Measurement of Flood Damage due to Climate Change by Dynamic Spatial Computable General Equilibrium Model *

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Abstract

In order to explain economic impacts of flood damages due to climate change over time in Japan, this study develops a dynamic spatial computable general equilibrium (SCGE) model, and measures flood damage costs through some numerical experiments.

To consider spillover effects of flood damage over time, this study proposes two indices as “dynamic damage cost in comparative statics” and “dynamic damage cost in transitional dynamics”. The former is the long-term damage caused as the result of shifting from a steady-state equilibrium to another by increasing in the frequency and the intensity of flood due to climate change. On the other hand, the latter is the difference between flood damage costs by a baseline scenario and by a flood scenario, on the transition path to a new steady-state equilibrium. As the transition path can be described, this study shows possible spillover effects of flood damage over time.

This study develops a spatial CGE model based on dynamic structure of the Ramsey model. Our model has 8 regions and 20 production sectors. The flood scenario is described as the increase in capital depreciation rate due to flood in simulation periods from 2000 to 2050. Also, in our simulations, 5 flood damage rates are used consisting of damage rates calculating by 4 climate model (CSIRO, GFDL, MIROC and MRI) and uniform damage rate throughout Japan.

The findings in this study are shown below. (1) In 2050, the total amount of flood damage cost is estimated to be from about US$0.4 billion to about US$5.6 billion. (2) The decrease in the rate of investment return by the long-term increase in flood damage causes decrease in savings and consumption, so that the dynamic multiplier of damage cost is estimated to be from 1.2 to 1.7 times.

JEL Classification: C68, H43, Q54

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1 Introduction

In order to explain economic impacts of flood damages due to climate change over time in Japan, this study develops a dynamic spatial computable general equilibrium model (SCGE), and measures flood damage costs through some numerical experiments.

It is inferred that the frequency and the intensity of flood are on the long-term increase. In the category of flood damage in Japan, there are serious flood damages to social capital, including in houses, buildings, roads and so on. These economic damages have been measured by a variety of methods, such as an econometric approach, a general equilibrium approach and an engineering approach. However, there remain questions regarding each approach. For instance, as a computable general equilibrium approach that is assumed to be a static economy does not consider a capital accumulation, it is inappropriate for traditional CGE model to evaluate the long-term flood damages due to climate change. Therefore, it is necessary to develop dynamic model that has an endogenous capital stock, and to evaluate economic impacts of flood damages.

This study develops a spatial computable general equilibrium model based on dynamic structure of the Ramsey growth model. Our model has 8 regions and 20 production sectors and goods in Japan. The flood scenarios in this study are described as the increase in capital depreciation rate due to flood in simulation periods from 2000 to 2050. Also, in our numerical experiments, 5 flood damage rates are used consisting of flood damage rates calculating by 4 climate model (CSIRO, GFDL, MIROC and MRI) and uniform damage rate throughout Japan. In order to consider spillover effects of flood damage over time, this study proposes two indices as dynamic damage costs in comparative statics and dynamic damage costs in transitional dynamics. The former is the long-term damage caused as the result of the shift from a steady-state equilibrium to another steady-state equilibrium by increasing in the frequency and the intensity of flood damage due to climate change. On the other hand, the latter is the difference between flood damage costs by a baseline scenario and by a flood scenario, on the transition path to a new steady-state equilibrium. As the transition path could be described, this study shows possible spillover effects of flood damage over time.

The two main findings in this study are shown. (1) In 2050, the total amount of flood damage cost is estimated to be from about US$ 0.4 billion to about US$ 5.6 billion. (2) The decrease in the rate of investment return by long-term increase in flood damage causes decreases in savings and consumption, so that the dynamic multiplier of damage cost is
estimated to be from 1.2 to 1.7 times.

The structure of this study is the following. Chapter 2 describes the Ramsey growth model and defines flood damage costs. Chapter 3 explains outline of our model and scenarios and Chapter 4 performs simplified numerical analyses. Finally, Chapter 5 presents some concluding remarks and topics for future study.

2 Definition of Flood Damage by Economic Growth Model

Using the Ramsey growth model, we formulate a steady-state with flood damage with respect to consumption and capital stock, and define flood damages as the change in equivalent consumption.

2.1 Ramsey Growth Model

According to Barro and Sala-i-Martin (2004), we explain the Ramsey model. We assume that the Ramsey model in this study is an aggregated closed economy with one sector and it consists of a representative household and a firm.

First, a representative household provides labor in exchange for wages, receives income on assets, consumes goods, and saves the rest of income. A household maximizes the present value of lifetime utility subject to the budget constraint in per capita term, as follows.

\[
\max_{c(t)} U = \int_0^\infty u(c(t)) \exp\{(n - \rho)t\} \, dt, \quad \text{where } u(c(t)) = \frac{c(t)^{(1-\theta)} - 1}{1 - \theta}
\]

\[\text{s.t. } \dot{a} = w + ra(t) - c(t) - na(t)\] (2)

where \(\rho\) is the rate of time preference, \(\theta\) is the inverse of the elasticity of inter-temporal substitution, \(n\) is a labor growth rate, \(c\) is consumption per capita, \(w\) is a wage rate, and \(r\) is an interest rate. And, a utility function is assumed to be the CRRA ((Constant Arrow&Pratt’s Relative Risk Aversion) and the CIES (Constant Inter-temporal Elasticity of Substitution). The necessary condition and the transversality condition of Hamiltonian dynamics for this optimization problem are well known as follows.

\[
\frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho) \quad \text{ (3)}
\]

\[
\lim_{t \to \infty} \left[ a(t) \exp\left\{-\int_0^\infty (r(v) - n)dv\right\}\right] = 0 \quad \text{ (4)}
\]

Secondly, we define \(L(t) = L(0)e^{nt}\) as the number of population in period \(t\) and \(L(t) = \hat{L}(t)e^{xt}\) as effective labor considering the Harrod neutral technology (\(x\) means the rate of
exogenous technological progress). On the other hand, capital stock per effective labor is represented as below.

\[ k = \frac{K}{L}, \dot{k} = \frac{K}{\dot{L}} = ke^{-xt}, k = \dot{ke}^{xt} \] (5)

A firm maximizes profit \( \pi \) under constraint that it has production function with constant return to scale as follows.

\[ \max_{\hat{k}} \pi = F(K, \hat{L}) - (r + \delta)K - wL = \hat{L} \left[ f(\hat{k}) - (r + \delta)\hat{k} - we^{-xt} \right] \] (6)

where \( \delta \) is the depreciation rate of capital stock. The first-order condition in the firm’s optimization problem is written as follows.

\[ f'(\hat{k}) = r + \delta \] (7)

\[ \left[ f(\hat{k}) - (r + \delta)\hat{k} \right] e^{xt} = w \] (8)

Thirdly, in order to show the equilibrium conditions according to Barro and Sala-i-Martin (2004), all variables are converted into effective labor unit. \( a = k \) with equations of (2), (3), (7) and (8) determines the equilibrium value of variables such as \( c, k, w, \) and \( r \). In order to express this economic system by only \( \hat{c} \) and \( \hat{k} \), substitute \( \dot{k} = \dot{ke}^{xt} + \dot{ke}^{xt} \) and equations (7) and (8) into equation (2).

\[ \dot{ke}^{xt} + x\dot{ke}^{xt} = e^{xt} \left\{ f(\hat{k}) - \hat{ke}^{xt} \right\} + \dot{ke}^{xt} \left\{ f'(\hat{k}) - \delta \right\} - c - \dot{nke}^{xt} \] (9)

Let \( \dot{c} = ce^{-xt} \). Then, equation (2) is rewritten as below.

\[ \dot{k} = f(\hat{k}) - \dot{c} - (n + x + \delta)\hat{k} \] (10)

Also, we substitute \( c = \hat{ce}^{xt} \) and \( \dot{c} = \hat{ce}^{xt} \) into Equation (3).

\[ \frac{\dot{c}}{\dot{c}} = \frac{1}{\theta} (r - \rho - \theta x) = \frac{1}{\theta} \left[ f'(\hat{k}) - \delta - \rho - \theta x \right] \] (11)

Hence, two differential equations of (10) and (11) determine the equilibrium path.

### 2.2 Steady-State and Comparative Statics

In accordance with Morisugi et al. (2012), we define flood damage costs based on the Ramsey model as below. We assume the long-term increases in flood damage costs due to climate
change, and that the increase in the annual disaster physical damage of capital stock loss is expressed as the change in depreciation rate from $\delta_0$ to $\delta_1$ and which is $\delta_0 < \delta_1$.

First, $\dot{k} = 0$ and $\dot{c} = 0$ in equations (10) and (11) lead to a steady-state which is expressed as below.

$$f(\hat{k}^*) - (n + x + \delta)\hat{k}^* = \hat{c}^*$$  \hspace{1cm} (12)

$$f'(\hat{k}^*) - \delta = r = \rho + \theta x$$  \hspace{1cm} (13)

From equation (13), on a steady-state, even if depreciation rate varies from $\delta_0$ to $\delta_1$, it can be seen that an interest rate is determined by a parameter. And, by substituting this condition into equation (3), it can be seen that the growth rate of consumption per effective labor is equal to the rate of technical progress $x$, and that the growth rates of income and capital also are the same rates.

Secondly, we consider comparative statics by the change in depreciation rate due to increase in flood damage. This can be carried as follows. From equation (13) and by the Inada condition,

$$\frac{d\hat{k}^*}{d\delta} = \frac{1}{f''(\hat{k}^*)} < 0$$  \hspace{1cm} (14)

From equation (12), we get as follows.

$$\left\{f'(\hat{k}^*) - (n + x + \delta)\right\} d\hat{k}^* - \hat{k}^* d\delta = d\hat{c}^*$$  \hspace{1cm} (15)

$$\Leftrightarrow \frac{d\hat{c}^*}{d\delta} = \frac{\{\rho + (\theta - 1)x - n\}}{f''(\hat{k}^*)}$$  \hspace{1cm} (16)

The graphs of equations (12) and (13) are shown in Figure.1. In this figure, the ex-ante steady-state for depreciation rate is indicated as $\delta_0$ on the point $E_0$. Then, the steady-state changes from $\delta_0$ on the point $E_0$ to $\delta_1$ on the point $E_1$, due to climate change, with the decrease in both capital stock per effective labor and consumption per effective labor, such as $\hat{c}$ and $\hat{k}$. Thus, by shifting of the steady-state equilibrium from the point $E_0$ to the point $E_1$ due to increase in flood damage, it can be seen that both $\hat{c}^*_1$ and $\hat{k}^*_1$ on the ex-ante steady-state get smaller than those on the ex-post.

Thirdly, by interpretation of equation (15), we define a direct damage cost (or disbenefit), a dynamic damage cost (or disbenefit) and a dynamic multiplier of damage cost. The first term of the left-hand side $\left\{f'(\hat{k}^*) - (n + x + \delta)\right\}$ in equation (15) shows the annual capital
return that capital stock lost in flood damage was supposed to produce, and it is multiplied by the change in capital per effective labor \( \delta k^* \) by shifting of the steady-state equilibrium. As a representative household changes a plan for consumption and savings over time, decrease in capital stock per effective labor \( \delta k^* \) results in a real decline in income and consumption per effective labor \( \delta c^* \) decreases on a steady-state equilibrium.

On the other hand, the second term of the left-hand side in equation (15) shows the increase in investment that covers capital stock loss affected by the change in depreciation rate due to flood damage. In the short-term, this investment increase is the direct damage cost that is described in statistical research on flood by Japanese government, and means a reconstruction investment or a disaster recovery activity. Although increase in this investment expenditure decreases in disposal income and consumption in each period, the level of recovery is on a new steady-state equilibrium. Therefore, to be exact, this investment is not for restoration before disaster. By two effects as mentioned above, since the right-hand side \( \delta c^* \) in equation (15) is negative, it is defined as the dynamic damage cost or the decrease in equivalent consumption due to the direct effect. Moreover, by rewriting equation (17), we get as follows.

\[
d\hat{c}^* = -\left[\frac{\{\rho + (\theta - 1)x - n\}}{-f''(\hat{k}^*) \cdot \hat{k}^*} + 1\right] \cdot \hat{k}^* d\delta
\]  

(17)

Morisugi et al. (2012) has estimated the angled brackets of the right-hand side in equation (17) as the multiplier of 1.357. And, it is defined as the dynamic multiplier of damage cost. The left-hand side in equation (17) means the dynamic damage cost, and the right-hand side is the product of the direct damage cost and the multiplier. By assumption of \( f'' < 0 \) and the transversality condition, it is ensured that the dynamic multiplier of damage cost is over 1.

Finally, reconsidering the differences between the dynamic situation and the static situation, it depends on whether a household makes a plan for consumption and savings over the future or not. In the static case in this model, as we assumes the economy in only one period, we can consider the economy as no changes in capital stock and savings, that is \( \delta k^* = 0 \). Thus, in equation (15), equation (18) holds. Therefore, in the static case, it can be seen that the direct damage cost in the left-hand side in equation (18) equals the decrease in equivalent consumption due to the direct effect in the right-hand side in this equation.

\[
-\hat{k}^* d\delta = \delta c^*
\]  

(18)
2.3 Dynamic Damage Cost as Decrease in Consumption

In order to consider spillover effects of flood damage over time, we propose two indices. One is the dynamic damage cost described in a comparative static situation. The other is the dynamic damage cost described in a transitional dynamic situation. We call the former “dynamic damage cost in comparative static” and the latter “dynamic damage cost in transitional dynamic” as below.

The system of differential equations in this study is given from equations (10) and (11).

\[
\dot{k} = f(\dot{k}) - c - (n + x + \delta)\dot{k}
\]

\[
\dot{c} \over c = \frac{1}{\theta}(r - \rho - \theta x) = \frac{1}{\theta} \left[f'(\dot{k}) - \delta - \rho - \theta x\right]
\]

Figure.2 shows that the economy without flood damage is on a steady-state equilibrium indicated on the point SS\textsubscript{0}. From this figure, as the economy without flood damage is on the point SS\textsubscript{0} independent on time, consumption on the steady-state equilibrium is constant level of \(\hat{c}_0^*\) in the future. On the other hand, since the economy with the increase in flood damage due to climate change means the economy with higher depreciation rate, the new steady-state equilibrium is moved from SS\textsubscript{0} to SS\textsubscript{1}.

First, the dynamic damage cost in comparative statics describes the difference between consumptions on SS\textsubscript{0} and SS\textsubscript{1}, that is \(\hat{c}_0^* - \hat{c}_1^*\). As mentioned above, this dynamic damage can be definitely expressed as a solution of the theoretical model and it is clear that the value of the dynamic multiplier of dynamic damage cost is over 1. Also, in this case, a variable of capital stock is treated as an endogenous variable.

Next, the dynamic damage cost in transitional dynamics describes the annual average cost that is derived from the sum of the difference of the annual consumption \(\hat{c}_0^* - \hat{c}_1^*(t)\) on the transition path for a new steady-state equilibrium SS\textsubscript{1}. Note that the increase in flood damage shifts instantaneously the steady-state equilibrium without flood damage SS\textsubscript{0} to the initial point with flood damage SA, which is represented as the change in investment adjusted in any year with flood damage and change in consumption in the Ramsey model. Then, the initial point SA with flood damage is on the stable-arm and the economy shifts toward a new steady-state equilibrium SS\textsubscript{1} over time. That is found in Figure.3 that shows a transition path of consumption, and the horizontal axis represents time in this figure. According to Novales et al. (2009) and Barro and Sala-i-Martin (2004), by the log-linear approximate representation around steady-state values, we show the time paths for consumption and
capital stock. For the details of derivation of time paths for consumption and capital stock, see appendix. Now, we log-linearize this system for the case in which the production function is the Cobb-Douglas type, \( y = Bk^\alpha \), \( 0 < \alpha < 1 \). For simplicity, we assume \( B = 1 \). Start by rewriting the system from equations (10) and (11) in terms of the logs of \( \hat{c} \) and \( \hat{k} \).

\[
\frac{d \ln \hat{c}}{dt} = \frac{1}{\theta} \left[ \alpha e^{-(1-\alpha)\ln \hat{k}} - (\delta + \rho + \theta x) \right] \tag{19}
\]

\[
\frac{d \ln \hat{k}}{dt} = e^{-(1-\alpha)\ln \hat{k}} - e^{(\ln \hat{c}-\ln \hat{k})} - (n + x + \delta)
\]

By log-linearizing equation (19) around the steady-state where \( d \ln \hat{c}/dt = d \ln \hat{k}/dt = 0 \), we have the following equation.

\[
\begin{pmatrix}
\frac{d \ln \hat{c}}{dt} \\
\frac{d \ln \hat{k}}{dt}
\end{pmatrix} =
\begin{pmatrix}
0 & -\eta \\
-\kappa & \xi
\end{pmatrix}
\begin{pmatrix}
\ln \hat{c} - \ln \hat{c}^* \\
\ln \hat{k} - \ln \hat{k}^*
\end{pmatrix}
\tag{20}
\]

with \( \eta = (1-\alpha)(\delta + \rho + \theta x)/\alpha > 0 \), \( \kappa = [(\delta + \rho + \theta x)/\alpha] - (n + x + \theta) > 0 \) and \( \xi = \rho - n - x + \theta x \). Since the coefficient matrix \( \Delta \) in equation (20) has determinant \(-\eta \kappa < 0\), the system holds a saddle point trajectory leading to the steady-state. And, eigenvalues of the transition matrix are as follows.

\[
\mu_1, \mu_2 = \frac{\xi \pm \sqrt{\xi^2 + 4\eta \kappa}}{2}
\tag{21}
\]

with \( \mu_1 > \xi > 0 \) and \( \mu_2 < 0 \). Then, for a level of consumption \( \hat{c}_0 \) chosen as a function of the initial condition on \( \hat{k}_0 \), the solutions to the system of linear differential equations are derived as follows

\[
\ln \hat{c}(t) - \ln \hat{c}^* = e^{\mu_2 t} (\ln \hat{c}_0 - \ln \hat{c}^*) = -e^{\mu_2 t} \frac{\eta}{\mu_2} \left( \ln \hat{k}_0 - \ln \hat{k}^* \right) \tag{22}
\]

\[
\ln \hat{k}(t) - \ln \hat{k}^* = e^{\mu_2 t} \left( \ln \hat{k}_0 - \ln \hat{k}^* \right) \tag{23}
\]

Finally, the solutions to the system, \( \ln \hat{c}(t) - \ln \hat{c}^* \), implies that the relationship between consumption \( \hat{c} \) and capital stock \( \hat{k} \) is the same at all time periods.

\[
\ln \hat{c}(t) = \ln \hat{c}^* - e^{\mu_2 t} \frac{\eta}{\mu_2} \left( \ln \hat{k}(t) - \ln \hat{k}^* \right), \ t = 0, 1, 2, 3, \cdots \tag{24}
\]

\[
\ln \hat{k}(t) = \left( 1 - e^{\mu_2 t} \right) \ln \hat{k}^* + e^{\mu_2 t} \ln \hat{k}_0, \ t = 0, 1, 2, 3, \cdots \tag{25}
\]
As can be seen from Figure.2 and Figure.3, since the dynamic damage cost in transitional dynamics has highly realistic descriptiveness, possible spillover effect of flood damage over time can be shown.

Finally, the direct damage cost describes the difference between consumptions on $SS_0$ and $F_j$, that is $\hat{c}_0^s - \hat{c}_0^{s'}$ in Figure.2 and Figure.3, and the decrease in consumption in the case of constant capital stock as an exogenous variable. That means equation (18) and the decrease in consumption is equal to the direct damage cost.

3 Outline of Model and Scenarios

3.1 Structure of Multi-Regional Computable General Equilibrium Model

Our SCGE model uses the 2000 Inter-regional Input-Output Table (47 prefectures and 45 sectors) that has been created by Miyagi et al. (2003) and Ishikawa and Miyagi (2004) as the reference data set. Table.1 and Table.2 show that our model integrates 47 prefectures into 8 regions and does 45 sectors into 20 sectors. Also, economic agents in our model are household sector, production sector, investment sector, export and import sector, and government.

3.1.1 Production Sector

As shown in Figure.4, all production functions in domestic production sector are assumed to be the nested CES (constant elasticity of substitution) style. For the first step, labor $L_j^s$ and capital $K_j^s$ are aggregated into the composite production factor $VA_j^s$ using a Cobb-Douglas production function, and the composite inputs $N_{ij}^s$ are made up of intermediate inputs $X_{rs}^{ij}$ from all regions using a CES production function. For the second step, in order to produce the gross domestic output $Y_j^s$ for the $j$-th production sector in the $s$-th region, the composite production factor $VA_j^s$ is combined with the composite inputs $N_{ij}^s$, using a Leontief production function.

3.1.2 Household Consumption Sector

Figure.5 shows the structure of household consumption. We assume that there is one representative household in each region. In order to yield utility $U_{iH}^s$ under a budget constraint, a household in $s$-th region demands composite household consumption goods $N_{iH}^s$ that are made up of intermediate household consumptions $X_{iH}^s$ from all regions using a CES function.
3.1.3 Government Consumption Sector

The structure of government consumption sector is described in the same way as that of household consumption sector in Figure.5. Also, we assume that government in each region earns revenue from income tax, production tax and indirect tax, and spends government consumption and investment.

3.1.4 Private Investment Sector and Government Investment Sector

The structure of private investment sector and government investment sector is the same as that of household consumption sector. And, we assume that there is a virtual investment sector in each region. While private investment sector demands investment goods over region, government sector demands investment goods in its own region.

3.1.5 Export and Import

In accordance with Hosoe et al. (2010), Figure.6 shows the structure of the substitution between imports and domestic goods and that of the transformation between exports and domestic goods. About imperfect substitution between imports and domestic goods, we assume the Armington’s assumption. The i-th Armington-composite-good-producing sector in the s-th region aggregates domestic goods $D_{irs}$ and imports $IM_{irs}$ into composite goods $Q_{irs}$ using a CES function. On the other hand, gross domestic output $Y_{irs}$ is transformed into domestic goods $D_{irs}$ and exports $EX_{irs}$ using a CET (constant elasticity of transformation) function. While parameters of elasticity of transformation $\sigma_{DEX}$ are assumed as 2.0 exogenously, parameters of elasticity of substitution $\sigma_{DIM}$ are set by values of GTAP7.1 and are shown in Table.3.

3.2 Structure of Dynamic Model

This study extends the way of describing the structure of dynamic model by Lau et al. (2002), Paltsev (2004) and Ban (2007). These studies have adopted a Ramsey growth model to develop a dynamic structure.

First, there are three assumptions in describing a neoclassical growth model in this study: 1) over all periods, an economy is on a steady-state equilibrium path, 2) in the initial period, an economy is on a steady-state, and 3) in the terminal period, under constraint that the growth rate of investment equals the growth rate of output, an economy is on a steady-state.
A representative household maximizes the present value of lifetime utility subject to three constraints that a production function in period $t$ is assumed to constant returns to scale in labor and capital, total output in period $t$ is divided into consumption and investment, and the capital stock in period $t+1$ is equal to the capital stock in period $t$ depreciated at rate $\delta$ plus investment in period.

\[
\max_{c(t)} \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t U(c(t))
\]

s.t. \( Y(t) = F(K(t), L(t)) \) \hspace{1cm} (27)

\( c(t) = Y(t) - I(t) \) \hspace{1cm} (28)

\( K(t+1) = K(t) \cdot (1-\delta) + I(t) \) \hspace{1cm} (29)

where $c(t)$ is consumption in period $t$, $Y(t)$ is output, $I(t)$ is investment, $K(t)$ is capital stock, $L(t)$ is labor, $F(\cdot)$ is production function, $U(\cdot)$ is utility function, $\rho$ is the time preference rate and $\delta$ is the annual depreciate rate, respectively. Solving the utility maximization problem results in the first-order conditions, and these conditions can be rewritten as:

\[
P(t) = \left( \frac{1}{1+\rho} \right)^t \frac{\partial U(c(t))}{\partial c(t)}
\]

\[
PK(t) = (1-\delta) \cdot PK(t+1) + P(t) \frac{\partial U(c(t))}{\partial c(t)}
\]

\[
P(t) = PK(t+1)
\]

where $P(t), PK(t)$ and $PK(t+1)$ are the values of the corresponding Lagrange multiplier, and they can be interpreted as that $P(t)$ is the output price in period $t$, $PK(t)$ is the capital price in period $t$ and $PK(t+1)$ is capital price in period $t+1$. According to Paltsev (2004), let $RK(t), W(t)$ and $M$ represent rental rate of capital, wage rate and consumer’s income, and denote unit cost function and demand function as $C(RK(t), W(t))$ and $D(P(t), M)$. Then, we can formulate the equilibrium conditions in terms of three classes of equations, i) zero profit conditions, ii) market clearance conditions, iii) income balance conditions, as the mixed complementarity problem.

i) zero profit conditions:

\[
P(t) \geq PK(t+1), I(t) \geq 0, I(t) (P(t) - PK(t+1)) = 0
\]
\[ PK(t) \geq RK(t) + (1-\delta) \cdot PK(t+1), K(t) \geq 0, \quad K(t) (PK(t) - RK(t) + (1 - \delta) \cdot PK(t + 1)) = 0 \]  
\[ C(RK(t), W(t)) \geq P(t), Y(t) \geq 0, Y(t) (C(RK(t), W(t)) - P(t)) = 0 \]

ii) market clearance conditions:

\[ Y(t) \geq D(P(t), M) + I(t), P(t) \geq 0, P(t) \left( Y(t) - D(P(t), M) + I(t) \right) = 0 \]  
\[ L(t) \geq Y(t) \frac{\partial C(RK(t), W(t))}{\partial W(t)}, W(t) \geq 0, W(t) \left( L(t) - Y(t) \frac{\partial C(RK(t), W(t))}{\partial W(t)} \right) = 0 \]

\[ K(t) \geq Y(t) \frac{\partial C(RK(t), W(t))}{\partial RK(t)}, RK(t) \geq 0, RK(t) \left( K(t) - Y(t) \frac{\partial C(RK(t), W(t))}{\partial RK(t)} \right) = 0 \]

iii) income balance conditions:

\[ M = PK(0) \cdot K(0) + \sum_{t=0}^{\infty} W(t) \cdot L(t), M > 0 \]  

In this study, equilibrium conditions in the statics can be shown as equations (33), (35), (36), (37) and (38), while those in the dynamics can be shown as two equations (34) and (39) in addition to these static conditions.

In accordance with Lau et al. (2002), Paltsev (2004) and Ban (2007), we introduce the level of the post-terminal capital stock as an endogenous variable and add a constraint that the growth rate of investment is equal to the growth rate of output in the terminal period \( T \). (the assumption 3))

\[ \frac{I(T)}{I(T-1)} = \frac{Y(T)}{Y(T-1)} \]

3.3 Dynamic Optimization Problem with Multiple Agents

When there exist multiple economic agents like a dynamic multi-regional model, it has been known that equilibrium solutions in a dynamic model with multiple infinitely lived agents must satisfy the Negishi condition by Negishi (1960). For instance, RICE (a Regional dynamic Integrated model of Climate and the Economy) model by Nordhaus and Boyer (2000) solves a dynamic multi-regional optimization problem by using the Negishi condition. On the other hand, by deriving an equilibrium solution from financial asset positions in the terminal period, Lau et al. (2002) solves a dynamic multi-regional optimization problem. Lau et al. (2002)
divides an optimization problem with infinite horizons into two distinct optimization problems that one is defined over the period $t = 0$ to $t = T$ and the other is defined over the period $t = T + 1$ to $t = \infty$, and puts these two periods together by financial assets in the terminal period. In accordance with Lau et al. (2002), we solve a dynamic optimization problem with multiple agents. See Lau et al. (2002) in details of this problem.

First, under the intertemporal budget constraint, the finite horizon problem for the representative household in region $s$ is shown as below.

\[
\max_{C^s(t)} \sum_{t=0}^{T} \left( \frac{1}{1 + \rho} \right)^t U(C^s(t)), \text{ where } U(C^s(t)) = \log C^s(t) \tag{41}
\]

\[
\text{subject to } \sum_{t=0}^{T} P^s(t)C^s(t) = \sum_{t=0}^{T} w^s(t)L^s(t) + A^s(0) - A^s(T + 1) \tag{42}
\]

where $A^s(t)$ is the stock of financial assets in region $s$ in period $t$. On the other hand, under the intertemporal budget constraint, the infinite horizon problem in region $s$ is shown as below.

\[
\max_{C^s(t)} \sum_{t=T+1}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \log C^s(t) \tag{43}
\]

\[
\text{subject to } \sum_{t=T+1}^{\infty} P^s(t)C^s(t) = \sum_{t=T+1}^{\infty} w^s(t)L^s(t) + A^s(0) + A^s(T + 1) \tag{44}
\]

By equation (43), we define the post-terminal asset position as below.

\[
A^s(T + 1) = \sum_{t=T+1}^{\infty} P^s(t)C^s(t) - \sum_{t=T+1}^{\infty} w^s(t)L^s(t) \tag{45}
\]

\[
= \sum_{t=T+1}^{\infty} [P^s(T)C^s(T) - w^s(T)L^s(T)] \left( \frac{1 + n_T}{1 + r_T} \right)^{(t-T)} \tag{46}
\]

where $n_T$ and $r_T$ represent the post-terminal growth and interest rate. In the terminal period, we represent the terminal asset value $\phi^s$ as a share of global assets.

\[
\phi^s = \frac{A^s(T + 1)}{\sum_r A^r(T + 1)} = \frac{[P^s(T)C^s(T) - w^s(T)L^s(T)]}{\sum_r [P^r(T)C^r(T) - w^r(T)L^r(T)]} \tag{47}
\]

And, with respect to region, the sum of the terminal assets equals the sum of the terminal capital stock. Hence,
\[
\sum_s A^s(T + 1) = \sum_s PK^s(T + 1) \cdot K^s(T + 1)
\]  \hspace{1cm} (48)

Then, from equation (47) and equation (48), we express the terminal asset position in region as below.

\[
A^s(T + 1) = \phi^s \sum_r PK^r(T + 1) \cdot K^r(T + 1)
\]  \hspace{1cm} (49)

A dynamic optimization problem with multiple agents can be solved using Equation (49).

### 3.4 Setting of Flood Damage Scenarios

As shown in Chapter 2, this study treats the change in flood damages as the change in the capital depreciation rate. The flood scenario due to climate change is assumed to increase in the capital depreciation rate of private capital stock by the flood damage rate calculated by a climate model. For calculations of the flood damage rate due to the future climate change, we use a total of 5 scenarios that consist of 4 calculation results made by CSIRO, GFDL, MIROC and MRI, and one result calculated in S8 scenario by Morisugi et al. (2012). Annual flood damage rate (% per year) calculated by these climate models is described as proportion of differences between flood damage costs in 1981 and in 2081 to the private capital stock in 2000.

### 4 Results from Simulation Analyses

#### 4.1 Changes in Household Consumption

Figure 7 shows the changes in household consumption and Table 4 shows the values of direct damage, calculated by five flood damage scenarios.

First, Figure 7 means dynamic damage costs in transitional dynamics. While flood damage costs in 2000 were estimated to be from about 0.23 billion US dollars per year to about 4.4 billion US dollars per year, those in 2050 were estimated to be from about 0.4 billion US dollars per year to about 5.6 billion US dollars per year. In 2050, the minimum value of flood damage was calculated by using the CSIRO scenario and the maximum value was calculated by the S8 Scenario.

Secondly, by calculating the dynamic multipliers of damage cost from dynamic damage costs in transitional dynamics, we estimated the values from about 1.1 to about 1.7. On the other hand, Morisugi et al. (2012) estimated the value of 1.357. We can confirm that our
results are close to the result of Morisugi et al. (2012) and our dynamic multipliers are over 1. Also, our results can be explained that when the increase in flood damages due to climate change is expected to reduce the rate of return on investment, the decreases in investment and savings by the long-term expectation results in the decrease in consumption.

Thirdly, Table.4 shows that direct damage costs were estimated to be from about 0.25 billion US dollars per year to about 5.0 billion US dollars per year. These are flood damages in constant capital stock and are equivalent to those of the comparative statics in the short-term. In comparison of direct damage costs to dynamic damage costs, it can be seen that each dynamic damage cost in all scenarios gets larger than direct damage costs over time. Since direct damage costs add incremental costs of asset damage with climate change and possible dynamic spillover effects of flood damage are not considered, direct damage costs are underestimated. Thus, Our results in this simulation analysis are consistent with those in this theoretical analysis indicated in Chapter.2 and Chapter.3.

4.2 Changes in Sectoral Output

Figure.8 shows the changes in sectoral outputs in 2030 and in 2050 calculated by the S8 scenario. In both periods, the primary industries (agriculture and fishery), the foods, the electricity, the gas, the water supply and the tertiary industries were affected by the decrease in output due to flood damage. On the other hand, many of the secondary industries and the construction sector were not affected. Especially, it can be seen that there was marked increases in the outputs of the construction sector in both period.

4.3 Changes in Regional and Sectoral Output

Figure.9 shows the change rate of the regional and sectoral outputs in 2050 of the S8 scenario. In this table, a cell in red indicates a positive change in output and another cell in blue does a negative change in output. In all regions, the primary industries (agriculture and fishery), the foods were affected by the decrease in output due to flood damage. On the other hand, outputs in many sectors of the secondary industries were affected by flood damage. Especially, it can be seen that there was marked increases in the outputs of the construction sectors in some regions such as Kanto, Chubu and Kinki.
5 Concluding Remarks

In order to explain economic impacts of flood damages due to climate change over time in Japan, this study measured flood damage costs through 5 flood scenarios by using a dynamic spatial computable general equilibrium model. The findings in this study are shown below.

1. In dynamic damage costs in transitional dynamics, in 2050, the total amount of flood damage cost was estimated to be from about US$ 0.4 billion to about US$ 5.6 billion, and the dynamic multiplier of damage cost was estimated to be from 1.2 to 1.7 times.

2. Results in our simulation analyses were shown to be consistent with results in theoretical analyses proposed by Morisugi et al. (2012) and this study.

3. The primary industry (agriculture sector and fishery sector), the foods sector, the electricity sector, the gas sector, the water supply sector and the tertiary industry were affected by the decrease in output due to flood damage. On the other hand, many of the secondary industry did not suffer output damage due to flood, and there were marked increases in outputs in the construction sector in Kanto region, Chubu region and Kinki region.

There are several works remaining for future. First, in order to evaluate regional and sectoral impacts of flood damage more precisely, we need to expand our CGE model; 8 regions to 47 regions (all prefectures in Japan) and 20 sectors to more sectors. Second, we need to apply our framework to economic evaluation of some adaptation strategies to climate change.
References


A Appendix

A.1 Solution to Differential Equations in the Time Paths for Consumption and Capital Stock

In accordance with Novales(2009), in deriving the time paths for consumption and capital stock in 2.3, the details of solution method of differential equations can be shown below. After equation (21), the continuous-time dynamic system can be written as follows.

\[ \dot{z}(t) \equiv \Delta \cdot z(t) \]  

(50)

where \( z(t) = (\ln \hat{c}(t) - \ln \hat{c}^*, \ln \hat{k}(t) - \ln \hat{k}^*) \) is the vector of deviations around a steady-state and \( \Delta \) is the coefficient matrix shown in equation (20). The solution to this system is as follows.

\[ z(t) \approx e^{\Delta t} \cdot z(0) \]  

(51)

Let \( \Gamma \) be the matrix having as columns the right-eigenvectors of \( \Delta \) and \( \Gamma^{-1} \) be the inverse matrix having as rows the left-eigenvectors of \( \Delta \).

\[
\Gamma = \begin{pmatrix} z_1 & y_1 \\ z_2 & y_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -\frac{\mu_1}{n} & -\frac{\mu_2}{n} \end{pmatrix} 
\]

(52)

\[
\Gamma^{-1} = \begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \end{pmatrix} = \begin{pmatrix} -\frac{\mu_2}{n} & 1 \\ -\frac{\mu_1}{n} & 1 \end{pmatrix} 
\]

(53)

Using the results on the spectral decomposition of a matrix and the representation of matrix exponential function, the solution (51) to the dynamic system can be shown as follows.

\[
z(t) \approx e^{\Delta t} \cdot z(0) = \left( \Gamma e^{\Lambda} \Gamma^{-1} \right)^t z(0) = \Gamma e^{\Lambda} \Gamma^{-1} z(0) \]

(54)

that is,

\[
\begin{pmatrix} \ln \hat{c}(t) - \ln \hat{c}^* \\ \ln \hat{k}(t) - \ln \hat{k}^* \end{pmatrix} = \frac{\eta}{\mu_1 - \mu_2} \begin{pmatrix} 1 & \frac{1}{\eta} \\ -\frac{\mu_1}{\eta} & -\frac{\mu_2}{\eta} \end{pmatrix} \begin{pmatrix} e^{\mu_1 t} & 0 \\ 0 & e^{\mu_2 t} \end{pmatrix} \begin{pmatrix} \frac{\mu_2}{n} & -1 \\ -\frac{\mu_1}{n} & 1 \end{pmatrix} \begin{pmatrix} \ln \hat{c}_0 - \ln \hat{c}^* \\ \ln \hat{k}_0 - \ln \hat{k}^* \end{pmatrix} \]

(55)

or,

\[
\ln \hat{c}(t) - \ln \hat{c}^* = e^{\mu_1 t} b_{11} + e^{\mu_2 t} b_{12} \]

(56)
\[ \ln \hat{k}(t) - \ln \hat{k}^* = e^{\mu_1 t}b_{21} + e^{\mu_2 t}b_{22} \] (57)

where

\[ b_{11} = -\frac{1}{\mu_1 - \mu_2} \left[ \mu_2 (\ln \hat{c}_0 - \ln \hat{c}^*) + \eta (\ln \hat{k}_0 - \ln \hat{k}^*) \right] \]

\[ b_{12} = \frac{1}{\mu_1 - \mu_2} \left[ \mu_1 (\ln \hat{c}_0 - \ln \hat{c}^*) + \eta (\ln \hat{k}_0 - \ln \hat{k}^*) \right] \] (58)

\[ b_{21} = \frac{\mu_1}{(\mu_1 - \mu_2)\eta} \left[ \mu_2 (\ln \hat{c}_0 - \ln \hat{c}^*) + \eta (\ln \hat{k}_0 - \ln \hat{k}^*) \right] \]

\[ b_{22} = -\frac{\mu_2}{(\mu_1 - \mu_2)\eta} \left[ \mu_1 (\ln \hat{c}_0 - \ln \hat{c}^*) + \eta (\ln \hat{k}_0 - \ln \hat{k}^*) \right] \]

Since the transversality condition implies \( b_{21} = 0 \), \( \mu_2 (\ln \hat{c}_0 - \ln \hat{c}^*) + \eta (\ln \hat{k}_0 - \ln \hat{k}^*) \), so that the initial consumption must be chosen by equation (59).

\[ \ln \hat{c}_0 = \ln \hat{c}^* - \frac{\eta}{\mu_2} \left( \ln \hat{k}_0 - \ln \hat{k}^* \right) \] (59)

Furthermore, notice that this condition also implies \( b_{11} = 0 \) and use equation (59) in the expression for \( b_{12} \) and \( b_{22} \), the following equations can be obtained.

\[ b_{12} = \frac{1}{\mu_1 - \mu_2} \left[ \mu_1 (\ln \hat{c}_0 - \ln \hat{c}^*) + \mu_2 (\ln \hat{c}_0 - \ln \hat{c}^*) \right] = \ln \hat{c}_0 - \ln \hat{c}^* \] (60)

\[ b_{22} = -\frac{\mu_2}{(\mu_1 - \mu_2)\eta} \left[ \mu_1 (\ln \hat{k}_0 - \ln \hat{k}^*) + \eta (\ln \hat{k}_0 - \ln \hat{k}^*) \right] = \ln \hat{k}_0 - \ln \hat{k}^* \]

Therefore, equation (22) can be derived.

**A.2 Derivation of the Initial Investment**

According to Paltsev (2004), we show the derivation of the initial investment. By three assumptions mentioned above, this model is ensured that there exist solutions in this dynamic optimal problem. Therefore, if a solution is on a steady-state growth path, some conditions are shown as below.

\[ P(t) = PK(t + 1) \] (61)

\[ (1 + r) \cdot P(t) = (1 - \delta) \cdot P(t) + RK(t) \] (62)

\[ I(t) = (\delta + n) \cdot K(t) \] (63)

\[ VK(t) = K(t) \cdot RK(t) \] (64)
where $VK(t)$ is the total of capital endowment. As we have these conditions from Equation (61) to Equation (64) in the initial period, investment in the initial period can be written as below.

$$I(0) = \frac{(\delta + n) \cdot VK(0)}{(\delta + r)}$$  \hspace{1cm} (65)

As an economy in this study is assumed to be on a steady-state in the initial period (the assumption 2), we need to determine if the value of investment in the initial period represented in Equation (65) corresponds with the value of investment in the social account matrix. We assume $\delta = 0.04$, $n = 0.001$ and $r = 0.05$. 


A.3 Figures and Tables

Figure 1: Steady-state with or without flood damage

Figure 2: Definition of flood damage
Figure 3: Definition of flood damage over time

Figure 4: Structure of production sector
Figure 5: Structure of household sector

Figure 6: Structure of export and import
Figure 7: Changes in household consumption due to flood damage

Figure 8: Sectoral outputs in 2030 and 2050 by S8 scenario
Figure 9: Regional and Sectoral outputs in 2050 by S8 scenario

Table 1: Regional classification

<table>
<thead>
<tr>
<th>Region</th>
<th>Code</th>
<th>Prefecture</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hokkaido</td>
<td>HKD</td>
<td>Hokkaido</td>
</tr>
<tr>
<td>2 Tohoku</td>
<td>THK</td>
<td>Aomori, Iwate, Miyagi, Akita, Yamagata, Fukushima</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Iwakuni, Tottori, Shimane, Okayama, Hiroshima, Yamaguchi</td>
</tr>
<tr>
<td>3 Kanto</td>
<td>KNT</td>
<td>Kanagawa, Niigata, Yamanashi, Nagano, Shizuoka</td>
</tr>
<tr>
<td>4 Chubu</td>
<td>CHB</td>
<td>Toyama, Ishikawa, Aichi, Gifu, Mie</td>
</tr>
<tr>
<td>5 Kinki</td>
<td>KIK</td>
<td>Fukui, Shiga, Kyoto, Osaka, Hyogo, Nara, Wakayama</td>
</tr>
<tr>
<td>6 Chugoku</td>
<td>CGK</td>
<td>Tottori, Shimane, Okayama, Hiroshima, Yamaguchi</td>
</tr>
<tr>
<td>7 Shikoku</td>
<td>SKK</td>
<td>Tokushima, Kagawa, Ehime, Kochi</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fukuoka, Saga, Nagasaki, Kumamoto, Oita, Miyazaki, Kagoshima, Okinawa</td>
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</table>

25
<table>
<thead>
<tr>
<th>Sector</th>
<th>Code</th>
<th>47 Prefectural Input-Output Table</th>
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<tbody>
<tr>
<td>1 Agriculture</td>
<td>AGR</td>
<td>Agriculture</td>
</tr>
<tr>
<td>2 Forestry</td>
<td>FRS</td>
<td>Forestry</td>
</tr>
<tr>
<td>3 Fishery</td>
<td>FSH</td>
<td>Fishery</td>
</tr>
<tr>
<td>4 Mining</td>
<td>MIN</td>
<td>Mining</td>
</tr>
<tr>
<td>5 Foods</td>
<td>FOD</td>
<td>Foods</td>
</tr>
<tr>
<td>6 Other manufacturing products</td>
<td>OMF</td>
<td>Textile products, Timber and wooden products, Furniture and fixtures, Pulp, paper, paperboard, building paper, Publishing, printing, Leather, fur skins and miscellaneous leather products, Ceramic, stone and clay products, Miscellaneous manufacturing products</td>
</tr>
<tr>
<td>7 Chemical products</td>
<td>CPR</td>
<td>Chemical products, Plastic products, Rubber products</td>
</tr>
<tr>
<td>8 Petroleum &amp; coal products</td>
<td>P.C</td>
<td>Petroleum and coal products</td>
</tr>
<tr>
<td>9 Iron &amp; steel</td>
<td>I.C</td>
<td>Iron and steel</td>
</tr>
<tr>
<td>10 Metal products</td>
<td>MTL</td>
<td>Non-ferrous metals, Metal products</td>
</tr>
<tr>
<td>11 Industrial machinery</td>
<td>MCH</td>
<td>General industrial machinery, Machinery for office and service industry, Motor Vehicles, Other transportation equipment</td>
</tr>
<tr>
<td>12 Electrical equipment</td>
<td>ELM</td>
<td>Household electronic and electric appliances, Electronic and communication equipment, Other electrical equipment, Precision instruments</td>
</tr>
<tr>
<td>13 Construction</td>
<td>CNS</td>
<td>Building construction and repair of construction, Public construction and Other civil engineering</td>
</tr>
<tr>
<td>14 Electricity</td>
<td>ELY</td>
<td>Electricity</td>
</tr>
<tr>
<td>15 Gas</td>
<td>GDT</td>
<td>Gas and heat supply</td>
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<td>16 Water supply</td>
<td>WTR</td>
<td>Water supply and waste management services</td>
</tr>
<tr>
<td>17 Commerce</td>
<td>COM</td>
<td>Wholesale and retail trade, Finance and insurance, Real estate</td>
</tr>
<tr>
<td>18 Transport</td>
<td>TRS</td>
<td>Transport</td>
</tr>
<tr>
<td>19 Medical service</td>
<td>MED</td>
<td>Medical service, health and social security and nursing care</td>
</tr>
<tr>
<td>20 Services</td>
<td>ANC</td>
<td>Activities not elsewhere classified</td>
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Table 3: Parameters of elasticity of substitution between imports and domestic goods

<table>
<thead>
<tr>
<th>Sector</th>
<th>Value</th>
<th>Sector</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Agriculture</td>
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<td>Industrial machinery</td>
<td>3.6</td>
</tr>
<tr>
<td>Forestry</td>
<td>2.5</td>
<td>Electrical equipment</td>
<td>4.4</td>
</tr>
<tr>
<td>Fishery</td>
<td>1.3</td>
<td>Construction</td>
<td>1.9</td>
</tr>
<tr>
<td>Mining</td>
<td>5.6</td>
<td>Electricity</td>
<td>2.8</td>
</tr>
<tr>
<td>Foods</td>
<td>2.5</td>
<td>Gas</td>
<td>2.8</td>
</tr>
<tr>
<td>Other manufacturing products</td>
<td>3.4</td>
<td>Water supply</td>
<td>2.8</td>
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<tr>
<td>Chemical products</td>
<td>3.3</td>
<td>Commerce</td>
<td>1.9</td>
</tr>
<tr>
<td>Petroleum &amp; coal products</td>
<td>2.1</td>
<td>Transport</td>
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<td>Iron &amp; steel</td>
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<td>Metal products</td>
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<td>Services</td>
<td>1.9</td>
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Table 4: Direct damage costs due to flood

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Direct Damage Cost (Billion US dollars)</th>
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<tbody>
<tr>
<td>CSIRO</td>
<td>-0.25</td>
</tr>
<tr>
<td>GFDL</td>
<td>-1.72</td>
</tr>
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<td>MIROC</td>
<td>-1.05</td>
</tr>
<tr>
<td>MRI</td>
<td>-1.87</td>
</tr>
<tr>
<td>S8</td>
<td>-5.00</td>
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