This is because of the finite size of the spatial setting. Because of this finite size, the expected number of potential destinations doesn’t increase quadratically with distance travelled, but slightly less and for very long distances the number of potential destinations hardly increases (see Figure 4). Note that in section 3 the density of potential destinations is proportional to distance (see equation 4), whereas Figure 4 yields a bell shaped pattern.

We conclude that although the assumptions on the spatial structure are different, the final result for the relationship between distance and utility of a trip is rather similar. Only when there is a very small number of potential destinations the curves are somewhat different. Thus, the shortcut applied in utility function (1) is defendable in the situation that no data are available at the individual level on the locations and qualities of the available destinations.

Figure 4. The density of distances to potential destinations, assuming a 1000 x 1000 grid.

Having established the relationship between distance travelled and average utility, it is also possible to derive some additional results on travel patterns. For example, one can easily derive the expected utility of the most preferred trip as a function of distance. For this purpose we have of course to take into account the transport costs. We assume that the cost per unit distance is such that the cost level at the maximum distance (d = 1998) equals 1. Thus we arrive at a cost of 1/1998 per unit distance.

For each iteration we determine the most preferred trip by solving the following problem in order to derive the distance j with with the highest net utility $u_j - cost_j$:

$$\max_j \{u_j - cost_j\} \qquad j=0,1,\ldots,1998$$

Thus we gather 10000 distances with associated net utilities. From these net utilities we can compute the level of expected net utility of the best destination (see Figure 5).

Figure 5. Expected net utility of the best destination as a function of distance and the number of potential destinations.

Note that the curves are declining with distance. With the given level of transport costs it appears that as the best alternative lies further away the transport costs increase faster than the 'gross' utility of the trip. Note, however, that this result depends on the level of the transport costs. When transport costs would be close to zero the resulting patterns of expected net utilities would be very similar to the ones found in Figure 3.

Another result that can be derived from these inputs is the distribution of destinations with the highest net utility according to distance. As Figure 6b shows the density is close to the origin when the number of potential destinations M is large (for example M=100). When M is small (for example M=1) the density has a very wide range (see Figure 6a). Thus, when people are searching for a scarce good or service the average distance travelled will be much higher. Note also that when M is small there will also be many persons who will not find a destination with a positive net utility: the utility of the feasible alternatives is always smaller than the costs of getting there[6].

Figure 6. The distribution of destinations with the highest net utility according to distance.

When we compare Figures 6 and 4, Figure 4 can be interpreted as the distribution of destinations with the highest net utility according to distance when transport costs were zero. We see that when transport costs are zero the density has a very wide range. This is due to the fact that in that case, people would consider every alternative and choose the one with the highest utility, even when the distance to that alternative is very large.

5. Utility of visits for an average consumer in a polycentric urban area.

The above analysis is based on the assumption that the relevant urban area has a uniform density and a rectangular space. One may wonder whether other urban forms would lead to different results. Therefore we also carry out an analysis for a rectangular polycentric urban

[6] This can be inferred from the mass of the density in d=0, reflecting the share of people that will not participate.

area.[7] We repeat the procedure as described in section 4 for a specific urban system with 4 centres (see Figure 7).

Figure 7. Example of a polycentric urban area.

When we construct the same figures as in section 4, we find that there are only small differences between Figures 3, 5 and 6 depicting the results for a monocentric system and the corresponding figures (Figures 8, 9 and 10) based on a polycentric system.

Figure 8. Expected maximum utility level of visiting a destination as a function of the distance and the number of potential destinations; based on a uniform distribution of utility (0.2 to 1) and on a polycentric urban system.

Figure 9. Expected net utility of visiting the best destination as a function of distance and the number of potential destinations, based on a polycentric urban system.

Figure 10. The distribution of destinations with the highest net utility according to distance, based on a polycentric urban system.

On the other hand, when Figure 4 is reproduced for this polycentric urban structure, one gets clear differences (see Figure 11). The difference between Figures 4 and 11 follows from the specific polycentric urban structure on which the latter is based. If one would carry out this procedure for a more fine-meshed and/or less systematic urban structure the differences would probably be less obvious. It is striking that although the spatial structure in sections 4 and 5 are rather different the fundamental relationship between distance and utility is not strongly affected.

[7] This means for the simulation that we consider the same 1000 x 1000 grid, but when we run the simulation we take care that when we draw the consumer and the set of M destinations some grids are drawn with chance zero (the empty areas). In this way one can simulate practically every urban structure.

Figure 11. The density of potential destinations, based on a polycentric urban system.

6. Conclusions

We conclude that the above approach yields a satisfactory basis for the common practice of including distance travelled in utility functions as a source of welfare. The spatial distribution of destinations and the distribution of their utilities are implicitly present in the formulations derived in sections 2 and 3. Thus, the utility function formulated in section 1 can be considered as a reduced form where these underlying distributions are taken on board. The conclusion is that in the context of estimation the parameter related to distance in equation (1) is not purely reflecting preferences, but that it also represents elements of the spatial distribution of destinations and of densities (as reflected by the parameter M). Also the quality level of destinations and variations in the quality of destinations (represented by the parameter a) play a role.

The obvious advantage of equation (1) is that it can be used without the need to specify the spatial distribution of destinations. For many applications where equation (1) is used data on the distances of relevant destinations of each consumer are not known. The disadvantage of the use of equation (1) is that it has a reduced form character so that the parameter related to distance reflects several things at the same time which cannot be disentangled without further information. An implication is that transferability of model parameters from one case to the other becomes a complex issue.

Equation (1) can be used for example for the analysis of the optimal commuting distance chosen by a job seeker who makes a choice out of a set of job openings. An interesting implication of this interpretation is that it helps one to understand why there is a general tendency that persons with higher education and higher incomes tend to commute at longer distances (Rouwendal and Rietveld, 1994, van Ommeren, 2000). Based on the notion of the value of time one would expect that these workers would opt for a commuting distance that is relatively short. The explanation of the fact that the opposite occurs in reality is that at higher levels of income and education the number of jobs offered within one's class of competence (M) is much smaller and the heterogeneity it higher. In terms of equation (9) this means that the marginal utility of longer commutes is much higher here than for most workers with lower qualifications.

In the present analysis we focussed on only one particular type of destination. When more than one transport motive is considered equation (1) should be generalised to become

$$U=U(d_1,d_2,..d_N,\ Y- c.[d_1+\ldots+d_N]\ ,\ T- t.[d_1+\ldots+d_N])$$

where d is the total distance travelled, defined as the sum of all distances d_n travelled for all motives n=1,2,...,N. The different parameters to be found for the different travel motives will represent both the priority attached to the respective activity, the spatial distribution of the destinations and the distribution of utilities across destinations and the absolute number M of destinations available.

Acknowledgement.
The authors thank Ken Small for constructive comments on an earlier draft of the paper.

References

De Jong, G.C., Some joint models of car ownership and car use, PhD. Thesis, University of Amsterdam, Amsterdam, 1989.

Golob, T.F., M.J. Beckmann, Y. Zahavi, A utility theory travel demand model incorporating travel budgets, Tra*nsportation Research B*, vol. 15B, 1981, pp. 375-389.

Mokhtarian, P., and I. Salomon, Travel for the fun of it, *Access*, no 15, 1999, pp. 20-31

Mood, A.M., and F.A. Graybill, *Introduction to the theory of statistics*, McGraw-Hill, New York, 1963.

McCarthy, P.S., *Transportation Economics,* Blackwell, Oxford, 2001.

Ommeren, J., *Commuting and relocation of jobs and residences, a search perspective*, Edward Elgar, Cheltenham, 2000.

Rouwendal, J. and P. Rietveld, Changes in commuting distances of Dutch households, *Urban Studies*, vol. 31, 1994, pp. 1545-1557.

K. A. Small, *Urban Transport Economics*, Harwood, Chur, 1992.

Figure 1. Maximum utility of visiting a destination within a certain distance d^+.

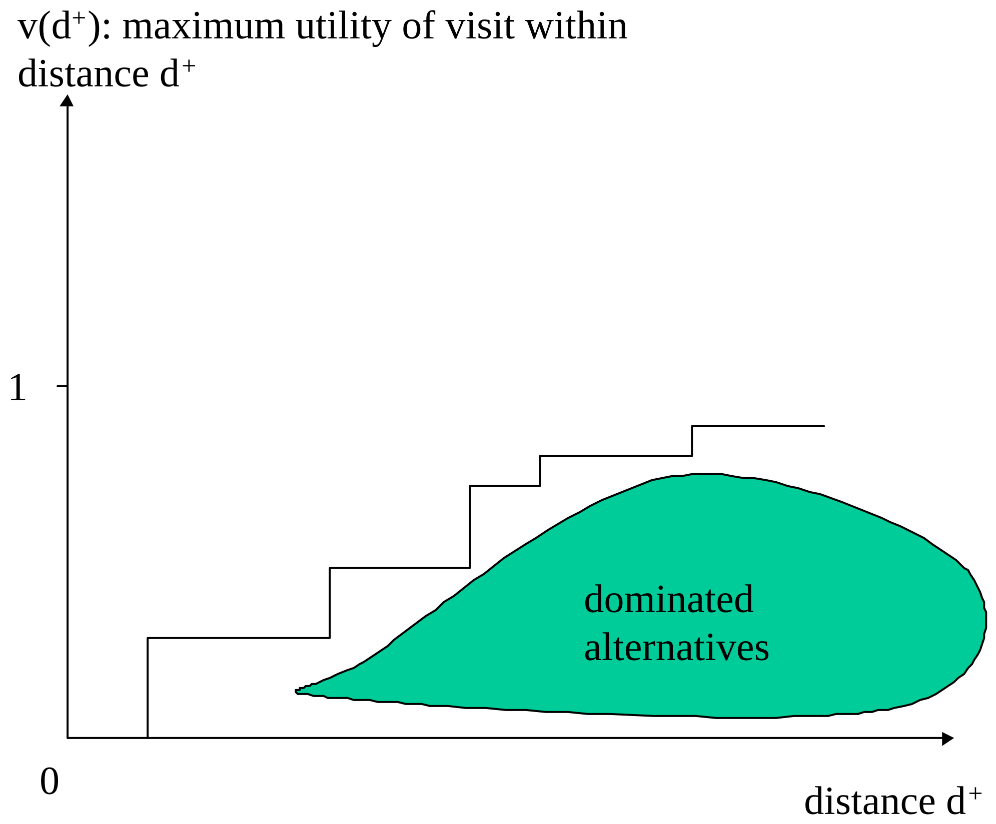


Figure 2. Utility of a trip as a function of distance and the number of potential destinations; based on uniform distribution of utility (0.2 to 1) and based on uniform density in space with circular distance function.

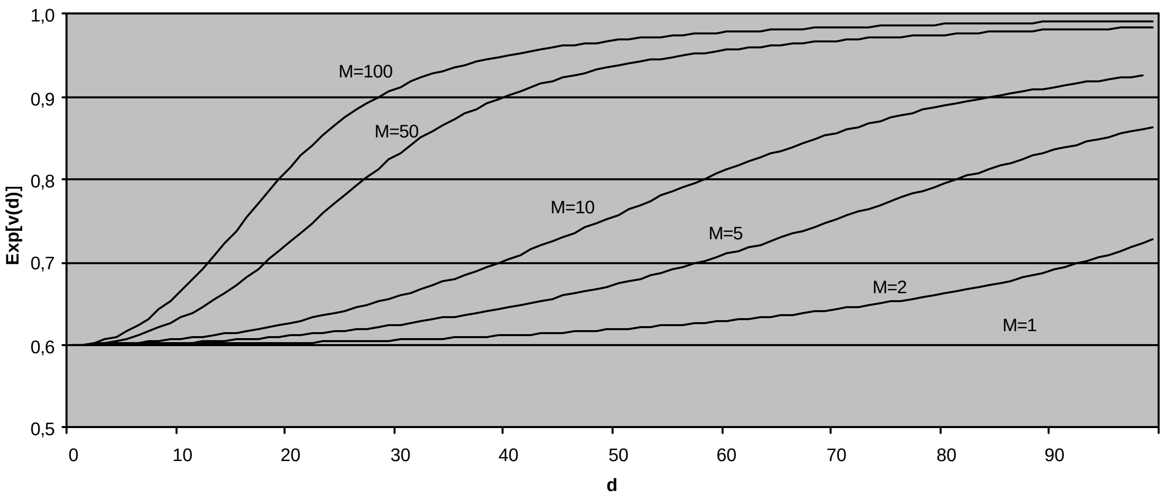


Figure 3. Expected maximum utility level of a destination, assuming there are one or more destinations, as a function of the distance and the number of potential destinations; based on uniform distribution of utility (0.2 to 1) and based on uniform density in space, which is a 1000 x 1000 rectangular city.

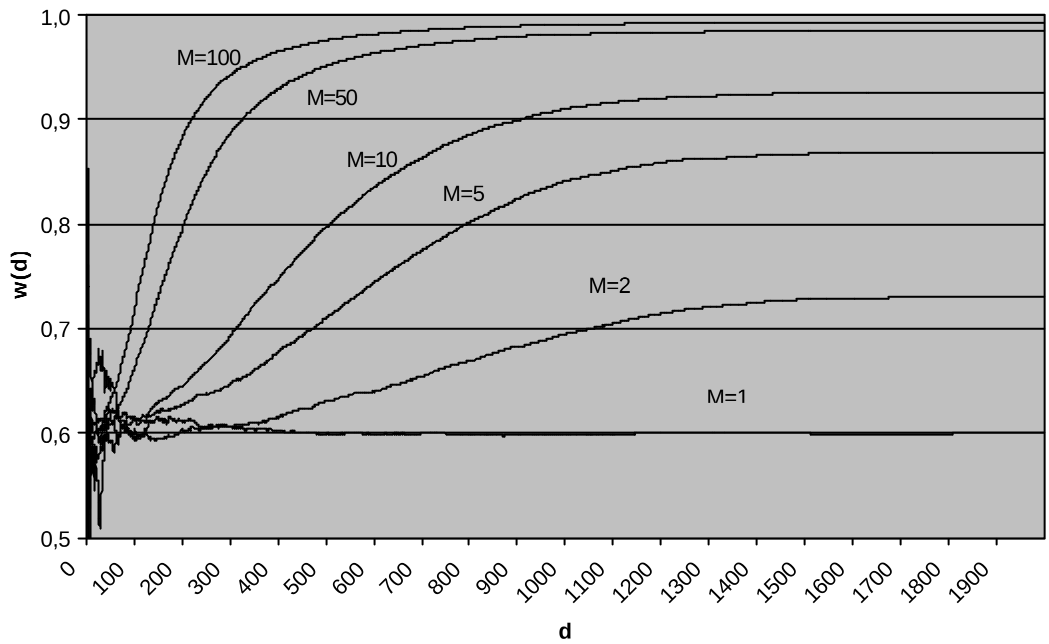


Figure 4. The density of potential destinations, assuming a 1000 x 1000 grid.

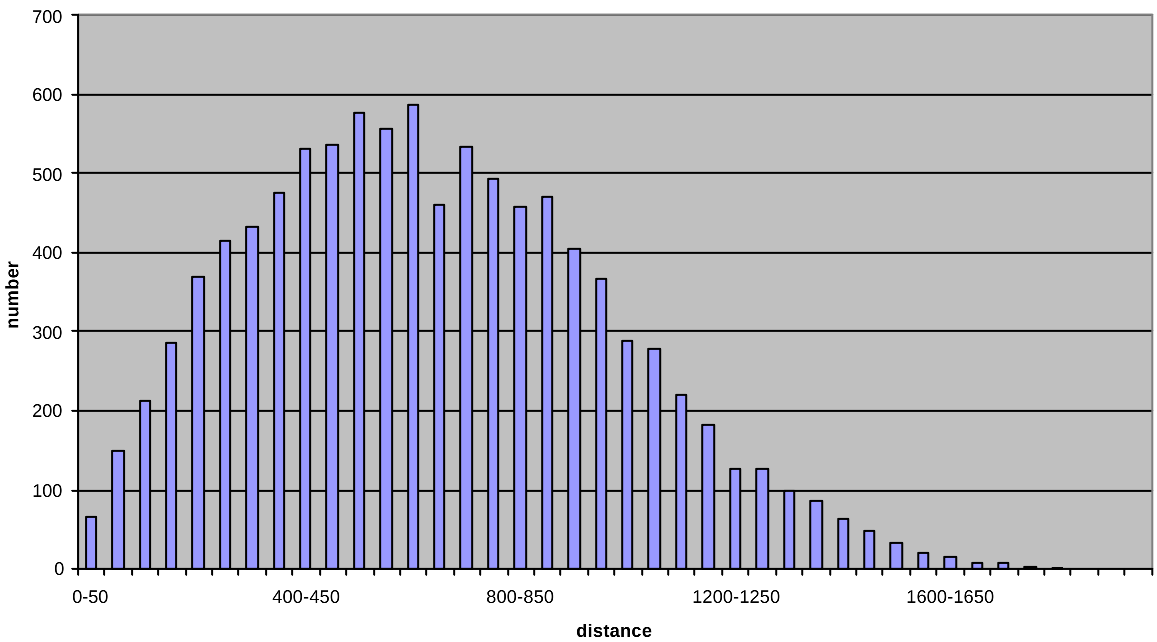


Figure 5. Expected net utility of the best destination as a function of distance and the number of potential destinations, based on uniform density in space, which is a 1000 x 1000 rectangular city.

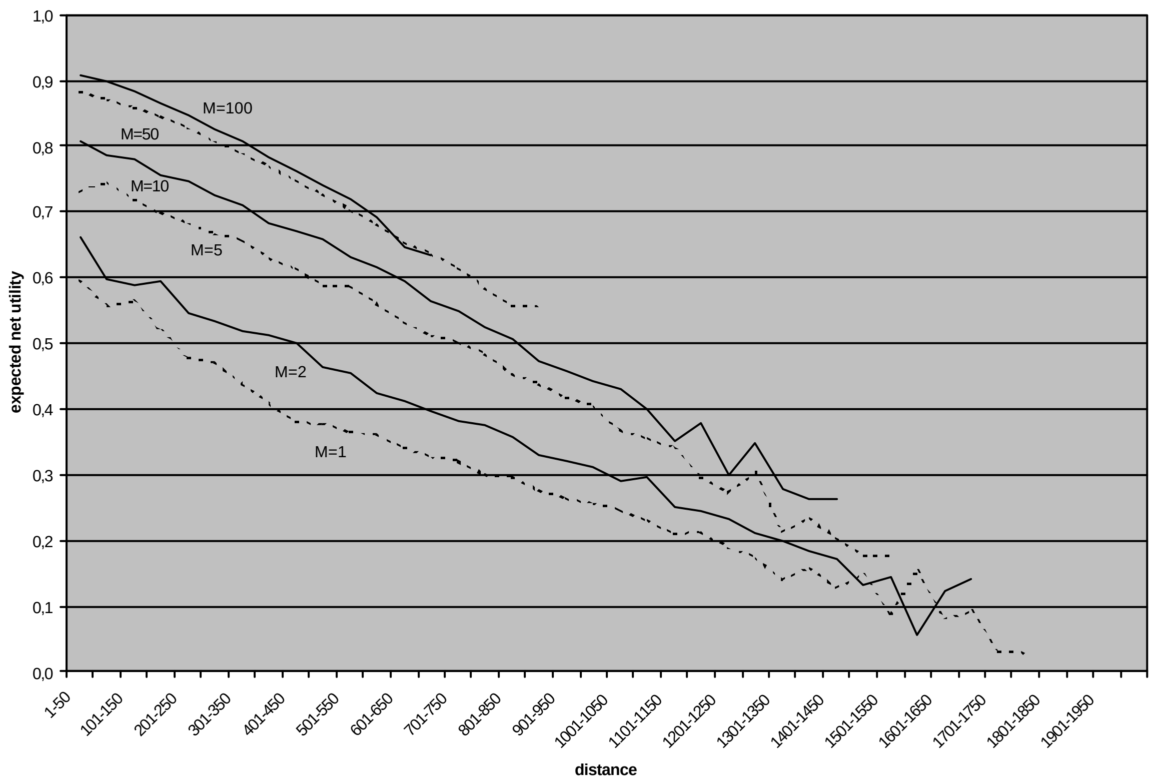
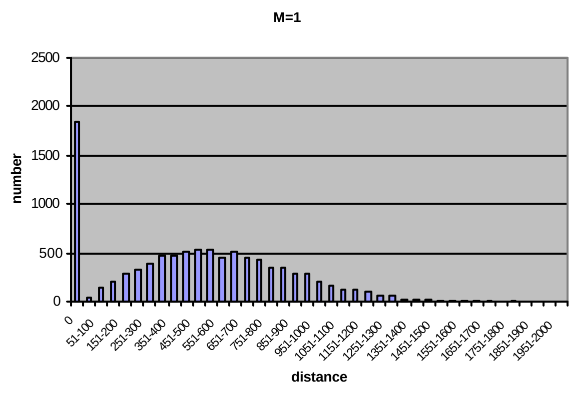


Figure 6. The distribution of destinations with the highest net utility according to distance, based on uniform density in space, which is a 1000 x 1000 rectangular city.

a.



b.

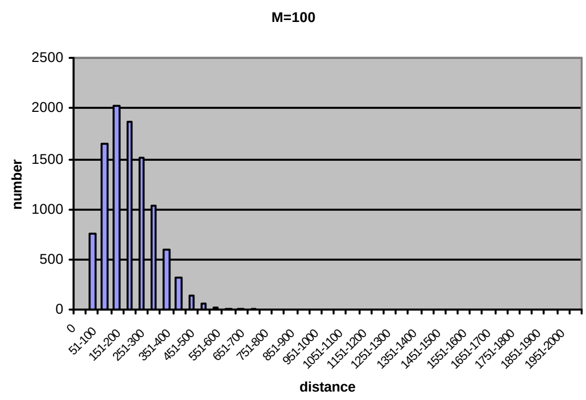


Figure 7. Example of a polycentric urban area.

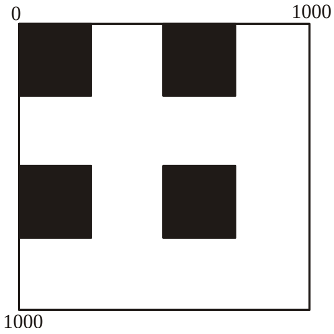


Figure 8. Expected maximum utility level of visiting a destination as a function of the distance and the number of potential destinations; based on a uniform distribution of utility (0.2 to 1) and on a polycentric urban system.

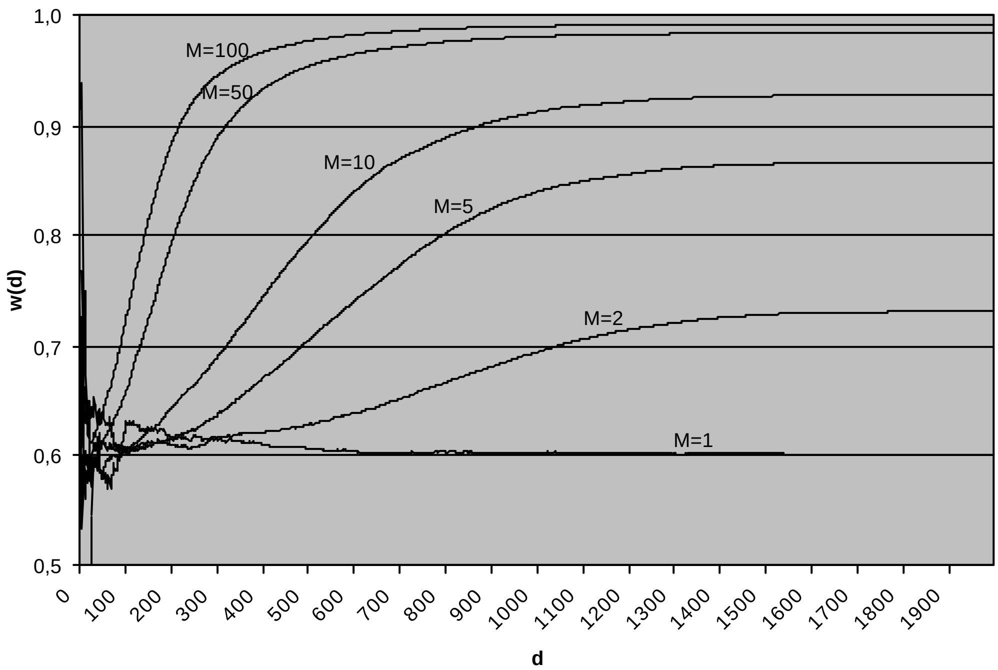


Figure 9. Expected net utility of visiting the best destination as a function of distance and the number of potential destinations, based on a polycentric urban system.

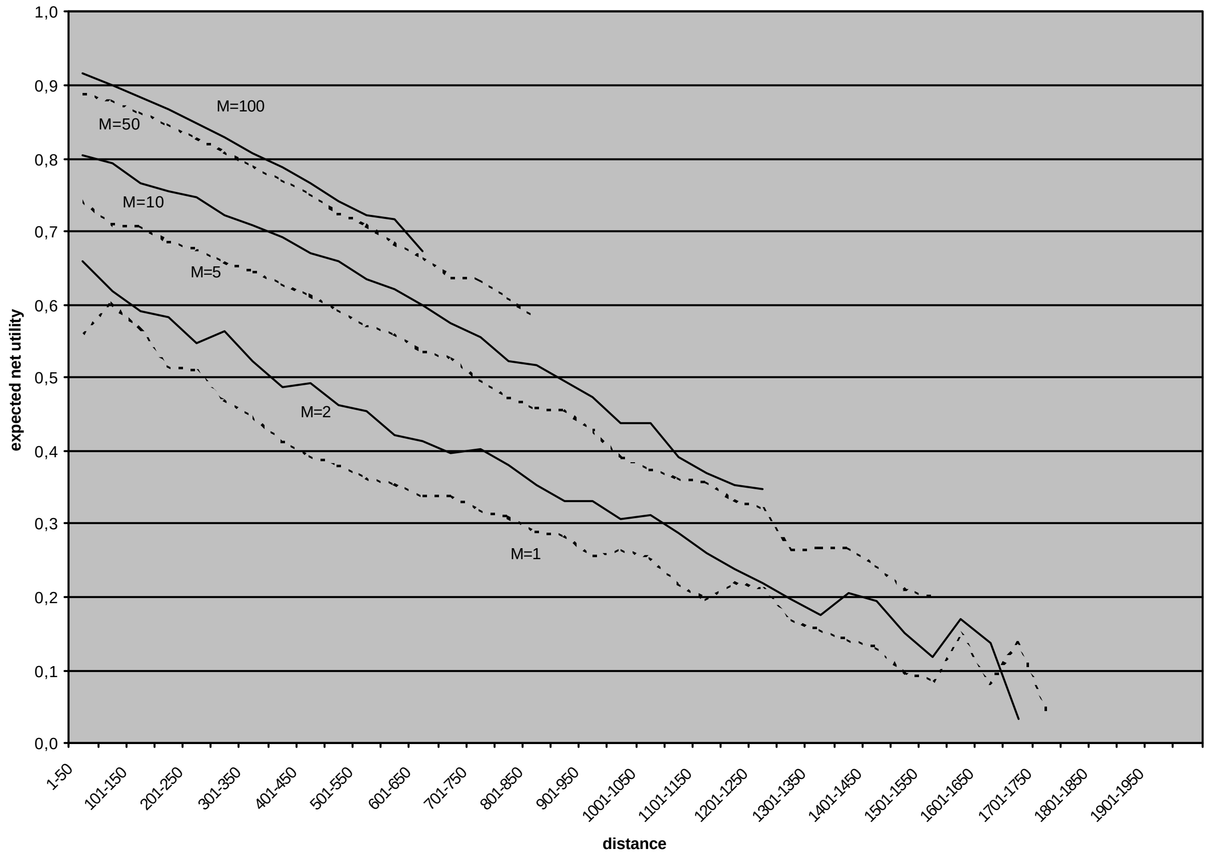
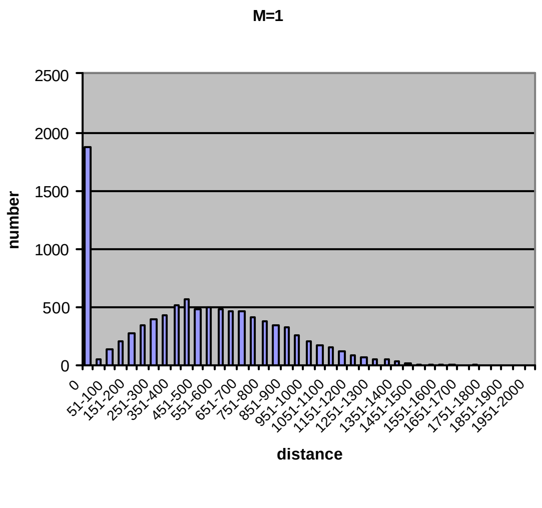


Figure 10. The distribution of destinations with the highest net utility according to distance, based on the urban system depicted on Figure 7.

a.



b.

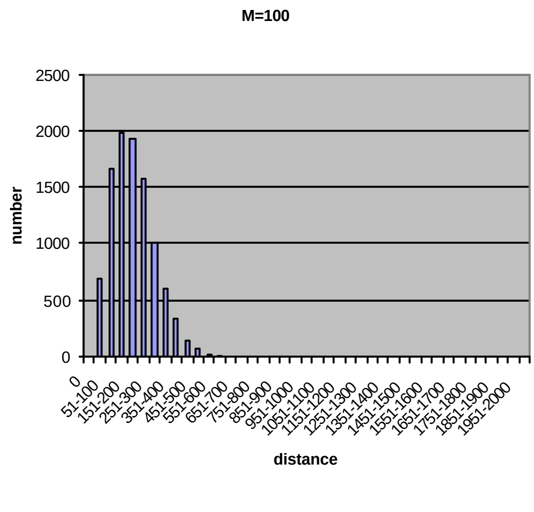


Figure 11. The density of potential destinations, based on the urban system depicted on Figure 7.