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LONG RUN URBAN CHANGE: SELECTED  
RESULTS FROM A SIMULATION FRAMEWORK

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## 1. Introduction

This contribution is to be viewed as a progress report of on-going research efforts by the authors. There are two lines of urban studies which are to be integrated in the course of the present research.

One of these lines is an effort to model the longer-run urban agglomeration and deglomeration processes for the city of Vienna. This effort is undertaken in the framework of an international co-operative project, called "Metropolitan Dynamics", which, in its initial phase was co-ordinated by IIASA in Laxenburg, Austria. (We gratefully acknowledge the financial support for a one year pilot study provided by the "Austrian Research Fund").

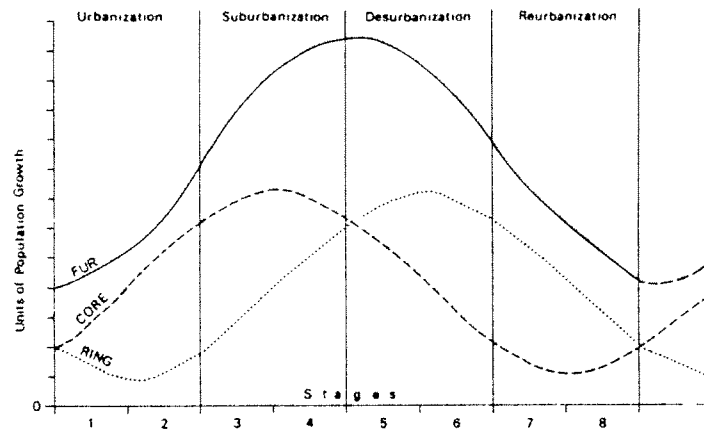
On the other hand the authors have been involved for years in a comparative project on the evolution of the European urban system (CURB). The results of this endeavor are now being published, one volume dealing with theoretical aspects of urban change (Schubert, forthcoming). The empirical results of this study have exerted a considerable influence on the way of thinking about urban change and should be briefly reviewed here.

One of the main theses of this comparative research is the existence of a cyclical pattern of the urbanisation process ("city life cycles") in which different stages can be distinguished (see v. d. Berg, et al. 1982). The simplest scheme of such stages enumerates four states of the system, the well known - urbanisation, suburbanisation, deurbanisation, and reurbanisation stage. (A more elaborate 8-way classification can be found in v. d. Berg, et al., 1982).

The indicator mostly used for these classifications is population. Many books and papers were published on the "life cycle debate", using data from the U.S., Australia, Japan, and Europe (for an overview see e.g. Nijkamp & Schubert, 1983).

Most of these studies are based on the concept of a "functional urban region" (F.U.R.), which regardless of administrative borders, distinguishes various urban zones (such as the "core", "ring", "hinterland", etc.). Urban population change expresses itself in a changing distribution of population over this F.U.R., the various constellations of which are considered characteristic of a stage of development. An idealised pattern of this process is shown in figure 1.

figure 1. stages of urban development



Another important observation made in empirical studies of urban change is the apparent existence of a strong feedback between urban and economic development, thus postulating that stages of economic development are typically linked with urban development stages.

It is further claimed (v. d. Berg, et al. 1982) that there are very strong synergistic forces at work which lead to strong **similarities** between cities in the **same stage** of development with regard to a whole host of important indicators (besides population which the classification is based on).

Among the variables are such significant magnitudes as the capacity and lay-out of infrastructure, often treated as (exogenous) instrument variables in policy oriented urban models.

On the modeling side the application of catastrophe, bifurcation and self-organising systems' theory (see e. g. Wilson, 1981; Allen & Sanglier, 1979, Sonis, 1983, Dendrinos, 1981) see a wave like pattern of urban change only as a special case of a wide variety of possible time paths, among them obviously states of the urban system such as long run stagnation and the emergence of ghost towns, etc.

Needless to say that this paper can only pick up this topic of "empirical regularity" (observed in the past!) versus the various possibilities modern urban development theory offers and claim it as a "leitmotiv" for a modest first attempt to cope with the scientific problem posed.

This paper proceeds by first outlining the theoretical background of a simple urban change simulation model in the second section. In the sequel an exact description of the model structure is presented and in the fourth section some simulation results are discussed. The contribution closes with a brief summary and conclusions.

## 2. The theoretical background

### 2.1. Model components and overall structure

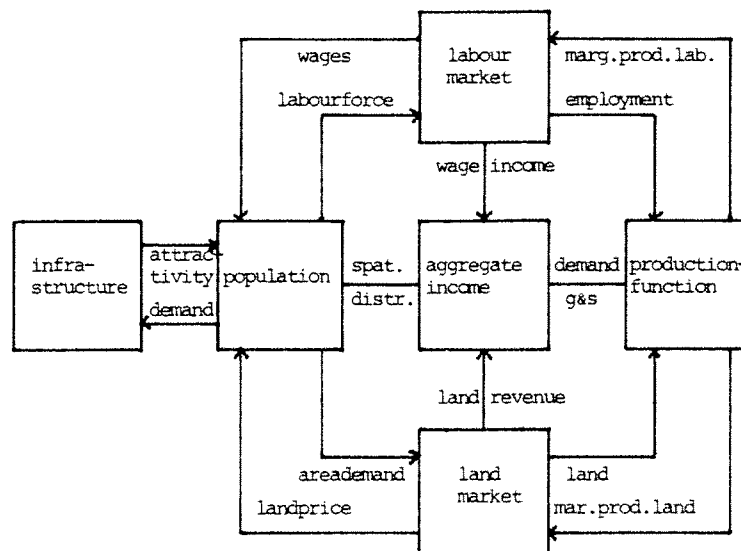
Four sets of key variables are contained in the model, around which the pertinent submodels cluster. In the light of the discussion in section 1 these variables are:

- residential population ("population model")
- jobs ("labour market and production model")
- infrastructure ("infrastructure model")
- land ("land market model")

Additional exogenous, key variables are the accessibility structure (average time distances between urban zones), which are, in a first step considered to remain constant and change only discretely.

The model is closed, hence no influences from outside are considered in the first simulation attempts made in this framework. To achieve this closure in the model two additional components become mandatory, i.e. income formation and production. The modular structure of the model is displayed in figure 2.

figure 2. the model components



The theoretical approach is based on the assumption of "representative" decision makers who plan rationally, reacting to changes in the decision variables. These "reaction functions" (supply, demand, migration, participation, commuting, etc.) are assumed to be valid for all decision makers concerned, with stochastic deviations from the representative reaction.

In the beginning of each time period the economic actors set up plans, which are based upon their experiences in the past and the prices formed in the period before. These plans are coordinated in the various markets (land, labour) resulting in a new set of prices. Since the model applies a disequilibrium framework the markets produce indicators of excess supply and demand as well (unemployed, vacancies). Finally, the representative firm determines its level of production and hence the marginal products of labour and land.

A macro approach is thus chosen, the behavioural relations of which are deduced from a micro-based theory.

Macro modeling, however, is to be based on a set of clearly defined accounting identities which need to be observed in aggregation procedures. These will be briefly mentioned when the individual components are sketched (an exact theoretical derivation is not presented here).

## 2.2. A complete specification of the simulation model

### a) The population submodel

The necessary accounting identity is the following stock-flow relation:

$$\text{Pop}_i^t = \text{Pop}_i^{t-1} + \text{Im}_i^t - \text{Em}_i^t + \text{B}_i^t - \text{D}_i^t \quad (1)$$

with    Pop .... Population  
          Im, Em . immigration and emigration respectively  
          B, D ... births and deaths respectively  
          t .... time index  
          i .... index of urban zones

In this contribution total population is assumed to be constant over time. We concentrate our attention on the redistribution of population within the F.U.R., so migration needs to be explained in the model. To do so migration of residential population from i

to  $j$  is formulated as a (variable) percentage of the residential population in the region of origin  $i$  at  $t-1$ . This percentage, the "propensity to migrate" from  $i$  to  $j$  ( $\mu_{j|i}^t$ ) is interpreted as a transition probability in a non-stationary Markov process and a nested logit model setting is used to derive the  $\mu$ .

$$M_{ij}^t = \mu_{j|i}^t \text{Pop}_i^{t-1} \quad (2)$$

with  $\mu_{j|i}^t$  .... probability that a person migrates from  $i$  to  $j$ .

$j$  .... index of urban zone

Additionally  $\sum_j M_{ij}^t = E_m^t$  and  $\sum_j M_{ij}^t = I_m^t$

The probability  $\mu$  is derived from an Alonso - v. Thünen type of decision model in which the representative individual (not the household in this simple version) has to trade-off the quantity of a consumption good, land and transportation cost, simultaneously. The location of the land to be acquired has to be decided. The "direct" utility function on which this decision is based, contains the above mentioned factors as arguments as well as attributes of the various locations (urban zones in this context) to be chosen from. Some of these attributes are "man-made", such as the capacity and quality of infrastructural facilities, some are naturally given (neglected here). This choice is constrained by the income available which has to be spent on consumption, land and transportation and an (exogenous) lump-sum for infrastructure (which will be neglected in this first round of simulations). In order to arrive at any discrete alternatives choice model two steps have to be taken; firstly, an indirect utility function has to be derived from the objective function and the constraints (Varian, 1978) and, secondly, a stochastic part has to be added to the deterministic indirect utility function.

Starting from a log-linear utility function and assuming the stochastic element to follow a Gumbel distribution the following (logit) migration propensity (residential choice in favor of  $j$  given  $i$ ) can be formulated:

$$\mu_{j|i}^t = \exp(U_{j|i}^t) / \sum_j \exp(U_{j'|i}^t) \quad (3)$$

$$U_{j|i}^t = u_1 * \ln(YC_j^t) - u_2 * \ln(P_j^{t-1}) + u_3 * \ln(INFRA_j^{t-1}) - u_4 * DIST_{ij}$$

with YC .... expected maximum per capita income (determined in commuting, participation, and land market submodels)

P .... land price

INFRA .. level of infrastructure

DIST ... distance

$u_1 - u_4$  ... positive parameters

$j'$  ... index of urban zones

Note that a multi-stage decision procedure is assumed, as income is considered given in this context, but is decided about in the "participation", "commuting", and "land market" submodels. It is derived from individual's optimal decisions in these markets, which are based upon prices and quantities of period  $t-1$ . It is further hypothesised as a first step that individuals are "myopic", i.e. that they do not explicitly take the future into account ("permanent income", etc.).

Additionally to the propensity to move, a decision about the amount of land to be purchased has to be made, leading to a demand function for the "representative" land area (see Alonso, 1964). The log-linear utility function mentioned above implies land-demand for residential purposes of the following form:

$$AHH_i^t = (Pop_i^t * YC_i^t / P_i^{t-1}) * u_2 / (u_2 + u_1) \quad (4)$$

where AHH .... total quantity of land demanded in  $i$  for residential purposes

#### (b) The labour market submodel

##### Supply:

The age structure of the residential population determines the number of people in the working age bracket. In this simple model the age structure remains exogenous (and identical for all regions) and is represented by a fraction ( $\Omega$ ) of the total residential population in each zone. When multiplying this "potential labour supply" by  $\pi$ , the (region specific) labour force partici-



pation rate, one obtains the labour force in (i), which by definition equals the number of people employed plus the number of unemployed at the place of residence.

In order to derive the labour supply at the place of work, commuting has to be introduced. In a logit approach a commuting probability  $\Gamma$  is formulated analogously to the migration decision. However, the decision of where to work is conditional of the participation decision and its consequence, i.e. the expected maximum wage net of commuting cost, is an important factor in the latter one.

$$\Gamma_{j|i}^t = \frac{\exp(a_1 * V_{j|i}^t)}{\sum_j \exp(a_1 * V_{j'|i}^t)} \quad (5)$$

$$V_{j|i}^t = (W_j^{t-1} - d_1 * DIST_{ij}) * (1 - ALR_j^{t-1}) + (ALR_j^{t-1} * UB) + \ln(LS_j^{t-1})$$

$$EW_i^t = (1/a_1) * \ln(\sum_j \exp(a_1 * V_{j'|i}^t))$$

where  $\Gamma$  .... commuting propensity from (i) to (j)  
 ALR .... unemployment rate  
 W .... wage rate  
 EW .... expected maximum income from labour market participation  
 UB .... unemployment benefits (invariant by time and zone)  
 LS .... labour supply (accounts for the size of the zone in this context)  
 a<sub>1</sub> .... positive parameter

The decision whether to participate in the labour market or not is considered a binary choice between two discrete, mutually exclusive alternatives (for a discussion of this problem see e.g. Isserman et al., 1986), The variable  $\pi$  can thus be seen as the probability to participate and a logit approach yields the following equation:

$$\pi_i^t = \alpha * 1 / (1 + \exp(-a_2 * EW_i^t)) + (1 - \alpha) * \pi_i^{t-1} \quad (6)$$

$$EY_i^t = (1/a_2) * \ln(1 + \exp(a_2 * EW_i^t))$$

where  $\pi$  .... labour force participation rate  
 $EY$  .... expected maximum labour income, over optimal participation and commuting decisions  
 $\alpha, a_2$  .... positive parameters ( $0 \leq \alpha \leq 1$ )

Furthermore:

$$LS_j^t = \sum_{j|i} r_{j|i}^t * \pi_i^t * \Omega * Pop_i^t \quad (7)$$

where  $LS$  ... labour supply at the place of work

Note that in this formulation the unemployed actively searching for a job are treated as "commuters" from (j) to (i), including the internal "commuters" (i.e.:  $i=j$ ). Moreover the formulation of the participation model allows for stickiness in the participation rate, the strength of which is determined by the parameter  $\alpha$ .

#### Demand:

The representative decision maker concerning the demand for labour is a firm that has its seat in an urban zone (the location of which it has decided about prior to the decision for labour demand). These urban firms are assumed to consider the longer term effects of their decision and thus are supposed to maximise the discounted value of returns accruing to them. Inputs and outputs are related by a production function consisting of 2 parts, a long term potential output type of relation and a short run "productivity loss" function due to frictional losses occurring because of "production detours" à la Böhm-Bawerk (1889), i.e. changes of production factors land and labour (for details see Brunner and Schubert, 1985).

$$PROD_i^t = S*(E_i^t)^\alpha *(AUN_i^t)^\beta - (p_1 * HF_i^t + p_2 * AC_i^t) \quad (8)$$

where  
 PROD .... Urban net production (NUP)  
 E .... employees  
 AUN .... land held by firms for production  
 HF .... hiring and firing by firms  
 AC .... area change for production purposes  
 P<sub>1</sub>, P<sub>2</sub> .. parameters which are positive for positive values of HF and AC, respectively, zero otherwise.

PROD ( $\geq 0$ ), HF, and AC represents flow variables, E and AUN are stocks. i.e.

$$LD_i^t = (1-m)*LD_i^{t-1} + HF_i^t \quad (9)$$

$$AUN_i^t = AUN_i^{t-1} + AC_i^t$$

where  
 m .... rate of replacement for labour demand  
 LD .... labour demand

Note that in this simple formulation capital stock is seen as a constant and there is no (exogenous) technical progress occurring (for a further version of this submodel see Brunner and Schubert, 1985).

The behavioral components of this model are constituted by HF ("hiring and firing" of labour) and AC (demand for extra land).

The dynamic approach hinted at above leads to a problem of optimal control (for details see Brunner and Schubert, 1985), the optimality conditions of which can (given certain plausible assumptions) be solved to yield the following demand equations:

$$HF_i^t = -b_1 * (W_i^{t-1} - W_i^{t-2}) + b_2 * (LD_i^{t-1} - LD_i^{t-2}) - \quad (10)$$

$$b_3 * (W_i^{t-1} - MPL_i^{t-1}) + b_4 * (YPOT_i^{t-1} - YPOT_i^{t-2})$$

where  
 MPL ... marginal product of labour (determined from the production function as  $(\alpha * PROD/E)$ )  
 YPOT .... income potential determining the effective spatial demand for goods and services in a zone

Furthermore the demand for land can be shown to be determined by:

$$AUN_i^t = AUN_i^{t-1} + AC_i^t \quad (11)$$

$$AC_i^t = -c_1 * (P_i^{t-1} - P_i^{t-2}) + c_2 * (AUN_i^{t-1} - AUN_i^{t-2}) -$$

$$c_3 * (P_i^{t-1} - MPA_i^{t-1}) + c_4 * (YPOT_i^{t-1} - YPOT_i^{t-2})$$

where MPL ... marginal product of land (determined from the production function as  $(E*PROD/AUN)$ )

Note that the changing income potential provides an "accelerator" type of element in this model.

#### Labour Market:

In this disequilibrium formulation of the urban labour market wages, employment, unemployment, and vacancies have to be determined. They result from labour supply and labour demand defined in the sections above.

Employment is determined by the following function, which is symmetric in excess supply and demand and ensures that actual employment is always less (or at most equal) than the minimum of labour supply and labour demand.

$$E_i^t = \frac{1}{2} \{ LS_i^t + LD_i^t - \text{SQRT} [ (LS_i^t - LD_i^t)^2 + c * (LS_i^t + LD_i^t)^2 ] \} \quad (12)$$

The parameter c determines the "natural" rate of unemployment, i.e. the unemployment rate when the market is in equilibrium.

Unemployment, vacancies, and unemployment rate are by definition:

$$AL_i^t = LS_i^t - E_i^t \quad (13)$$

$$OS_i^t = LD_i^t - E_i^t$$

$$ALR_i^t = AL_i^t / LS_i^t$$

Wages are assumed to be more flexible upwards than downwards, the equation for wage determination also allows for temporal stickiness ( $\tau < 1$ ).

$$W_i^t = \tau * w_1 * \exp(w_2 * (LD_i^t - LS_i^t) / E_i^t) + (1-\tau) * W_i^{t-1} \quad (14)$$

(c) The land market submodel

The demand for land equations were already briefly mentioned above. In the first step, the land market itself is assumed an equilibrating market, where demand is always fully satisfied. The land price is determined by

$$P_i^t = p_1 * \exp(p_2 * (AD_i^t - AS_i^t)) \quad (15)$$

with 
$$AD_i^t = AHH_i^t + AUN_i^t$$

AS is an exogenously given indicator for the supply of land. AUN and AHH correspond to (10) and (4), respectively.

(d) Infrastructure

Again an extremely simple model was formulated for the first attempt to use the model for simulation. A stock of infrastructural capital is assumed to exist in all zones, which is added to by investment (Replacement and the age-structure of the stock of infrastructure is neglected at the moment)

$$INFRA_i = (1-i_1) * INFRA_i^{t-1} + INVEST_i^t \quad (16)$$

$$INVEST_i = i_2 + i_3 * (Pop_i^{t-1} - Pop_i^{t-2}) \quad INVEST_i \geq 0$$

where INFRA .. stock of infrastructure capital  
 INVEST . investment in infrastructure  
 $i_1, i_2, i_3$  ... positive parameters

Demand for infrastructure is simply expressed in terms of changes of residential population. Expectations are hypothesised to be a simple extrapolation of past experience.

Transportation infrastructure is, as already mentioned, treated

as exogenous.

(e) Income formation

Income accrues to owners of labour and land in this model, wages are determined in the labour market of the zone in which supply and demand meet. A similar statement holds for the land market. In this contribution we assume that income from land does not go to "absentee land lords" and thus becomes only effective in the zone where the land is supplied.

Labour income is made in the zone of work but is assumed to become effective in the zone of residence exclusively, i.e. wages paid to workers at their location of job have to "commute" back to the zones of residence. The same holds for unemployment benefits. Thus, we obtain:

$$Y_i^t = AD_i^t * P_i^t + \sum_j (\Gamma_{j|i}^t * \Omega * \pi_i^t * Pop_i^t) * (ALR_j^t * UB + (1-ALR_j^t) * (W_j^t - d_1 * DIST_{ij}^t)) \quad (17)$$

The income potential representing the effective spatial demand for goods and services in a zone is defined as

$$YPOT_i^t = \sum_j Y_j^t * f(DIST_{ij}^t) \quad (18)$$

where  $f$  is a distance decay function. It assumes the value one at distance zero and approaches zero as distance increases.

Expected maximum per capita income as used in equation (3) results from prices and quantities of period  $t-1$  and the optimal commuting and participation decisions computed at the beginning of period  $t$ . Since we do not divide population by any structural characteristics explicitly (e.g. land owner), land revenues are ascribed to all residents of the respective urban zone.

$$YC_i^t = AD_i^{t-1} * P_i^{t-1} / Pop_i^{t-1} + EY_i^t \quad (19)$$

Summarizing it can be stated that the model outlined above yields highly non-linear, dynamic relations. These relations yield a reduced form the dynamic properties of which cannot directly be evaluated by the conventional methods, hence simulation is used

to assess the possible time path of the key indicators. The parameters for the simulation runs were partly taken from preliminary econometric tests with Austrian data used in the above mentioned pilot study and some comparative econometrics undertaken in the framework of the CURB project (see introduction).

### 3. Some simulation results

It is the aim of the first simulation runs to gain some insight into the functioning of the model outlined in section 2 and to investigate the influence exerted by the various feedback loops. We do not intend at this point to reproduce the development pattern of any existing city. Thus, we use an extremely simple, abstract urban system consisting of only four zones. The spatial structure of this "city" will be the first influence investigated. Moreover, we start with an extremely simplified version of the model. By setting the appropriate parameters equal zero we eliminate INFRA from the migration model (3), HF and AC from the production function (8), all but the third term from the "hiring and firing"-function (10) as well as the "area change"-function (11). The parameter  $m$  in (9) was set to zero,  $\alpha$  in (6) and  $\tau$  in (14) were set to one. In this version the model consists of the nodes population and production connected by income formation, labour and land market. Only price information is passed between the model components.

When we start with the most simple spatial structure, i.e. equal distances between all zones, we obtain the same stable time path for all spatial units. Since the adjustment process assumed is of a cobweb type, with a change of parameter values one can produce instability and fluctuations as well.

The distances between urban zones have a strong influence on the spatial distribution of population and jobs. By appropriately adjusting the distance matrix one zone can be defined as the core of the agglomeration. In the equilibrium this zone has more population and more employment than the other zones. This results from the better accessibility of jobs in the core making it more attractive. A dampening effect stems from the land market with higher land prices resulting from the increased demand for land. However, reducing all distances by the same factor (lower transport cost, improved transport technology, etc.) first yields a more concentrated equilibrium distribution of population and jobs, later on it results in an unstable time path.

figure 3

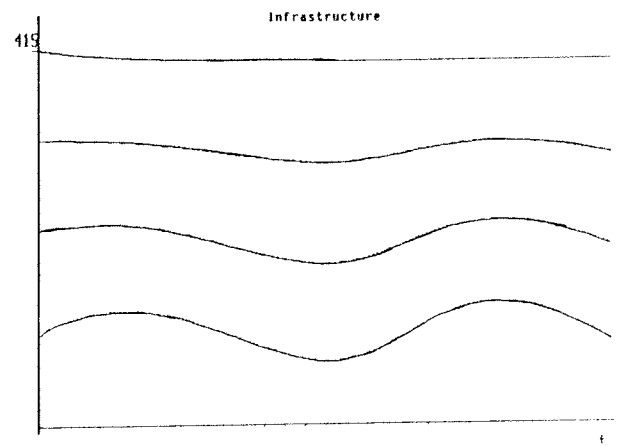
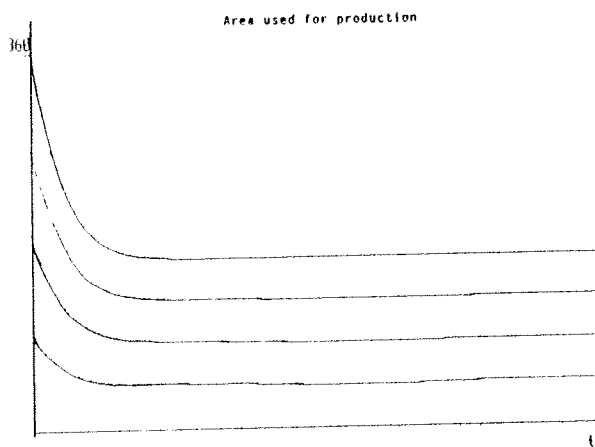
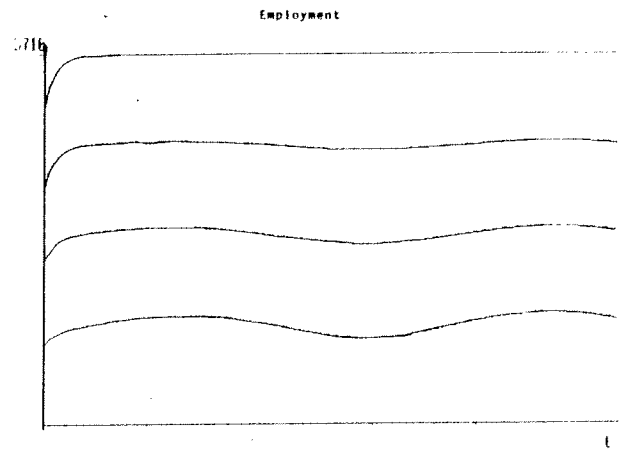
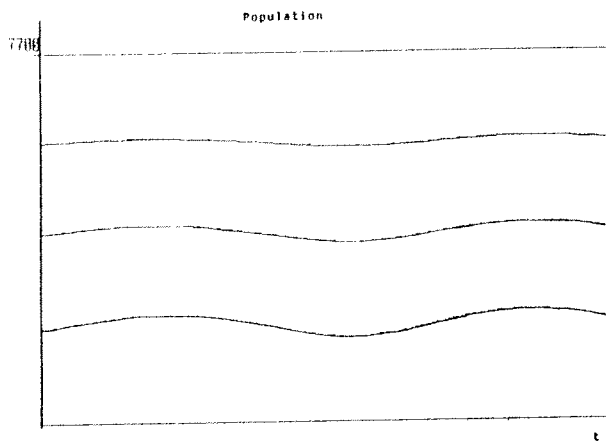




figure 4

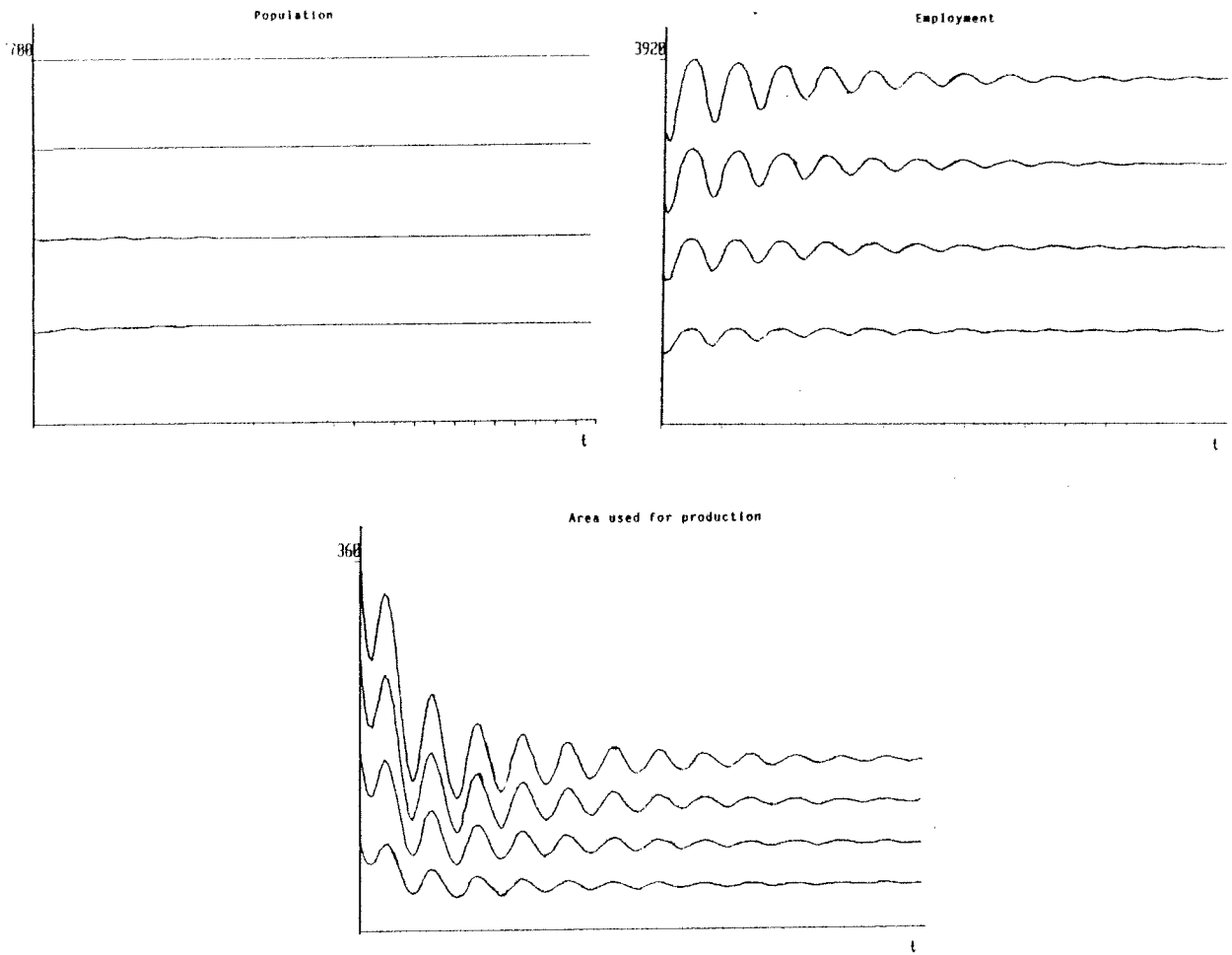
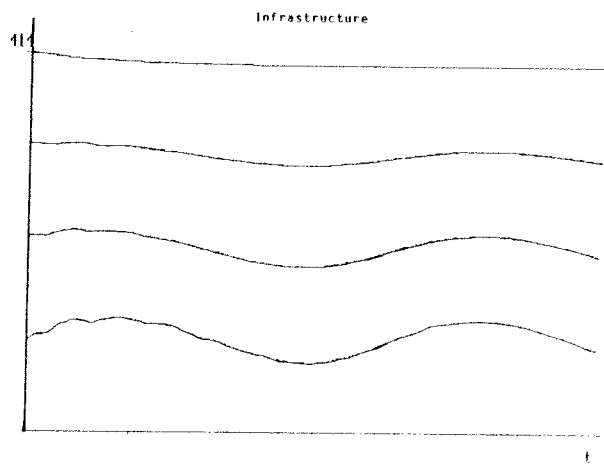
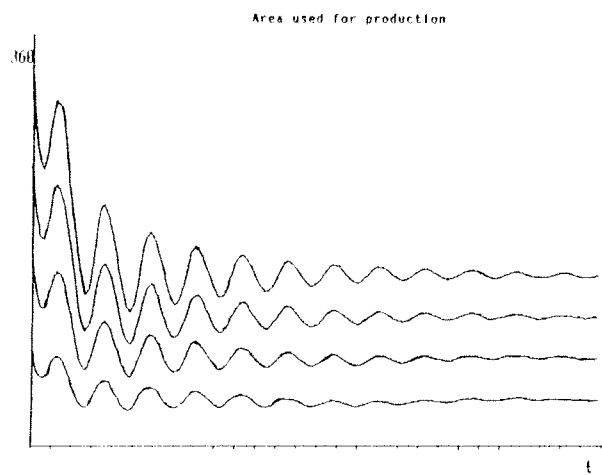
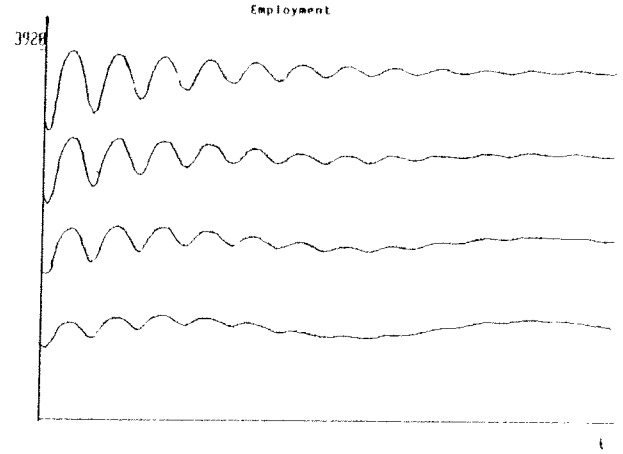
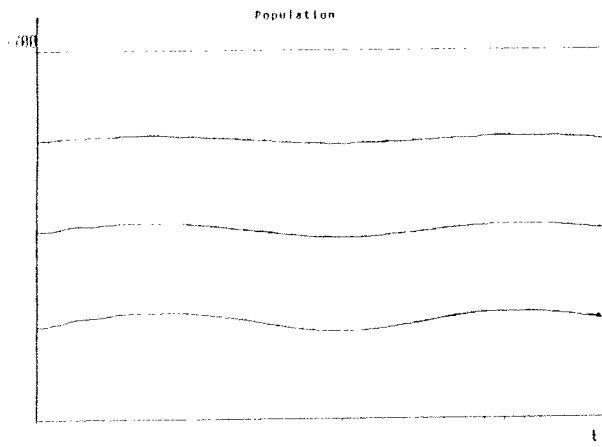


figure 5



The infrastructure submodel as formulated in eq.(16) might add to the concentration tendency of the model. Investment in (social) infrastructure is assumed to be stimulated by population growth, while on the other hand infrastructure attracts population. Recall, however, that we do not account for the aging of infrastructure. When adding this single link to the model it produces a very interesting time path. As can be seen from figure 3 it yields long waves with a wavelength of more than 70 time periods. All elements of the urban system are infected by these long swings. Note that we didn't assume any complex infrastructure aging and replacement mechanisms. The displayed dynamics result from a constant deterioration rate, and an infrastructure investment function consisting of a constant part and a part depending on population change.

Another positive link in the model presented in section 2 is the income potential. Its change has an "accelerator" type of impact on the representative firm's demand for labour and land (see equ. 10 and 11). While infrastructure of a zone reacts only to a population change in the same zone the income loop has a spatial dimension as well (income potential). When adding the income potential link instead of infrastructure to the model, it produces short, business cycles like fluctuations. The wavelength is about 10 time periods (see figure 4).

When using both feedback loops (infrastructure and income potential) the two fluctuations described simply overlay each other. The system starts with short period fluctuations and after they have faded out swings in long waves (see figure 5).

#### 4. Summary and outlook

The simulations made with the model outlined seem to indicate that the model specified exhibits some of the desired properties, i.e. it tends to produce wave-like population patterns, long run stability and short run variability of unemployment and vacancies, etc. These results point in the direction of the "empirical regularity" found in post war urban development. These results are produced with an extremely simple model lacking any explicit long run adjustment processes. Neither technical progress, innovation, and innovation diffusion nor accumulation and long run deterioration of physical structures is implemented in the model. Nevertheless, it is able to produce long as well as short waves

in all key indicators. Other time paths, however, are produced by this model when using other parameter constellations.

The model still contains many oversimplifications and gaps. Prominent among these is the oversimplified infrastructure submodel and the lack of a precise, endogenous bridge to the income generated in the urban economy. Similarly, steps will have to be undertaken to introduce capital accumulation (and technical progress) explicitly into the labour demand formulation. The homogeneity of population, labour, land, production, etc. is another oversimplifying assumption which has to be relaxed prior to any attempts of applying the model to real urban development. Using a "fully" specified model from the very beginning, however, would blur the effects investigated in this paper. Nevertheless the "demo-economic" approach utilized for urban analysis in this exercise seems to be fruitful in the future.

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