The spatial selection of heterogeneous quality: 
An approach using different demand elasticities

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Abstract

To explain the spatial selection of firms with differentiated quality, this paper incorporates heterogeneous preferences and firm heterogeneity in demand elasticity. We employ a footloose capital model, in which workers have different skill. We find that, when trade becomes freer, high-quality firms agglomerate in the region that accommodates more high-skilled labor, whereas low-quality firms move to the region that hosts more low-skilled labor. If trade freeness is high, the spatial separation of high- and low-quality firms occurs. We further show that the individual welfare of either type of labor in either region is better off when trade becomes freer.

Keywords quality heterogeneity, heterogeneous demand elasticity, spatial selection, trade liberalization, economic geography

JEL classifications F12, F15, F21, F22, R12

1 Introduction

This paper aims to explain the spatial selection of heterogeneous quality firms in terms of differentiated demand elasticities. Many previous firm-level empirical studies have shown that considerable gaps in productivity and quality exist among firms even within narrowly defined product categories. However, in general the firms that produce higher-quality products charge higher prices (Bagwell and Riordan, 1991) and are more likely to enter foreign markets.

For instance, Greenaway et al. (1995) disentangle horizontal and vertical intra-industry trade in the United Kingdom using a “unit value dispersion criterion” and a range of industry characteristics. Similarly, by analyzing product-level import data from the United States, Schott (2004) finds that US increasingly imports the same products from both high- and low-wage countries and that this intra-industry specialization results in intra-industry trade.

Based on the analysis of the import flows of six countries from the rest of the world, Hummels and Skiba (2004) find strong evidence of “shipping the good apples out.” In other words, firms tend to export high-quality goods out. Further, Clerides et al. (1998), Bernard and Jensen (1999), Aw et al. (2000), Bernard et al. (2003), Hummels and Klenow (2005), and Hallak (2006) also offer clear evidence of the increasing importance of
intra-industry trade based on vertically differentiated products. These empirical findings suggest that international specialization has shifted away from industries (e.g., apparel versus machinery) to varieties within industries (e.g., mass-market car versus luxury car). They also imply that spatial selection occurs in terms of product quality.

After the major theoretical breakthroughs associated with the findings presented by Melitz (2003), Helpman et al. (2004) and Bernard et al. (2003) among others, the literature on firm heterogeneity has rapidly expanded. In this literature, firms are always assumed to be horizontally differentiated and heterogeneity among firms is based on their relative productivity only (e.g., Melitz, 2003; Helpman et al., 2004; Melitz and Ottaviano, 2008; Okubo et al., 2010). Consequently, the selection of firms that different levels of productivity in a specific region has been thoroughly examined. However, this heterogeneity in productivity cannot explain the abovementioned empirical findings on the spatial selection of firms because not only the productivity but also the product quality of firms affects their export strategies and location decisions in response to trade liberalization or globalization.

Some previous papers have studied product quality as an output of the R&D process based on a quality-ladders growth model. For example, to analyze the long-run impacts of trade on welfare and growth in economies that present quality heterogeneity, Dinopoulos and Unel (2011) show that higher-quality firms charge higher prices, enjoy higher markups and profits, and engage in exporting. These authors also conclude that trade liberalization reallocates resources from low-quality to high-quality products. However, because they focus on the effects of trade on growth in the presence of quality heterogeneity in two structurally identical countries, the spatial selection of quality differentiated firms was not discussed.

It is natural to consider high-quality products to be produced by higher-skilled workers. Based on this concept, Mori and Turrini (2005) study the role of skill heterogeneity in an uneven spatial distribution of skill and industry. Their presented economy comprises heterogeneously skilled agents who offer horizontally (variety) and vertically (quality) differentiated goods. The incomes of mobile skilled workers come from the rents associated with their skills. To produce goods of higher quality requires workers who have higher skills. In addition to transport costs, Mori and Turrini (2005) also include communication costs in their proposed model in order to capture the quality loss in exporting goods. They show that most skilled workers are attracted to regions that have an abundance of wealth and human capital.

Moreover, in the real world, the substitution elasticity of high-quality goods is lower
than that of low-quality goods. Take the comparison of the markets for mass-market and luxury vehicles as an example. It is natural to imagine that the competition among different mass-market cars (such as Camry, Altima, Fusion, Accord, Corolla, and Cruz) is greater than that among luxury vehicles (e.g., BMW 3 series, Mercedes C class, Audi A4, and Infinity G). Further, producing luxury cars needs more advanced technology and better design requiring skilled labor. Moreover, wealthier people are more likely to be able to afford expensive luxury cars (high-quality goods), whereas ordinary households consume cheaper mass-market ones (low-quality goods). Therefore, firms that produce higher-quality products are able to charge higher prices and enjoy higher markups and profits.

In addition to the literature on the migration of heterogeneously skilled labor, this paper suggests an alternative approach to study the spatial selection of vertically heterogeneous firms across regions in response to trade liberalization. Because labor mobility is low in regions where migration costs are high (e.g., in Europe), we employ a framework based on the footloose capital model (Martin and Rogers, 1995), in which labor is immobile. We assess the production and consumption of quality goods, taking account of the following assumptions: (i) the demand elasticity (or the elasticity of substitution) of high-quality goods is always lower than that of low-quality goods (e.g., Amiti, 1998); (ii) producing goods of higher quality requires workers with higher skill levels, and a high-skilled worker earns a higher wage than a low-skilled worker (e.g., Mori and Turrini, 2005); and (iii) the consumption of high- and low-quality goods is segmented (e.g., Zeng, 2008).

Based on these considerations, we retain the analytical tractability of the model and thus successfully capture the following two main findings. First, high-quality firms more aggressively take advantage of scale economies in response to trade liberalization. Second, the spatial separation of high- and low-quality firms between regions emerges if trade freeness is high enough. In addition to the spatial sorting of differentiated quality firms, we also illustrate the dynamics of capital flows between regions and the impacts on individual welfare. We conclude that trade liberalization is beneficial for individual welfare of either type of labor in either region in economies that present spatial specialization in heterogeneous quality production.

The results of this paper explain the distribution of production for differentiated quality goods, such as the spatial configurations seen in the European automobile industry. Bordenave and Lung (1996) show that luxury and upper-range car assembly are attracted by the center of gravity in southern Germany, whereas assembly of small cars and produc-
tion of generic components are decentralized to southern and eastern European countries. This paper also points out the positive effect of integration on welfare owing to the specialization of product quality.

Compared with related studies of this topic, this paper is more in line with Okubo et al. (2010), who study the location decisions of heterogeneous firms that have different production costs. However, these authors assume that the allocation of capital between heterogeneous firms (i.e., low- and high-cost firms) is exogenously given and that the specification of firm heterogeneity is only based on productivity. By contrast, our model endogenously considers the distribution of capital between vertically differentiated firms and between regions. Therefore, we demonstrate the existence of interior equilibria of both high- and low-quality firms, whereas Okubo et al. (2010) conclude that an interior configuration of low- and high-cost firms cannot be a long-run equilibrium. Moreover, Mori and Turrini (2005) explain the spatial sorting of heterogeneous quality firms based on the different migration patterns of workers that possess differentiated skill levels. However, we assume that labor is immobile. Thus, by contrast, our model explains the spatial sorting of heterogeneous quality firms based on the market segmentation of low- and high-quality products.

The remainder of this paper is organized as follows. The model is specified in Section 2. Then, the analysis of long-run equilibria and spatial selection patterns are characterized in Section 3. In Section 4, we examine the impact of trade liberalization on welfare. Section 5 compares the presented findings with the existing literature. Finally, Section 6 concludes.

2 The model

Consider a world comprising just two regions \((i = 1, 2)\). These two regions have the same physical geographical constraints and technologies; however, their populations and compositions differ. The total mass of workers is normalized to unity. The share of total workers who live in region 1 is denoted by \(\lambda\), while the share of total workers living in region 2 is \((1 - \lambda)\). Among the workers living in region \(i\), a \(\theta_i\) \((\leq 1/2)\) share of them are high-skilled and a \((1 - \theta_i)\) share of them are low-skilled. Each worker owns one unit of labor and one unit of capital, which are both supplied inelastically. Workers (i.e., capital owners) are immobile; however, they can invest their capital across the two regions.

There are two sectors: the agricultural sector \((A\text{-sector})\) and the manufacturing sector. In the manufacturing sector, there are two types of manufacturing firms: \(H\)-firms produc-
ing high-quality (low-elasticity) goods and $L$-firms producing low-quality (high-elasticity) goods. High-skilled workers can work in either the $A$-sector or $H$-firms, while low-skilled workers can work in either the $A$-sector or $L$-firms.

To produce one unit of the homogeneous agricultural good ($A$-good), the $A$-sector employs either one unit of low-skilled labor or $1/\beta$ (where $\beta > 1$) unit of high-skilled labor under perfect competition and constant returns to scale. The $A$-good is chosen as the numéraire and is assumed to be traded costlessly between regions. Thus, the price of the $A$-good $p_i^A$ in region $i$ (where $i = 1, 2$) is $w_i^l = w_i^h / \beta = 1$, where $w_i^l$ is the wage of low-skilled labor and $w_i^h$ is the wage of high-skilled labor. Hereafter, the superscripts $l$ and $h$ represent low- and high-skilled labor, respectively, whereas another pair of superscripts $H$ and $L$ denote high- and low-quality firms, respectively.

In the manufacturing sector, the products comprise a continuum of horizontally differentiated varieties; each variety is produced by a single firm under monopolistic competition and increasing returns to scale. Each manufacturing firm requires one unit of capital as its fixed input to start up. The total mass of manufacturing firms in the world is normalized to unity. Let $k$ denote the share of all manufacturing firms located in region 1 and $(1 - k)$ be the share of all manufacturing firms in region 2. Therefore, $k$ and $(1 - k)$ can also be regarded as the distribution of capital between both regions. Of the manufacturing firms located in region $i$, an $\alpha_i$ share of them are $H$-firms and a $(1 - \alpha_i)$ share of them are $L$-firms. Figure 1 shows the definitions of the variables $k$ and $\alpha_i$.

![Figure 1: Schematic diagram of the distribution of firms](image)

To produce one unit of output, an $L$-firm requires one unit of low-skilled labor, while an $H$-firm needs one unit of high-skilled labor as its marginal input. Each variety of $H$-goods (resp. $L$-goods) can be traded across regions at an iceberg-form transport cost $\tau^H > 1$ (resp. $\tau^L > 1$). This means that $\tau^H$ (resp. $\tau^L$) units of $H$-goods (resp. $L$-goods) are needed to transport one unit of goods from one region to the other.

Let $y_i^l$ (resp. $y_i^h$) denote the total disposable income of each low-skilled (resp. high-
skilled) labor household in region $i$. Since capital is internationally mobile with profit repatriation, their incomes are given by

$$ y_i^l = w_i^l + [r_i^H \alpha_1 + r_i^L (1 - \alpha_1)] k + [r_i^H \alpha_2 + r_i^L (1 - \alpha_2)] (1 - k) $$

$$ = 1 + [r_i^H \alpha_1 + r_i^L (1 - \alpha_1)] k + [r_i^H \alpha_2 + r_i^L (1 - \alpha_2)] (1 - k), \quad (1) $$

$$ y_i^h = w_i^h + [r_i^H \alpha_1 + r_i^L (1 - \alpha_1)] k + [r_i^H \alpha_2 + r_i^L (1 - \alpha_2)] (1 - k) $$

$$ = \beta + [r_i^H \alpha_1 + r_i^L (1 - \alpha_1)] k + [r_i^H \alpha_2 + r_i^L (1 - \alpha_2)] (1 - k), \quad (2) $$

where $r_i^H$ (resp. $r_i^L$) denotes the capital return paid by $H$-firms (resp. $L$-firms) in region $i$, and $i = 1, 2$.

### 2.1 Preferences and demands

Each low-skilled labor household in region $i$ ($i = 1, 2$) consumes $L$-goods and the $A$-good; thus, the utility is

$$ \max_{M_i^l, A_i^l} U_i^l(M_i^l, A_i^l) = (M_i^l)^\mu (A_i^l)^{1-\mu}, \quad \mu < 1 $$

s.t. $y_i^l = P_i^L M_i^l + p_i^A A_i^l$, where $A_i^l$ is each low-skilled labor’s consumption of the homogeneous $A$-good and $M_i^l$ is each low-skilled labor’s consumption of $L$-goods in region $i$:

$$ M_i^l = \left[ n_i^L \left( m_{ii}^L \right)^{\sigma^L \over \sigma^{L-1}} + n_j^L \left( m_{ji}^L \right)^{\sigma^L \over \sigma^{L-1}} \right]^{1 \over \sigma^L}, $$

where $n_i^L$ (resp. $n_j^L$) is the number of $L$-firms in region $i$ (resp. $j$):

$$ n_i^L \equiv k (1 - \alpha_1), \quad n_j^L \equiv (1 - k) (1 - \alpha_2), \quad (3) $$

and $m_{ii}^L$ (resp. $m_{ji}^L$) is the demand of each low-skilled labor household for each $L$-variety produced in region $i$ (resp. $j$) and sold in region $i$. Since firms of the same type in the same region are symmetrical, they charge the same prices and earn the same profits. Parameter $\mu$ represents the share of each low-skilled labor’s total disposable income spent on differentiated manufacturing goods, while $\sigma^L$ denotes the demand elasticity and the elasticity of substitution of $L$-goods. $P_i^L$ denotes the CES price index of the $L$-variety in
region $i$, calculated as
\[
P_i^L = \left[ n_i^L \left( p_{ii}^L \right)^{1-\sigma^L} + n_j^L \left( \tau_i^L p_{jj}^L \right)^{1-\sigma^L} \right]^{\frac{1}{1-\sigma^L}},
\]  
where $p_{ii}^L$ (resp. $p_{jj}^L$) is the unit mill price of the $L$-goods produced and consumed in region $i$ (resp. $j$). Finally, $p_i^A = 1$ is the price of the homogeneous $A$-good.

By maximizing utility subject to budget constraints, we obtain the demand of each low-skilled household for $L$-goods and the $A$-good as follows:
\[
M_i^l = \frac{\mu y_i^l}{P_i^L}, \quad A_i^l = (1-\mu) y_i^l.
\]  

By contrast, each high-skilled labor household in region $i$ ($i = 1, 2$) consumes $H$-goods and the $A$-good; thus, the utility is written as
\[
\max_{M_i^h, A_i^h} U_i^h(M_i^h, A_i^h) = \left( M_i^h \mu (A_i^h)^{1-\mu}, \quad \mu < 1 \right.
\]
\[\text{s.t. } y_i^h = P_i^H M_i^h + p_i^A A_i^h,\]
where $A_i^h$ is each high-skilled labor’s consumption of the homogeneous $A$-good and $M_i^h$ is each high-skilled labor’s consumption of $H$-goods in region $i$:
\[
M_i^h = \left[ n_i^H \left( m_{ii}^H \right)^{\sigma^H \frac{1}{\sigma^H-1}} + n_j^H \left( m_{jj}^H \right)^{\sigma^H \frac{1}{\sigma^H-1}} \right]^{\frac{\sigma^H}{\sigma^H-1}}.
\]
where $n_i^H$ (resp. $n_j^H$) is the number of the $H$-firms in region $i$ (resp. $j$):
\[
n_i^H \equiv k \alpha_1, \quad n_j^H \equiv (1-k) \alpha_2.
\]  
and $m_{ii}^H$ (resp. $m_{jj}^H$) is the demand of each high-skilled labor household for each $H$-variety produced in region $i$ (resp. $j$) and sold in region $i$. $\sigma^H$ denotes the demand elasticity and the elasticity of substitution of $H$-goods, $\mu$ represents the share of each high-skilled labor’s total disposable income spent on $H$-goods, and $P_i^H$ denotes the CES price index of the $H$-variety in region $i$, calculated as
\[
P_i^H = \left[ n_i^H \left( p_{ii}^H \right)^{1-\sigma^H} + n_j^H \left( \tau_i^H p_{jj}^H \right)^{1-\sigma^H} \right]^{\frac{1}{1-\sigma^H}},
\]
where $p_{ii}^H$ (resp. $p_{jj}^H$) is the unit mill price of the $H$-goods produced and consumed in
By maximizing utility subject to budget constraints, we have

\[ M^h_i = \frac{\mu y^h_i}{P^H_i}, \quad A^h_i = (1 - \mu) y^h_i. \]  

(8)

The demands for all varieties are

\[ m^l_{ii} = M^l_i \left( \frac{P^L_{li}}{P^L_{ii}} \right)^{\sigma^L}, \quad m^l_{ji} = M^l_i \left( \frac{P^L_{ji}}{P^L_{ii}} \right)^{\sigma^L}, \]  

(9)

\[ m^h_{ii} = M^h_i \left( \frac{P^H_{li}}{P^H_{ii}} \right)^{\sigma^H}, \quad m^h_{ji} = M^h_i \left( \frac{P^H_{ji}}{P^H_{ii}} \right)^{\sigma^H}, \]  

(10)

where \( p^L_{ji} \) (resp. \( p^H_{ji} \)) is the unit mill price of the \( L \)-goods (resp. \( H \)-goods) produced in region \( j \) and consumed in region \( i \).

Then, we have the indirect utility functions of low-skilled and high-skilled labor in region \( i \), i.e., \( V^l_i \) and \( V^h_i \) as follows:

\[ V^l_i = (1 - \mu)^{1 - \mu} \mu \mu y^l_i (P^L_i)^{\mu}, \]  

(11)

\[ V^h_i = (1 - \mu)^{1 - \mu} \mu \mu y^h_i (P^H_i)^{\mu}. \]  

(12)

### 2.2 Prices and profits

The profit functions of the manufacturing firms are

\[ \pi^l_1 = p^L_{11}(1 - \theta_1)m^l_{11} + p^L_{12}(1 - \lambda)(1 - \theta_2)m^l_{12} \]
\[ - w^l_1[\lambda(1 - \theta_1)m^l_{11} + \tau^l(1 - \lambda)(1 - \theta_2)m^l_{12}] - r^l_1, \]

\[ \pi^l_2 = p^L_{22}(1 - \lambda)(1 - \theta_2)m^l_{22} + p^L_{21}(1 - \lambda)m^l_{21} \]
\[ - w^l_2[\lambda(1 - \lambda)(1 - \theta_2)m^l_{22} + \tau^l\lambda(1 - \theta_1)m^l_{21}] - r^l_2, \]

\[ \pi^H_1 = p^H_{11}\lambda \theta_1 m^h_{11} + p^H_{12}(1 - \lambda)\theta_2 m^h_{12} \]
\[ - w^h_1[\lambda \theta_1 m^h_{11} + \tau^H(1 - \lambda)\theta_2 m^h_{12}] - r^H_1, \]

\[ \pi^H_2 = p^H_{22}(1 - \lambda)\theta_2 m^h_{22} + p^H_{21}\lambda \theta_1 m^h_{21} \]
\[ - w^h_2[(1 - \lambda)\theta_2 m^h_{22} + \tau^H\lambda \theta_1 m^h_{21}] - r^H_2. \]
In the short-run equilibrium, firms maximize profits, and product and factor markets clear. The equilibrium mill prices of $L$-firms and $H$-firms are

\[ P_{11}^L = \frac{\sigma^L}{\sigma - 1} = P_{22}^L, \quad P_{12}^L = \frac{\tau \sigma^L}{\sigma - 1} = P_{21}^L, \]  
(13)

\[ P_{11}^H = \frac{\beta \sigma^H}{\sigma^H - 1} = P_{22}^H, \quad P_{12}^H = \frac{\tau \beta \sigma^H}{\sigma^H - 1} = P_{21}^H. \]  
(14)

For convenience, we assume that $\left(\frac{L}{1\; L}\right) = \left(\frac{H}{1\; H}\right) \phi$, which greatly simplifies later calculations. This can be partly supported by the fact that the elasticity of substitution among differentiated low-quality goods ($\sigma^L$) is larger than that among differentiated high-quality goods ($\sigma^H$). Moreover, the transport costs of high-quality goods ($\tau^H$) are higher than those of low-quality goods ($\tau^L$) because high-quality goods need extra care or insurance fees to guarantee customers against any loss.

Next, using (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (13), and (14), the capital returns paid by the $L$-firms and $H$-firms in both regions can be derived based on the free-entry condition. Thus, we obtain

\[ r_1^L = G_1 \left[ \frac{\lambda (1 - \theta_1)}{k (1 - \alpha_1) + (1 - k) (1 - \alpha_2) \phi} + \frac{\phi (1 - \lambda) (1 - \theta_2)}{k (1 - \alpha_1) + (1 - k) (1 - \alpha_2) \phi} \right], \]  
(15)

\[ r_2^L = G_1 \left[ \frac{\phi \lambda (1 - \theta_1)}{k (1 - \alpha_1) + (1 - k) (1 - \alpha_2) \phi} + \frac{\phi (1 - \lambda) (1 - \theta_2)}{k (1 - \alpha_1) + (1 - k) (1 - \alpha_2) \phi} \right], \]  
(16)

\[ r_1^H = G_2 \left[ \frac{\lambda \theta_1}{k \alpha_1 + (1 - k) \alpha_2 \phi} + \frac{\phi (1 - \lambda) \theta_2}{k \alpha_1 + (1 - k) \alpha_2 \phi} \right], \]  
(17)

\[ r_2^H = G_2 \left[ \frac{\phi \lambda \theta_1}{k \alpha_1 + (1 - k) \alpha_2 \phi} + \frac{\phi (1 - \lambda) \theta_2}{k \alpha_1 + (1 - k) \alpha_2 \phi} \right], \]  
(18)

where

\[ G_1 \equiv \frac{\mu \left\{ (\beta - 1) \mu [\lambda \theta_1 + (1 - \lambda) \theta_2] + \sigma^H \right\}}{(\sigma^L - \mu) \sigma^H - \mu (\sigma^L - \sigma^H) [\lambda \theta_1 + (1 - \lambda) \theta_2]} > 0, \]

\[ G_2 \equiv \frac{\mu \left\{ \beta \sigma^L - (\beta - 1) \mu [1 - \lambda \theta_1 - (1 - \lambda) \theta_2] \right\}}{(\sigma^L - \mu) \sigma^H - \mu (\sigma^L - \sigma^H) [\lambda \theta_1 + (1 - \lambda) \theta_2]} > 0. \]

### 3 Equilibrium and spatial selection

Since capital is mobile, capital seeks the location and type of firms that provide the highest capital returns. We assume that capital owners first choose a region (i.e., region 1 or region 2) and then choose a type of manufacturing firm (i.e., $L$-firms or $H$-firms) in which to invest. In other words, this decision on firm types will not cause a feedback on
the previous decision on investment locations. Therefore, in the first stage, the adjustment
equation of capital between regions is

\[
\frac{dk}{dt} = (r_1 - r_2)k(1 - k),
\]  

(19)

where \( r_1 \) and \( r_2 \) denote the capital returns earned from region 1 and region 2, respectively, given by

\[
r_1 \equiv r_1^H \alpha_1 + r_1^L(1 - \alpha_1),
\]

\[
r_2 \equiv r_2^H \alpha_2 + r_2^L(1 - \alpha_2).
\]

In the second stage, the adjustment equations of capital between types of firms in either region are

\[
\frac{d\alpha_1}{dt} = (r_1^H - r_1^L)\alpha_1(1 - \alpha_1),
\]  

(20)

\[
\frac{d\alpha_2}{dt} = (r_2^H - r_2^L)\alpha_2(1 - \alpha_2).
\]  

(21)

To capture the essence of quality heterogeneity, hereafter we focus on the case in which \( \lambda = 1/2 \) and \( 1/2 \geq \theta_1 > \theta_2 \). We examine the effect of trade liberalization on the spatial equilibrium of firms for regions with identical population sizes but different population compositions.

Let

\[
\phi = \frac{\theta_2}{\theta_1}, \quad \bar{\phi} = \frac{1 - \theta_1}{1 - \theta_2}
\]

Then, \( 0 < \phi < \bar{\phi} < 1 \) holds according to \( 0 < \theta_2 < \theta_1 \leq 1/2 \). To see how the spatial distribution of firms depends on trade freeness, we consider the following three cases:

- **Phase I** (small \( \phi \)): \( \phi \in [0, \phi] \);
- **Phase II** (intermediate \( \phi \)): \( \phi \in [\phi, \bar{\phi}] \);
- **Phase III** (large \( \phi \)): \( \phi \in [\bar{\phi}, 1] \)
3.1 Small \( \phi \)

At an interior equilibrium, \((19), (20), \text{and} \ (21) \) are equal to zero. By using \((15), (16), \ (17), \text{and} \ (18) \), we have

\[
\begin{align*}
\alpha_1(\phi) &= \frac{\theta_1 [2\beta \sigma_L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)] (1 - \phi)}{2G_3 - (\beta - 1) \mu (\theta_1 - \theta_2) (1 + \phi)}, \quad (22) \\
\alpha_2(\phi) &= \frac{\theta_1 [2\beta \sigma_L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)] (1 - \phi)}{2G_4 + (\beta - 1) \mu (\theta_1 - \theta_2) (1 + \phi)}, \quad (23)
\end{align*}
\]

where

\[
G_3 \equiv (1 - \theta_2) (\phi - \phi) \sigma^H + \theta_1 \beta (1 - \phi) \sigma^L, \\
G_4 \equiv (1 - \theta_2) (1 - \phi) \sigma^H + \theta_1 \beta (\phi - \phi) \sigma^L.
\]

It is easy to check that functions \((22), (23), \text{and} \ (24) \) have the following properties:

\[
\begin{align*}
k(0) &= \frac{2 [\beta \sigma_L \theta_1 + \sigma^H (1 - \theta_1)] - (\beta - 1) \mu (\theta_1 - \theta_2)}{2(\theta_1 + \theta_2) (\beta \sigma_L - \sigma^H) + 4\sigma^H} > \frac{1}{2} = \lambda \\
\frac{\partial k(\phi)}{\partial \phi} &= \frac{\theta_1 [2\beta \sigma_L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)]}{(1 - \phi)^2 [(\theta_1 + \theta_2) (\beta \sigma_L - \sigma^H) + 2\sigma^H]} > 0, \\
\alpha_1(0) &= \frac{2 [\beta \sigma_L \theta_1 + \sigma^H (1 - \theta_1)] - (\beta - 1) \mu (\theta_1 - \theta_2)}{\theta_1 [2\beta \sigma_L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)]} > \theta_1, \\
\frac{\partial \alpha_1(\phi)}{\partial \phi} &= \frac{2G_3 - (\beta - 1) \mu (\theta_1 - \theta_2) (1 + \phi)^2}{[2\beta \sigma_L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)] [(\beta - 1) \mu (\theta_1 + \theta_2) + 2\sigma^H]} \\
&> 0, \\
\alpha_2(0) &= \frac{\theta_1 [2\beta \sigma_L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)]}{2[\beta \sigma_L \theta_2 + \sigma^H (1 - \theta_2)] + (\beta - 1) \mu (\theta_1 - \theta_2)} > \theta_2, \\
\frac{\partial \alpha_2(\phi)}{\partial \phi} &= -\frac{\theta_2 [2\beta \sigma_L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)]}{[2G_4 + (\beta - 1) \mu (\theta_1 - \theta_2) (1 + \phi)]^2}
\end{align*}
\]

\(^1\)We exclude two other solutions that violate the assumption that both high- and low-quality goods have to be produced and consumed in the economy.
\[
\times [2\beta \sigma^L - (\beta - 1)\mu(2 - \theta_1 - \theta_2)] \left[(\beta - 1)\mu(\theta_1 + \theta_2) + 2\sigma^H\right] < 0.
\]

Furthermore,
\[
\alpha_2(\phi) = 0,
\]
\[
\alpha_1(\phi) = \frac{(\theta_1 + \theta_2)[(1 - \beta)(2 - \theta_1 - \theta_2)\mu + 2\beta \sigma^L]}{2(1 - \theta_1 - \theta_2)\sigma^H + 2(\theta_1 + \theta_2)\beta \sigma^L + (\theta_1 + \theta_2)(1 - \beta)\mu} \in (0, 1),
\]
\[
k(\phi) = \frac{2(1 - \theta_1 - \theta_2)\sigma^H + (\theta_1 + \theta_2)[(1 - \beta)\mu + 2\beta \sigma^L]}{2[(2 - \theta_1 - \theta_2)\sigma^H + \beta(\theta_1 + \theta_2)\sigma^L]} \in (0, 1).
\]

Therefore, when \(\phi\) increases from 0 to \(\phi\), \(H\)-firms agglomerate in region 1, whereas some \(L\)-firms move to region 2. We have interior equilibria of (22), (23), and (24) in this case.

### 3.2 Intermediate \(\phi\)

For \(\phi > \phi\), we have
\[
r_1^H - r_2^H = \frac{\theta_1 \mu \left[2\beta \sigma^L - (\beta - 1)\mu(2 - \theta_1 - \theta_2)\right]}{2k\alpha_1 \left\{2\sigma^L\sigma^H - \mu[(\theta_1 + \theta_2)\sigma^L + (2 - \theta_1 - \theta_2)\sigma^H]\right\}} \frac{(1 - \phi)(\phi - \phi)}{\phi} > 0.
\]

Thus, all \(H\)-firms agglomerate in region 1 and \(\alpha_2 = 0\).

For \(L\)-forms to disperse into the two regions, \(r_1^L - r_2^L = 0\) and \(r_1^H - r_1^L = 0\) need to be true. Solving them, we obtain\(^2\)
\[
k(\phi) = 1 - \frac{2\sigma^H + (\beta - 1)\mu(\theta_1 + \theta_2)}{2[\beta(\theta_1 + \theta_2)\sigma^L + (2 - \theta_1 - \theta_2)\sigma^H]} \left[(1 - \theta_1) + \frac{\theta_1 - \theta_2}{1 - \phi}\right],
\]
\[
\alpha_1(\phi) = \frac{1}{1 + \frac{2\sigma^H + (\beta - 1)\mu(\theta_1 + \theta_2)}{2\beta \sigma^L - (\beta - 1)\mu(2 - \theta_1 - \theta_2)(\theta_1 + \theta_2)(1 - \phi)}(\phi - \phi)}.
\]

Since
\[
\frac{\partial k(\phi)}{\partial \phi} = -\frac{(\theta_1 - \theta_2) \left[2\sigma^H + (\beta - 1)\mu(\theta_1 + \theta_2)\right]}{2(1 - \phi)^2 \left[(\theta_1 + \theta_2) (\beta \sigma^L - \sigma^H) + 2\sigma^H\right]} < 0,
\]
\[
\frac{\partial \alpha_1(\phi)}{\partial \phi} = \frac{\theta_1 - \theta_2}{(G_3)^2} \left[2\beta \sigma^L - (\beta - 1)\mu(2 - \theta_1 - \theta_2)\right][2\sigma^H + (\beta - 1)\mu(\theta_1 + \theta_2)]
\]

\(^2\)We also exclude the solution of \((k, \alpha_1) = (1, 1)\), which violates the assumption that both high- and low-quality goods have to be produced and consumed in the economy.
\[
\alpha_1(\phi) = 1, \\
> 0,
\]
where
\[
G_5 \equiv (1 - \phi) (\theta_1 + \theta_2) \left[ 2 \beta \sigma^L - (\beta - 1) \mu (2 - \theta_1 - \theta_2) \right] \\
\quad + (\phi - \phi) (1 - \theta_2) \left[ (\beta - 1) \mu (\theta_1 + \theta_2) + 2 \sigma^H \right] \\
> 0,
\]
we know that, in the case of intermediate \( \phi \), region 2 hosts only \( L \)-firms, while region 1 hosts both \( H \)-firms and \( L \)-firms. Furthermore, although the share of \( H \)-firms in region 1 increases in trade freeness, the total share of capital invested in region 1 decreases.

### 3.3 Large \( \phi \)

Next, we consider the case of \( \phi \in (\phi, 1] \), in which we have
\[
r_1^L - r_2^L = \frac{\mu (1 - \theta_2) \left[ 2 \sigma^H + (\beta - 1) \mu (\theta_1 + \theta_2) \right]}{2(1 - k) \{ 2 \sigma^L \sigma^H - \mu \left[ (\theta_1 + \theta_2) \sigma^L + (2 - \theta_1 - \theta_2) \sigma^H \right] \} \phi} \\
< 0.
\]
Therefore, \( r_1^L \) is always smaller than \( r_2^L \) and thus no \( L \)-firms are located in region 1. Thus, in Phase III, \( H \)-firms and \( L \)-firms separate into different regions: all \( H \)-firms are located in region 1 with more high-skilled labor, while all \( L \)-firms agglomerate in region 2 with more low-skilled labor.

For both types of firms to coexist, there should be no difference in the capital returns generated by each region (i.e., \( r_1^L - r_2^L = 0 \)). We then obtain the firm share in region 1:
\[
k(\phi) = \frac{(\theta_1 + \theta_2) [2 \beta \sigma^L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)]}{2(\theta_1 + \theta_2) (\beta \sigma^L - \sigma^H) + 4 \sigma^H}.
\]

In the case of \( \sigma^L > \mu (2 - \theta_1 - \theta_2) \), Region 1 hosts more firms (i.e., \( k(\phi) > \lambda = 1/2 \)) only if
\[
\beta > \frac{(2 - \theta_1 - \theta_2) \left[ \sigma^H - \mu (\theta_1 + \theta_2) \right]}{(\theta_1 + \theta_2) \left[ \sigma^L - \mu (2 - \theta_1 - \theta_2) \right]}.
\]
Since \( \mu (2 - \theta_1 - \theta_2) \) is always less than 2, we know that in usual cases (e.g., \( \sigma^L \geq 2 \)), region 1, whose population of wealthier households (i.e., high-skilled workers) is larger,
will not necessarily host more capital or firms. The gap between the wages of high- and low-skilled labor has to be big enough to make region 1 more attractive than region 2.

By contrast, if \( \sigma^L < \mu (2 - \theta_1 - \theta_2) \), then \( \sigma^L \) is small enough that the difference between \( \sigma^L \) and \( \sigma^H \) becomes unimportant. In this case, region 1, the wealthier region, always hosts more capital.

Firm distribution is illustrated in Figure 2 and summarized in Propositions 1 and 2.

![Figure 2: Spatial selection of high- and low-quality firms](image-url)
Proposition 1  When $\phi$ increases from 0 to 1, H-firms fully agglomerate in region 1 (i.e., $\alpha_2 = 0$) earlier than L-firms fully agglomerate in region 2 (i.e., $\alpha_1 = 1$).

Proposition 2  The equilibrium location of H-firms and L-firms is characterized by:

(i) In autarky, the share of H-firms in each region is larger than its skilled labor share, and the share of H-firms in region 1 is larger than that in region 2.

(ii) For $\phi \in (0, \phi)$, the share of H-firms in region 1 rises, whereas the share of H-firms in region 2 falls in trade freeness. H-firms and L-firms coexist in both regions.

(iii) For $\phi \in [\phi, \phi]$, no H-firms are located in region 2, whereas H-firms and L-firms coexist in region 1.

(iv) For $\phi \in [\phi, 1]$, H-firms and L-firms separate into the two regions. Only H-firms exist in region 1, whereas only L-firms are located in region 2.

When the two regions have identical population sizes and when trade becomes freer, high-quality firms agglomerate in the region that accommodates more high-skilled labor, whereas low-quality firms move to the region that hosts more low-skilled labor. Because firms that produce higher-quality products are able to charge higher prices and enjoy higher markups and profits, high-quality firms more aggressively take advantage of scale economies in response to trade liberalization. As a result, the full agglomeration of high-quality firms occurs earlier than that of low-quality firms. If trade freeness is high enough, the spatial separation of high- and low-quality firms between the two regions emerges. We present the existence of interior equilibria of both high- and low-quality firms, whereas Okubo et al. (2010) conclude that an interior configuration of low- and high-cost firms cannot be a long-run equilibrium. This is because our model endogenizes the distribution of capital between vertically differentiated firms and between regions. By contrast, Okubo et al. (2010) assume that the allocation of capital between low- and high-cost firms is exogenously given and thus that the profit gap for high-cost firms between countries is always strictly smaller than the profit gap for low-cost firms between countries. Our model explains the specification of heterogeneous quality firms from the home market effect based on the market segmentation between low- and high-quality products.

Next, Proposition 3, illustrated in Figure 3, demonstrates how capital is allocated between regions when trade becomes freer.

Proposition 3  The equilibrium distribution of capital is characterized by:

(i) In autarky, the region that has more skilled labor hosts more capital.

(ii) When $\phi$ is small, more-than-proportionate capital moves into the region in which more skilled workers reside.
(iii) When $\phi$ is intermediate, the share of capital invested in the region that has more skilled labor decreases in trade freeness.

(iv) When $\phi$ is large, the distribution of capital remains constant. Whether the share of capital invested in the region in which more skilled labor resides is larger than $1/2$ depends on the productivity differential $\beta$.

In autarky and Phase I, this model shows similar results to the asymmetrical footloose capital model of Baldwin et al. (2003). Indeed, the total market size (disposable income) in region 1 is also larger because

$$Y_1 = \frac{1}{2}[(\beta \theta_1 + (1 - \theta_1)) + r^*], \quad Y_2 = \frac{1}{2}[(\beta \theta_2 + (1 - \theta_2)) + r^*],$$

$$Y_1 - Y_2 = \frac{1}{2}[(\beta - 1)(\theta_1 - \theta_2)] > 0,$$

where $Y_i$ denotes the total disposable income of region $i$ and $r^*$ is the equilibrium capital returns. However, it is noteworthy that in this study household consumption is segmented and that the larger market for each type of firm is located in different regions. Thus, using (3), (6), (22), (23), and (24), we know that in autarky region 1 hosts more capital because

$$n_1^H(0) = \frac{\theta_1 \left[ 2\beta \sigma^L - (\beta - 1) \mu (2 - \theta_1 - \theta_2) \right]}{2(\theta_1 + \theta_2) (\beta \sigma^L - \sigma^H) + 4\sigma^H},$$
As a result of the home market effect, when trade is freer, the number of \( H \)-firms in region 1 increases, while that of \( L \)-firms decreases. In region 2, the opposite tendency emerges. Finally, the net effect of trade liberalization on the equilibrium share of capital invested in region 1 is positive (i.e., \( \partial k(\phi) / \partial \phi > 0 \)) because in region 1 the loss of fewer \( L \)-firms is compensated by the gain of more \( H \)-firms. According to (3), (6), (22), (23), and (24), the change in firm numbers in region 1 is given as

\[
\frac{\partial n^H_1(\phi)}{\partial \phi} = \frac{(\theta_1 - \theta_2) \left[ 2\beta \sigma^L - (\beta - 1) \mu (2 - \theta_1 - \theta_2) \right]}{2 (1 - \phi)^2 \left[ (\theta_1 + \theta_2) (\beta \sigma^L - \sigma^H) + 2\sigma^H \right]} > 0,
\]

\[
\frac{\partial n^L_1(\phi)}{\partial \phi} = -\frac{(\theta_1 - \theta_2) \left[ (\beta - 1) \mu (\theta_1 + \theta_2) + 2\sigma^H \right]}{2 (1 - \phi)^2 \left[ (\theta_1 + \theta_2) (\beta \sigma^L - \sigma^H) + 2\sigma^H \right]} < 0,
\]

\[
\frac{\partial n^H_1(\phi)}{\partial \phi} + \frac{\partial n^L_1(\phi)}{\partial \phi} = \frac{\partial k(\phi)}{\partial \phi} > 0.
\]

However, when all \( H \)-firms move to region 1 (i.e., no \( H \)-firm stays in region 2) in Phase II, the number of \( H \)-firms in each region remains constant (i.e., \( \frac{\partial n^H_1(\phi)}{\partial \phi} = \frac{\partial n^H_2(\phi)}{\partial \phi} = 0 \)). Consequently, the change in equilibrium capital distribution directly reflects the movement of \( L \)-firms in progress:

\[
\frac{\partial k(\phi)}{\partial \phi} = \frac{\partial n^L_1(\phi)}{\partial \phi} = -\frac{(\theta_1 - \theta_2) \left[ (\beta - 1) \mu (\theta_1 + \theta_2) + 2\sigma^H \right]}{2 (1 - \phi)^2 \left[ (\theta_1 + \theta_2) (\beta \sigma^L - \sigma^H) + 2\sigma^H \right]} < 0.
\]

Then, after all \( L \)-firms have also clustered in region 2 (in Phase III), \( H \)-firms and \( L \)-firms are geographically separated from each other. Perfect spatial selection thus occurs and persists. The equilibrium allocation of capital is also fixed. Proposition 4 summarizes the results about capital flows between regions, which is illustrated in Figure 4.

**Proposition 4** The \( H \)- and \( L \)-industry sizes in each region are characterized by:

(i) In autarky, region 1 hosts more firms.

(ii) If \( \phi \) is small, in region 1 (resp. region 2), the number of \( H \)-firms (resp. \( L \)-firms)
increases, whereas that of L-firms (resp. H-firms) decreases in trade freeness.

(iii) If \( \phi \) is intermediate, when trade becomes freer, the number of H-firms in each region remains constant, while the number of L-firms decreases in region 1 and increases in region 2.

(iv) If \( \phi \) is large, the industry size of either type of firms in either region is constant.

Figure 4: Equilibrium industry sizes of both types of firms in each region

When trade freeness is low (i.e., in Phase I), it is clear that the share of capital in region 1 increases in trade freeness. However, when trade freeness takes an intermediate value (i.e., in Phase II), the share of capital in region 1 starts to decline. Proposition 4 explains that the inward move of high-quality firms ends, whereas the outward escape
of low-quality firms continues. When trade freeness is large enough (i.e., in Phase III), high- and low-quality firms are geographically separated from each other. Hence, both the movements of high- and low-quality firms toward region 1 and region 2 stop. Therefore, the industry size of either type of firm in the respective region is fixed.

4 Welfare analysis

In this section, we examine how the spatial selection of quality goods influences individual welfare during the process of trade liberalization. By substituting (1), (2), (3), (4), (6), (7), (13), and (14) into (11) and (12), we obtain the general expressions of the welfare levels of the two types of households living in these two regions as follows:

\[
V_h^1(\phi) = [k\alpha_1 + \phi (1-k) \alpha_2] \frac{\sigma^\mu}{\beta^\sigma H - 1} (1 - \mu)^{1-\mu} \mu^\mu \left( \frac{\sigma^H - 1}{\beta \sigma^H} \right)^\mu \\
\times [2 \beta \sigma L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)] \\
\times \left[ \sigma^H [2 \beta \sigma L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)] + (2 - \theta_1 - \theta_2) \sigma^H \right],
\]

\[
V_h^2(\phi) = \left[ \phi k \alpha_1 + (1-k) \alpha_2 \right] \frac{\sigma^\mu}{\beta^\sigma H - 1} (1 - \mu)^{1-\mu} \mu^\mu \left( \frac{\sigma^H - 1}{\beta \sigma^H} \right)^\mu \\
\times [2 \beta \sigma L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)] \\
\times \left[ \sigma^H [2 \beta \sigma L - (\beta - 1) \mu (2 - \theta_1 - \theta_2)] + (2 - \theta_1 - \theta_2) \sigma^H \right],
\]

\[
V_l^1(\phi) = [k (1 - \alpha_1) + \phi (1-k) (1 - \alpha_2)] \frac{\sigma^\mu}{\beta^\sigma L - 1} (1 - \mu)^{1-\mu} \mu^\mu \left( \frac{\sigma^L - 1}{\beta \sigma^L} \right)^\mu \\
\times \left[ \sigma^L [\beta - 1) \mu (\theta_1 + \theta_2)] + 2 \sigma^H \right] \\
\times \left[ \sigma^L [\beta - 1) \mu (\theta_1 + \theta_2)] + 2 \sigma^H \right],
\]

\[
V_l^2(\phi) = [\phi k (1 - \alpha_1) + (1-k) (1 - \alpha_2)] \frac{\sigma^\mu}{\beta^\sigma L - 1} (1 - \mu)^{1-\mu} \mu^\mu \left( \frac{\sigma^L - 1}{\beta \sigma^L} \right)^\mu \\
\times \left[ \sigma^L [\beta - 1) \mu (\theta_1 + \theta_2)] + 2 \sigma^H \right] \\
\times \left[ \sigma^L [\beta - 1) \mu (\theta_1 + \theta_2)] + 2 \sigma^H \right].
\]

In autarky, the welfare levels of the two types of households living in these two regions are written as

\[
V_h^1(0) = \frac{G_6(\theta_1)^{\sigma^\mu_{H-1}}}{\beta^\mu (\sigma^H - 1)^{1-\mu} (\sigma^H)^{-1+\mu}}, \quad V_l^1(0) = \frac{G_7(1 - \theta_1)^{\sigma^\mu_{L-1}}}{(\sigma^L - 1)^{1-\mu} (\sigma^L)^{-1+\mu}},
\]

\[
V_h^2(0) = \frac{G_6(\theta_2)^{\sigma^\mu_{H-1}}}{\beta^\mu (\sigma^H - 1)^{1-\mu} (\sigma^H)^{-1+\mu}}, \quad V_l^2(0) = \frac{G_7(1 - \theta_2)^{\sigma^\mu_{L-1}}}{(\sigma^L - 1)^{1-\mu} (\sigma^L)^{-1+\mu}}.
\]
where

\[ G_6 \equiv \frac{\mu^{\frac{1}{1+\mu}} (1 - \mu)^{1-\mu}}{2\sigma^H \sigma^L - \mu [(\theta_1 + \theta_2) \sigma^L + (2 - \theta_1 - \theta_2) \sigma^H]} \times \left[ 2\beta \sigma^L - (\beta - 1) \mu (2 - \theta_1 - \theta_2) \right]^{\frac{\mu}{\mu - 1}} \times \left[ 2 (\theta_1 + \theta_2) (\beta \sigma^L - \sigma^H) + 4\sigma^H \right]^{\frac{\mu}{\mu - 1}}, \]

\[ G_7 \equiv \frac{\mu^{\frac{1}{1+\mu}} (1 - \mu)^{1-\mu}}{2\sigma^H \sigma^L - \mu [(\theta_1 + \theta_2) \sigma^L + (2 - \theta_1 - \theta_2) \sigma^H]} \times \left[ 2\sigma^H + (\beta - 1) \mu (\theta_1 + \theta_2) \right]^{\frac{\mu}{\mu - 1}} \times \left[ 2 (\theta_1 + \theta_2) (\beta \sigma^L - \sigma^H) + 4\sigma^H \right]^{\frac{\mu}{\mu - 1}}. \]

Similarly, we analyze the changes in welfare in Phase I, Phase II, and Phase III according to the equilibrium values of \( k(\phi), \alpha_1(\phi), \) and \( \alpha_2(\phi). \)

(i) Phase I: \( \phi \in [0, \bar{\phi}) \)

\[ \frac{\partial V^h_1}{\partial \phi} = G_6 \frac{\mu (\theta_1) \sigma^L \frac{\mu}{\mu - 1} (1 + \phi) \sigma^L^{\frac{\mu}{\mu - 1}}}{\beta^L (\sigma^L - 1)^{1-\mu} (\sigma^H)^{-1+\mu}} > 0, \]

\[ \frac{\partial V^h_2}{\partial \phi} = G_6 \frac{\mu (\theta_2) \sigma^L \frac{\mu}{\mu - 1} (1 + \phi) \sigma^L^{\frac{\mu}{\mu - 1}}}{\beta^L (\sigma^L - 1)^{1-\mu} (\sigma^H)^{-1+\mu}} > 0, \]

\[ \frac{\partial V^l_1}{\partial \phi} = G_7 \frac{\mu (1 - \theta_1) \sigma^L \frac{\mu}{\mu - 1} (1 + \phi) \sigma^L^{\frac{\mu}{\mu - 1}}}{(\sigma^L - 1)^{1-\mu} (\sigma^L)^{-1+\mu}} > 0, \]

\[ \frac{\partial V^l_2}{\partial \phi} = G_7 \frac{\mu (1 - \theta_2) \sigma^L \frac{\mu}{\mu - 1} (1 + \phi) \sigma^L^{\frac{\mu}{\mu - 1}}}{(\sigma^L - 1)^{1-\mu} (\sigma^L)^{-1+\mu}} > 0, \]

(ii) Phase II: \( \phi \in [\bar{\phi}, \bar{\phi}) \)

\[ \frac{\partial V^h_1}{\partial \phi} = 0, \]

\[ \frac{\partial V^h_2}{\partial \phi} = G_6 \frac{\mu (\theta_1 + \theta_2) \sigma^L \frac{\mu}{\mu - 1} \phi \sigma^L^{\frac{\mu}{\mu - 1}}}{\beta^L (\sigma^L - 1)^{1-\mu} (\sigma^H)^{-1+\mu}} > 0, \]

\[ \frac{\partial V^l_1}{\partial \phi} = G_7 \frac{\mu (1 - \theta_1) \sigma^L \frac{\mu}{\mu - 1} (1 + \phi) \sigma^L^{\frac{\mu}{\mu - 1}}}{(\sigma^L - 1)^{1-\mu} (\sigma^L)^{-1+\mu}} > 0, \]

\[ \frac{\partial V^l_2}{\partial \phi} = G_7 \frac{\mu (1 - \theta_2) \sigma^L \frac{\mu}{\mu - 1} (1 + \phi) \sigma^L^{\frac{\mu}{\mu - 1}}}{(\sigma^L - 1)^{1-\mu} (\sigma^L)^{-1+\mu}} > 0, \]
(iii) Phase III: $\phi \in [\bar{\phi}, 1]$

\[
\frac{\partial V_1^h (\phi)}{\partial \phi} = \frac{\partial V_1^l (\phi)}{\partial \phi} = 0, \\
\frac{\partial V_2^h (\phi)}{\partial \phi} = G_6 \frac{\mu (\theta_1 + \theta_2) \sigma_{H-1}^{-\mu} \phi^{\sigma_{H-1}^{-\mu} - 1}}{\beta^\mu (\sigma_H - 1)^{1-\mu} (\sigma_H)^{-1+\mu}} > 0, \\
\frac{\partial V_2^l (\phi)}{\partial \phi} = G_7 \frac{\mu (2 - \theta_1 - \theta_2) \sigma_{L-1}^{-\mu} \phi^{\sigma_{L-1}^{-\mu} - 1}}{(\sigma_L - 1)^{1-\mu} (\sigma_L)^{-1+\mu}} > 0.
\]

These are summarized in the following Proposition 5.

**Proposition 5** The individual welfare of either type of labor in either region improves when trade becomes freer.

In autarky (i.e., $\phi = 0$), the welfare of high-skilled labor living in region 1 is higher than that of high-skilled labor in region 2. However, the welfare of low-skilled labor living in region 1 is lower than that of low-skilled labor in region 2 (i.e., $V_1^h (0) > V_2^h (0)$ and $V_2^l (0) > V_1^l (0)$). If $\phi \in [0, \bar{\phi})$ (i.e., Phase I), the welfare levels of high-skilled and low-skilled labor living in each region are all better off when trade becomes freer (i.e., $\partial V_1^h (\phi) / \partial \phi > 0$, $\partial V_2^h (\phi) / \partial \phi > 0$, $\partial V_1^l (\phi) / \partial \phi > 0$, and $\partial V_2^l (\phi) / \partial \phi > 0$). In Phase II, $\phi \in [\bar{\phi}, \bar{\phi})$, the welfare of the high-skilled labor in region 1 remains constant, while the welfare levels of the other groups of labor are all better off when trade becomes freer (i.e., $\partial V_1^h (\phi) / \partial \phi = 0$, $\partial V_2^h (\phi) / \partial \phi > 0$, $\partial V_1^l (\phi) / \partial \phi > 0$, and $\partial V_2^l (\phi) / \partial \phi > 0$). This is because all $H$-firms have fully agglomerated in region 1 and thus the high-skilled labor in region 1 no longer benefits by the decreasing price index caused by the inward movement of $H$-firms. Finally, when $\phi$ is large (i.e., $\phi \in [\bar{\phi}, 1]$), the high-skilled labor in region 1 and the low-skilled labor in region 2 have obtained their best welfare levels, which results from the full agglomeration of $H$-firms in region 1 and $L$-firms in region 2, respectively. Thus, $\partial V_1^h (\phi) / \partial \phi = \partial V_2^l (\phi) / \partial \phi = 0$. By contrast, the high-skilled labor in region 2 and low-skilled labor in region 1 keep enjoying the advantage of lower transport costs for importing goods (i.e., $\partial V_2^h (\phi) / \partial \phi > 0$ and $\partial V_1^l (\phi) / \partial \phi > 0$), since they have to consume manufacturing goods by importing. To summarize, trade liberalization is beneficial for the individual welfare of either type of labor in either region in the case of spatial specialization in heterogeneous quality production.

In general, measuring the overall effect of decreasing trade costs on welfare is a formidable task. Helpman and Krugman (1985, p.179) declare that it is difficult to generalize that countries gain from trade in the differentiated products model, although their
arguments are limited to the cases of free trade and autarky. Takahashi et al. (2012) obtain a clear result for the smaller region, while the result for the larger region is ambiguous because in their model firms produce horizontally differentiated goods by employing homogeneous labor. In other words, there is no specialization of product quality.

5 Discussion

Since we apply differentiated demand elasticities to examine the trade patterns of quality goods, it is worth comparing our results with the findings of the existing literature on demand elasticity. Although our framework comprises two identical countries, both high-quality (low-elasticity) and low-quality (high-elasticity) firms display the market size effect in our setup. In other words, regardless of high-quality (low-elasticity) or low-quality (high-elasticity) production, firms tend to agglomerate in bigger markets. However, high-quality (low-elasticity) firms, which enjoy higher markups, show stronger incentives to agglomerate in bigger markets when the economy is more integrated. This finding is consistent with Baldwin et al.’s (2003, p.79) claim that “the smaller the elasticity, the smaller is the relative weight of market crowding.” A similar argument was also put forward by Fujita et al. (1999, p.75). Low-elasticity firms are less concerned about the locations of their competitors. Thus, the lower the elasticity of demand the stronger is the pull of firms toward clustering in the country that hosts more of their consumers.

Amiti (1998) also studies the trade patterns of goods that have different demand elasticities using a pair of countries with different population sizes and suggests that the larger country is a net exporter of high-elasticity goods, whereas the smaller country is a net exporter of low-elasticity goods at an integration level close to free trade. This means that the smaller country specializes in the production of high-quality (low-elasticity) goods. This outcome contradicts the analysis of Fujita et al. (1999) and Baldwin et al. (2003), who consider that high-quality (low-elasticity) firms are located in the larger country, which is our conclusion.

The reason for the different results lies in the production functions. Amiti (1998) assumes that firms use a Leontief composite of capital and labor as their fixed and marginal inputs. By contrast, we assume that firms use capital as their fixed input and labor as their marginal input.

On one hand, when firms use a Leontief composite of capital and labor as their fixed and marginal inputs to produce, as Amiti (1998) points out, the tension between the market access effect and the production effect determines the pattern of specialization
and trade. At integration levels close to free trade, where wage differences between the two countries are smaller, the market access effect rather dominates (Amiti, 1998, p.247). Moreover, Amiti (1998) also shows that the market access effect of high-elasticity firms is higher than that of low-elasticity firms. In other words, high-elasticity firms display the home market effect, while low-elasticity firms do not. Thus, relatively more high-elasticity firms are located in the larger country.

On the other hand, if firms use capital as their fixed input and labor as their marginal input to produce, as assumed in the footloose capital model, low-elasticity firms show stronger agglomeration forces than do high-elasticity firms. Therefore, at an integration level close to free trade, low-elasticity firms, which enjoy higher markups, agglomerate in the larger country. When the elasticity of substitution reaches infinity, which can be regarded as the agricultural good in the footloose capital model, our result is also consistent with the outcome of the footloose capital model.

Further, by using two manufacturing industries as the context of this paper, this study is also consistent with Krugman’s (1980) two-industry model. Thus, each country will be a net exporter in the industry for whichever products have the higher demand. However, this study endogenizes the size of each industry using mobile capital as the fixed production input and differentiates these two manufacturing industries in terms of the elasticity of substitution.

6 Concluding remarks

Based on a setting of segmented preferences and firm heterogeneity in demand elasticity, this paper demonstrates an evolution of the spatial sorting of high- and low-quality firms in order to explain intra-industry quality trade. In two regions that have identical population sizes, high-quality firms agglomerate in the region that accommodates more high-skilled labor, whereas low-quality firms move to the region that hosts more low-skilled labor in response to trade liberalization. However, because high-quality firms charge higher prices and enjoy higher markups and profits, they more aggressively take advantage of scale economies in response to trade liberalization. The partial agglomeration of both types of firms prevails in both regions until high-quality firms fully agglomerate. After the full agglomeration of high-quality firms, low-quality firms keep moving to the other region and they finally cluster along with decreasing transport costs. Therefore, the spatial separation of high- and low-quality firms between regions occurs.

With respect to the equilibrium allocation of worldwide capital, the freer the trade the
more capital is located in the region that has more high-skilled labor until all high-quality firms have clustered. In turn, along with this increasing trade freeness, the region that hosts all high-quality firms witnesses an outflow of capital since low-quality firms in that region keep being sorted. As the spatial separation of high- and low-quality firms between regions emerges, the distribution of capital between regions becomes independent of trade freeness. Finally, the individual welfare of either type of labor in either region is better off when trade becomes freer. This is because all households benefit from the advantage of industry agglomeration and the subsequent lower prices.

References


