Migration as Driving Force for the Dynamics of Housing Rent

Yuri Yegorov, University of Vienna

Started: 7 October 2010 This draft: 14 February 2011

Abstract

The goal of this paper is to link models of urban formations (urban studies) with models of housing rent. While housing market models are essentially static, model of urban formation is presented in dynamic set up. The driving force for dynamics comes from migration theory and includes chain migration model. An important concept of integral land rent of a city is introduced and the question of its ownership is addressed. The models with different types of rationality and migration are studied. In the case of rational market, the convergence may take place to either optimal city size (without chain migration), or towards too large city (with chain migration). It is shown that chain migration reinforces the price bubble effect in housing market.

1 Introduction

In economics and social studies, we often observe collective phenomena that produce externality effects and that cannot be described by classical market theory. On one hand, we observe mass migration to urban agglomerations despite congestion effect. This is a typical disequilibrium phenomenon (if we depart from the equilibrium distribution of population across different cities to maximize utility of each agent). On the other hand, we observe such phenomena as housing bubbles that can lead to huge macroeconomic consequences (like financial crisis of 2008 driven by explosion of the US bubble, or contemporary recession of Spain also driven by housing bubble).
To model these effects it is important to depart from migration model, where dynamics is driven by the difference in potentials of attraction of different locations (dynamic law being similar to one derived by Omm for electric current). This process may start by some shock. For example, adopting the law on free migration perturbs old equilibrium where migration has been impossible even with the difference in potentials. Another shock may be related to new technology of massive inflow of capital to a particular city that makes labour there relatively scarce and wages high, motivating migration flow to this city. In fact, industrialization process, with rural-urban migration, has been driven by this mechanism.

It is also important to recall the spatial structure of a city in order to introduce heterogeneous land rent in a model. The easiest way to do so comes from urban economics and its CBD model. Then it is possible to decompose the price of real estate into construction costs and land rent. While such model is essentially static, it can be used as a “working horse” for dynamic model involving migration flow. Adding construction sector and housing market can close the model set up. Thus, the focus of this paper will be on elaboration of transition dynamics of a city to a new equilibrium after shock involving the behaviour of migrants, constructors and housing market.

1.1 Literature

Real phenomena are often complex and cannot be referred to only one trend in economic literature. In the case of this paper, there are several scientific backgrounds. The first trend in literature is linked to urban economics (see Alonso, 1964; Fujita, 1989). These models are essentially static. They use microeconomic assumptions in application to spatial structure of a city. Here the concept of land rent is used as a kind of potential for location that drives land price for urban use.

There is also literature on housing markets. The main assumption that is used here is durability of housing. Following Lopez Garcia (1992), price of housing (as a consumption and investment good) reflects the characteristics of demand and supply through the supply of land, income levels, interest rate, tax laws, etc. These factors put price index into dynamic equilibrium, but it does not explain the differences of house prices across locations. This is the first challenge. Traditionally the schools of housing market analysis use
either this dynamic approach for price index, or compare spatial structure of prices in static format.

There are many factors that influence attractiveness of cities. While location of capital creates jobs and raise wages through labour demand, attractiveness also depends on life quality and access to public goods. Trullen (2001) adds such factor as "knowledge formation" to driving urban forces.

How is it possible to capture spatial heterogeneity and dynamics in one model? The present paper does it but aggregate index of land rent of a city influences housing price index in a dynamic set up.

**Paper structure.** Several blocks and model derivation will be considered in section 2. Section 3 deals with the analysis of the model. Section 4 considers policy implications. Section 5 concludes.

## 2 Model Derivation

### 2.1 Agglomeration and the Dynamics of Land Rent

Suppose that agglomeration force emerges at some moment and leads to growth of a city (see section 2). Urban and housing economics (see CBD model) suggests that the equilibrium price of housing consists of construction price and location rent. However, these theories are essentially static. Urban dynamics is driven by temporal imbalance of centrifugal and centripetal forces. The cause of such disequilibrium may be some social innovation making city more attractive. For example, after 1991 Moscow has gained more attraction in comparison to other Russian cities due to new distribution of financial capital and political power. Huge financial flows to Moscow (including practically all FDI) caused higher wages (via positive jump in coefficient $\xi$ in (4)), and thus attraction for migration. This caused a shift from old equilibrium city size $N^*$ to a larger one, $N^{**}$. The new border of the city expanded, and land rent in its center has grown. This caused the price of all existing real estate to rise, and also has created the demand for building new real estate between old and new city borders$^1$. First of all, the prices

---

$^1$Here we ignore the development of some central areas, that give substantial premium in land rent for developer
of old housing went up, and the owners were satisfied. Second, there was a substantial construction boom. Third, the process of transition to new equilibrium caused a positive trend in housing price (via the change of land rent) and some speculators took their position. The result was price overshooting with substantial price crash after the world financial crisis of 2008.

Price appreciation due to the growth of land rent is a complex collective phenomenon, where different groups of people are affected. The old city dwellers get not only appreciation of their housing, but also utility loss from growing congestion and higher competition with migrants on labour markets. But the most interesting question is: to whom newly created rent should belong? Note that in this process we have quite complicated accounting. It includes utility loss of other Russian cities (sending migrants to Moscow). Thus, they might have a right for some compensation. The additional land rent created in Moscow city thus belongs not only to citizens but also to other Russian people (via compensation for negative externality effect). And clearly it cannot belong to politicians or construction companies. In an "hon-est" framework (that only can be created by corresponding laws) construction should take place in a competitive framework, slots for construction should be sold at open auctions, and the revenue should be distributed across citizens, after payment of some tax to state budget (that later can be used for subsidies of regions-losers).

2.2 Behavior of migrants

The dynamics of migration here is based on the ideas from (Helmenstein & Yegorov, 2000) and (Mascarilla & Yegorov, 2005). It is assumed that centrifugal and centripetal forces may be in balance, giving a city of optimal size. Let this equilibrium be described by city size $N_0$ and obtained utility in it, $U(N_0) \equiv U_0$. In general, utility in a city, $U(N, \xi)$, depends also on some external factor $\xi$ (like capital, or technology in it) that is subject to (random) shocks. It is assumed that initial equilibrium has been perturbed by such shock. For example, the variable (one can think of it as proxy for wages) shifted from $\xi_0$ to $\xi = \xi_0 + \Delta$. This gives rise to transitional process of migration to this city.

The paper (Helmenstein & Yegorov, 2000) models the process of chain migration, assuming that single migrant are driven by the differences in ex-
pected utility obtained from living in different locations, while chain migrants are "pulled" by those who has already migrated. If we assume zero natural population growth (quite typical nowadays), then the dynamics of population in a city, \( \dot{N} \), is equal to the dynamics of migration. The migration equation has two parts, corresponding to single and chain components:

\[
\dot{N} = a(U(N) - U_0) + b(N - N_0).
\]  

(1)

Here it is assumed that before transition process there were no chain migrants. Then the term \( N - N_0 \) is the total population of migrants, while \( b \) is the strength of "pulling" force.

### 2.3 Equilibrium structure of a city

Assume classical CBD (central business district) model (see Fujita, 1989), with radially symmetric city, surrounded with agricultural land, with exogenously given land rent \( R_a \). For simplicity, it is assumed that all citizens have standard 1-story houses of equal size. The construction market is assumed to be perfectly competitive, and thus the cost of constructing a standard house equals to its "no-location" price, \( H \). All of them work in city center and have to commute distance \( r \) to their work, with linear transport cost \( \tau \) per unit of distance. In equilibrium, all of them get the same utility. The equality of costs for a citizen living in center and at some distance gives (after subtracting \( H \) and equal benefits):

\[
R(r) = R^* - \tau r.
\]  

(2)

Let \( r^* \) denotes the city border. The equilibrium in land market there requires equality of land prices for urban and agricultural use: \( R(r^*) = R_a \).

Next formula comes from geometry: the city of radius \( r^* \) has the area \( S = \pi (r^*)^2 \), where \( \pi = 3.14... \). Let every citizen uses the same territory \( l_1 \) (assumption of constant population density across city), including one for public use (roads, parks, etc). This brings the link between city radius and population, \( N = c(r^*)^2 \), from where \( r^* = \sqrt{N/c} \). Here \( c = \pi / l_1 \). Putting all formulae of this subsection together, we get:

\[
R(r) = R_a + \tau \sqrt{N/c} - \tau r.
\]  

(3)
Utility of citizen. Assume that utility is the difference between benefits (assumed to be $B\xi$, where $\xi$ is some factor of attractiveness that can vary via jumps, and $B$ is constant coefficient) and costs (that include housing price and commuting costs). Due to indifference across locations we can calculate this only for citizen in center. Then,

$$U(N, \xi) = B\xi - H - R_a - \tau\sqrt{N/c}. \quad (4)$$

Here it was assumed that benefits are exogenous and do not depend on city size. With endogenous benefits, there exists some optimal city size (see, for example, the model in (Mascarilla-i-Miro, Yegorov, 2005)). Suppose that utility maximum is obtained in the city of finite size $N_0$. Then, using Taylor expansion, we can write locally near maximum:

$$U(N, \xi) = U_0(\xi) - \epsilon(N - N_0(\xi))^2. \quad (5)$$

If external factors (like wage) are kept constant, we can forget about $\xi$. In this case, there exists repulsive force pushing citizens away as soon as the threshold size of a city is reached. But if $\xi$ has positive shock, $N_0$ moves to new, higher equilibriums, and migration continues.

Integral land rent. Let us calculate the sum of all land rents in a city, $IR$. We use $R(r)$ as a density function. In polar coordinates we use radial symmetry to eliminate integral over angle (note that elementary area is $\phi r dr d\phi$):

$$IR \equiv \int_0^{2\pi} \phi \int_0^{r^*} R(r) r dr = 2\pi \left[ \left( R_a + \tau\sqrt{N/c} \right) \frac{r^2}{2} - \frac{\tau r^3}{3} \right] \bigg|_{r_0}^{r^*} = \pi \left( \frac{R_a}{c} N + \frac{\tau}{3c^{3/2}} N^{3/2} \right). \quad (6)$$

It is also useful to calculate the derivative of integral rent with respect to population:

$$\frac{d(IR)}{dN} = \pi \left( \frac{R_a}{c} + \frac{\tau}{2c^{3/2}} N^{1/2} \right). \quad (7)$$

Note that the integral rent is growing super-linearly with population. This is a collective effect of scale economy. It also might grow faster then the sum of utilities of all citizens. This explains a typical phenomenon, when
hedonic utility of a citizen in expensive city may be lower that one derived from valuation of housing.

It is also possible to write the expression for average rent, \( AR \equiv IR/N \):

\[
AR = \pi c R_a + \frac{\pi \tau}{3c^{3/2}} \sqrt{N}.
\]

Both aggregate and average rent depend on city attractiveness, that determines its population in equilibrium.

**Proposition 1** Both aggregate and average land rent in a city depend on its population, which depends on city attractiveness. It is a collective phenomenon, that comes from nonlinear interaction between citizens with some specific assets of a city.

### 2.4 Construction sector and housing market

The ideas here are a bit idealistic and do not correspond to some of real markets. But understanding them serve a normative purpose.

If construction sector is perfectly competitive and all land is initially distributed across owners, then constructors can get only zero profits. In a "normal" set up, land rent belongs to all citizens. New entrant pays higher price than construction cost. He also increases marginally the value of land rent of all citizens. At the same time, he brings them marginal dis-utility associated with higher congestion.

In a "normal" market (existence of which is subject to "normal" laws) collective of citizens decides whether to allow new entrant to the city (in fact, this law operates in some rural areas of Austria). If they come to conclusion that collective benefits are higher than costs, then they allocate new land for this construction (priced by an auction). As a result, construction firm will get zero profits, and all revenue from the auction will go to city budget. If the country’s government decides that migration process brings some dis-utility to sending regions, it may redistribute the revenue from auction also between those regions.
It makes sense to consider two housing markets, that have exhibited price bubble recently, one in Russia (Moscow) and other in Spain. Both markets have some similarities and some differences.

**Application to Russia.** In 1991, Moscow became even more attracting center than it has been in Soviet times. More than 60% of all financial flows and almost all FDI was coming to Russian capital, moving labour market and wages up. This produced a significant disequilibrium across locations and created incentive for migration. The housing price index in Moscow has been quite volatile during the last two decades, having price growth up to 40% in certain years. After the crisis of 2008 the housing market in Moscow has lost a substantial part of its value. During 1 year from the beginning of the crisis housing of economic class lost 32-25% of its value, while the value loss for business class was even larger, 45%. Some hidden mechanics of Moscow housing market (in particular, ownership of land rent) has became public only recently. The company "Inteco" owner by the wife of former Moscow major Luzhkov got access to land for construction for the price well below market value and was able to make super profits not on construction skills, but through the ownership of corresponding land rent.

**Application to Spain.** Spain has eased its migration policy close to 2000, and the share of foreign population began to grow fast. This has created additional demand for housing, and contributed positively to housing bubble. While the share of migrants has changed from 1% in early 1990s to about 5%, the prices for housing went up by factor 2-3, and it was a clear bubble. However, contrary to the USA and Moscow, bubble did not explode much, and the loss of prices (comparing to maximal) was only 13%. Now Spain has 700,000 unsold new homes, a lot of unemployment in construction sector (that has employed 13% of labour force at construction peak). The incomplete explosion of Spainish housing bubble (contrary to one in Moscow and in the USA, with 33% of price fall) has caused severe macroeconomic problems, with 20% of unemployment rate and about 10% of budget deficit. Although Yegorov (2003) has explained the mechanism of bubble formation

---

4See "The Economist", January 22-28, 2011, "A great burden for Zapatero to bear".
and forecasted a possibility of negative macroeconomic consequences of bubble explosion and although Spanish press has been talking about housing bubble well before its maximal peak, no policy measures have been taken. This Spanish bubble phenomenon needs further explanation. It might be related with the behavior of strategic investors (speculators), but it also might be state policy to save banks from mass bankruptcy on bad mortgages.

Assumptions about housing market. It is possible to make different assumptions about housing market with different level of rationality. At the basic level, it is assumed that market is rational, competitive and efficient. Then it simply provides the good (standard housing) at cost level $H$, while the income from land goes to city community and is used for public goods. In this case we will have one dynamic equation for $\dot{N}$, describing the evolution of city population.

If some land rent (for example, its nonlinear part) goes to profits of construction firms (that can form a kind of cartel), then the sector gets positive profit, and there is clear incentive for its growth (this fact is observed in reality). In this case, the capacity of the sector will expand, and we can write some dynamic equation for change in housing stock of a city, $(\dot{M})$: $\dot{M} = F(M, N)$. If the demand for houses comes only from migrants, then we should have $\dot{M} = \dot{N}$. However, in the environment of positive trend in housing price (driven by change in land rent by migration process) additional demand from strategic investors (trend followers) may also come. In fact, this phenomenon was modeled and empirically confirmed in the paper (Carreras, Mascarilla, Yegorov, 2004).

2.5 Putting all together

We consider several models here. They differ in the assumption about rationality and in the structure of migration process.

Rational housing market. If there is rational housing market without speculators, only equation for migration drives the dynamics,

$$\dot{N} = a(B(\xi - \xi_0) - H - R_a - \tau\sqrt{N/c}) + b(N - N_0).$$  \hspace{1cm} (9)
while the other processes (like housing price dynamics) are determined by this equation:

\[ P(N) = H + R_a + \tau \sqrt{N/c}. \]  

(10)

Here we can distinguish two sub-cases, that are considered in the next section. If there is no chain migration \((b = 0)\), then there will be smooth convergence to new equilibrium, and migration will stop. Demand for new housing will stop also.

The presence of chain migration effect never stops the process, and we should consider other externalities (like growth of unemployment rate, leading to fall in city’s attractiveness) explicitly. In general, this may include the process of return migration.

**Bubble in housing market.** Bubble phenomenon in the housing market has been recognized by everybody after the crisis of 2008. What can be the micro driving forces for its emergence? Let us focus on new housing (ignoring relocations in existing stock). As it has been suggested in [6], housing demand contains two terms. While residential demand depends negatively on price (and interest rate), speculative demand is proportional to price derivative. It was shown that periodic interest rate (typical for macroeconomic cycles) results in price dynamics, containing exponential trend and cycles.

3 Solution to the Model

We consider several cases, differing in market rationality and presence of chain migrants.

3.1 Rational Market. No Chain Migration

Suppose that factor \(\xi\) (no shocks in attractiveness) is fixed and parameter \(b = 0\) (no chain migration). Then the equation for migration dynamics is as follows:

\[ \dot{N} = \alpha(N - N^*)^2, \quad N < N^*; \]  

(11)

\[ \dot{N} = -\alpha(N - N^*)^2, \quad N > N^*, \]  

(12)

10
where $\alpha \equiv a\epsilon$. This equation can be integrated:

$$
\int \frac{dN}{(N - N^*)^2} = \alpha(t - t_0) \Rightarrow N(t) = N^* - \frac{1}{\alpha t}.
$$  \hfill (13)

City size will asymptotically grow to its optimal level, but will never surpass it. If it is initially larger than optimal, it will decrease to optimal size.

### 3.2 Rational Market with Chain Migration

The dynamics of migration is given by the equation

$$
\dot{N} = \alpha(N - N^*)^2 + b(N - N_0), \quad N < N^*,
$$

$$
\dot{N} = -\alpha(N - N^*)^2 + b(N - N_0), \quad N > N^*.
$$  \hfill (14)

Here $N_0 = N(t_0)$ is the city population before migration starts, while $N^*$ is the optimal city size. Assume that $N_0 < N^*$. The first term describes single (non-chain) migration that is driven by attractive potential of a city. Whether the city size is below or above the optimal level, this force moves it towards optimality. The chain term is different: it is always proportional to the stock of migrants in the city.

Note that in this case the city will reach its optimal level in finite time. Indeed, for all $N < N^*$, $\dot{N} > 0$. The case of $N > N^*$ is more interesting, since two driving forces provide opposite effects. Let us find equilibrium point. The dynamic equation for $N > N^*$ can be written as:

$$
\dot{N} = -\alpha(N - N_1)(N - N_2),
$$

$$
N_{1,2} = b/2\alpha + N^* \pm \sqrt{D}/2, \quad D \equiv (b/\alpha)^2 + 4b(N^* - N_0)/\alpha.
$$  \hfill (15)

Since the r.h.s. of dynamic equation (2-nd order polynomial) is negative for large $N > N_2$ and positive for $N_1 < N < N_2$, the point $N = N_2$ represents a unique stable equilibrium. However, its level is far beyond the optimal level of $N^*$, and the difference $N_2 - N^* = b/(2\alpha) + \sqrt{D}$ positively depends on strength of chain effect $b$.

### 3.3 Housing Price Bubble. No Chain Migration

**Model of housing market.** Suppose that native population in a city is constant and does not change its living conditions. The movement in housing
market is driven by housing demand of newly coming migrants and by speculative demand of strategic investors (trend chasers), which is proportional to price derivative, \( \dot{P} \). Thus, the (annual) demand for new housing can be written as
\[ ND = \dot{N} + \gamma \dot{P}. \]
New housing supply comes from construction and can be modelled as follows. Since the profit of construction firm can be derived from price appreciation during the construction period, the growth of this sector (denoted as \( \dot{K} \)) is proportional to price derivative:
\[ \dot{K} = \theta \dot{P}. \]
Integration over time gives \( K(t) = \theta P(t) + \text{some constant} \). Equating demand and supply of new housing, we get the following equation:
\[ \dot{N} + \gamma \dot{P} = \theta P. \tag{16} \]

**Dynamical system.** Together with the equation for population dynamics, we get the system (for \( N < N^* \)):
\[ \dot{N} = \alpha (N - N^*)^2, \tag{17} \]
\[ \gamma \dot{P} - \theta P = -\dot{N}. \tag{18} \]
The second equation in the system is similar to one in [6]; its solution is a sum of general \( (P_0(t) = Aexp(\theta t/\gamma)) \) and particular solution (that can be constructed using explicit solution for \( N \)).

Suppose that we can neglect speculative term. Then price will be proportional to \( \dot{N} \), which will decline as city approaches its optimal level. The construction sector will also decline. However, we rarely observe such phenomenon.

### 3.4 Housing Price Bubble with Chain Migration

In the case of chain migration the dynamics will be driven by the system:
\[ \dot{N} = -\alpha (N - N_1)(N - N_2). \tag{19} \]
\[ \gamma \dot{P} - \theta P = -\dot{N}. \tag{20} \]
The solution algorithm is similar to one in the previous subsection, and it describes the emergence of bubble. Due to chain effect, migration inflow will be positive and high when city passes its optimal level. Housing price and construction sector will both grow at this stage, although utility of citizens...
already decline. When city will reach its new equilibrium (stable, but sub-
optimal), migration inflow will approach zero, pushing prices down. Before
this stage, prices and the size of construction sector will pass its peak.

4 Policy Implications

4.1 City Land Rent and its Ownership

There are several important policy issues, addressed in this paper. The first
one is the ownership of land rent and externalities coming from use of this
land rent by construction business and speculators in housing market. In
general, population of the city may be not interested in its growth, since
it brings negative externalities (congestion) which may not be compensated
fully by growth of land rent (especially for owners of one unit of real estate,
who also live there). Interest may be gained by large business, and here we
may have a conflict of interests.

The ownership and division of land rent also determines the dynamics of
housing market and construction sector. We have seen that arrival of new
residents has a negative externality on old dwellers through congestion. If all
residents would jointly own construction firms, then this externality could be
materialized in their dividends. In reality, construction sector has typically
other owners. In an extreme case, they gain all surplus of city rent and do
not pay old dwellers for externality. The unique possibility for them to be
compensated for externality is to sell their houses on the secondary market.
However, due to indivisibility, the majority cannot capitalize this rent with-
out losing residential location in this city.

That is why the origin and split of city’s location rent should be an issue
of public debates, with raising the question about the split of its ownership
and introduction of corresponding laws. In ideal situation, this rent should
belong to a city and be used for public goods or split across individuals.
Only a fraction of it might go to developers. In this case, the ”gray” part of
their profit will be reduced, and this will partly prevent emergence of price
bubbles for real estate.
4.2 Effect of Chain Migration on City Dynamics

The second issue is related to the effect of chain migration as de-stabilizer of the economy and one of sources for bubble emergence. Migration process has two components: individual and chain. Individual migrants are typically skilled workers who rationally choose to a city with higher attractiveness (which comes for them in a form of wage surplus). In an environment of no legal restrictions, individual (rational) migration stops when potential migrants do not find them better off after migration. Even if wage is high, living costs (or negative externalities) might offset them completely, and city size will stay in optimal point for its residents.

Chain migrants are normally those migrants who do not search individually for their optimal location, but are pulled (attracted) by their network who is already residing there. Mostly they consist of family members, but can also include long distance relatives and friends. Why do we observe overshooting effect with chain migration? The first reason is that this network grows unboundedly, and has some pull effect even when city has surpassed its optimal size.

The second effect is demographic. It is well known that the number of children in cities is typically smaller than in rural areas (due to higher cost of raising children). This rule typically holds for native population. In the case of international migration, we typically observe the phenomenon of higher fertility in sending countries comparing to receiving (European Union). That is why the potential number of chain migrants brings an imbalance to demographic structure of a city, raising also such issues as poverty and criminality.

5 Conclusions

1. Combination of city dynamics with the static theory of urban and housing economics makes it necessary to understand the issue of ownership in land rent. This is a complex collective phenomenon with externalities, and special laws have to be introduced to avoid power abuse and dishonest profits.

2. The phenomenon of chain migration can destabilize the process of convergence to optimal city size. If a city (country) introduces some restric-
tions (regulation) on chain migration, there will be less negative externalities (congestion, price bubble, ghetto with poverty, criminality) for city residents.

3. The construction sector can have different levels of rationality. If it is trend chaser (construct more if last periods were bringing profits), then a bubble in housing price is more likely to emerge. There are two more drivers of it. The first is incorrect split of land rent across developers, old residents and city as a community. If developers get too high share, profitability of construction business continues till the moment of bubble explosion. Chain migration is another amplifier of bubble process in construction.

4. The attractiveness of different cities changes over time, and often with jumps. This process (sometimes caused by extra capital investment, legal changes or even advertisement) can generate the flows of relocation of citizens and can cause emergence of bubbles in construction sector and on housing markets. After explosion of this price bubble there are typically negative macroeconomic consequences leading to long recession and stagnation of economy. Currently this phenomenon is observed in Spain.

6 Literature