The Regional Inequality Frontier: Brazil (1872-2000)
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ABSTRACT
Milanovic, Lindert and Williamson (2007) introduced the concept of the “Inequality Possibility Frontier”. Their starting point is that very poor societies will never display high Gini indexes of personal distribution of income because there is very little surplus to be appropriated by the upper classes of these societies. The Inequality Possibility Frontier is the maximum level of inequality possible at each level of income. This paper extends the concept to cover regional cases. Countries with populations close to subsistence level inevitably display low regional inequality of income per capita. Rising levels of wealth imply higher attainable degrees of regional inequality. The concepts of Regional Inequality Frontier and Regional Inequality Ratio are presented in this paper, and are illustrated by the case of Brazil between 1872 and 2000.

Introduction
Jeffrey Williamson (1965) identified an inverted U-curve relationship between regional inequalities and income per capita. In the initial stages of economic development, certain regions were able to attract private capital, better qualified workers, and even more public investment. Centripetal forces lead to an increase in inequalities between geographical regions (Myrdal, 1963). From a certain point onwards, the classic mechanisms of movement of capital and labour (and, in some cases, of government policies) caused a reduction in the inequalities of per capita income between the regions.

More than forty years after his article first appeared, Williamson (together with Milanovic and Lindert) established the concepts of the Inequality Possibility Frontier and the Inequality Extraction Ratio (Milanovic, Lindert and Williamson, 2007; hereinafter MLW). The purpose of these concepts was to analyse the evolution of the distribution of personal income, stressing that there are maximal limits to inequality at each income level. This paper sets out to demonstrate that the same insight of MLW (2007) – with adaptations - can be applied to the analysis of long-term regional inequality. It is thus possible to arrive at the concepts of a Regional Inequality Frontier (RIF) and a Ratio of Regional Inequality (RIR). For the purposes of illustration, this study calculates the regional Gini indexes and

1 I would like to thank CNPq and Nemesis FAPERJ for their support (Proc. E52 168.171/2006/Pronex).
2 Barrios and Strobl (2009) were not able to refute the inverted U-curve hypothesis for more recent international data, even though they applied more sophisticated econometric techniques.
Williamson’s Vw index for the states of Brazil between 1872 and 2000, together with their respective RIF and RIR indexes.

How exactly did long-term regional inequality evolve in Brazil? First of all, it is important to remember that Brazil was an extremely poor country in 1872. Per capita income was only 1.8 times higher than subsistence level, and only one third of the level observed in Uruguay for the same year (Maddison, 2010). It is also well-known that there were marked regional disparities in Brazil in 1872. The relative under-development of the northeast region in comparison to the central and southern regions had continued. In the 20th century, the rapid industrialisation of the country was considered to be responsible for the maintenance and expansion of regional inequality. Williamson (1965) affirmed that Brazil was a classic example of the U-curve theory during the period from 1939 to 1959. Inequality in the country peaked in 1952 (an international record at the time), but has been falling ever since. Azzoni (1997) demonstrated that, in general terms, inequality between the states continued to fall in subsequent years, and stabilized from 1985 onwards. In more recent times, a myriad of studies about the convergence of per capita income have produced varying results concerning the tendency to regional inequality among the states.3

1 The Frontier of the Possibility of Inequality

The best way to present the contribution made by MLW (2007) is to give a simple example. Let us suppose a community of 100 inhabitants which produces the equivalent to 110 subsistence salaries. 90 inhabitants survive on just one subsistence salary each, while the remaining 10 inhabitants earn the remaining disposable income of 20 subsistence salaries. The Gini index for this community is therefore 0.08, which suggests an egalitarian society, whereas in fact the remaining disposable income is appropriated by only one very small group. MLW (2007) noted that, with very low levels of per capita income, the Gini index is low due to a mathematical restriction.

The Inequality Possibility Frontier (IPF) shows the possible Gini index for each level of per capita income in a particular society (MLW, 2007). It is quite easy to estimate, and is based on the calculation of the maximum possible income of a small group of high-earning individuals (elite).

\[ s = \text{subsistence income} \]

3 See Mossi et al. (2003) for references to studies of convergence between Brazilian states.
\[ N = \text{number of persons in the society} \]
\[ \mu = \text{per capita income of the whole society} \]
\[ \varepsilon = \text{number of high-earning individuals/number of persons in the society (N)} \]

Therefore:
\[ y_h = \frac{\mu N - sN(1 - \varepsilon)}{\varepsilon N} \] (1)
\[ y_h = \frac{1}{\varepsilon} [\mu - s(1 - \varepsilon)] \] (2)

When individuals are organized according to ascending levels of income, the Gini index is simplified when we assume full equality within the levels and no overlap between levels. In other words, no one at a lower level earns more than anyone at a higher level. Thus:
\[ G = \frac{1}{\mu} (y_j - y_i) p_j p_i \] (3)

In which \( p_i \) and \( p_j \) represent the number of inhabitants in classes \( i \) and \( j \) (\( y_j > y_i \)). Through \( y_i = s \), \( \varepsilon = p_j \) and \( (1-\varepsilon) = p_i \), we can calculate the maximum Gini (\( G^* \)):
\[ G^* = \frac{1 - \varepsilon}{\mu} (\mu - s) \] (4)

\( \mu = \alpha s \) gives per capita income as a multiple of subsistence income.
\[ G^* = \frac{\alpha - 1}{\alpha} (1 - \varepsilon) \] (5)

When \( \varepsilon = 0.10 \), the values of \( G^* \) are in line with Table 1 below, i.e. in a country where the per capita income is twice the subsistence income, and the group of highest earners constitute 10% of the population, the maximum Gini index possible is equal to 0.45.
Table 1 – Simulation of the maximum Gini index (G*) in accordance with per capita income.

<table>
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<tr>
<th>α</th>
<th>G*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
<td>1,5</td>
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<td>4,5</td>
<td>0.70</td>
</tr>
<tr>
<td>5</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Source: author based on MLW (2007)

The Inequality Possibility Frontier proposed by MLW is derived directly from equation (5) above. MLW then proposed the concept of the Inequality Extraction Ratio, i.e. the ratio between the observed Gini and G*. Any values close to one mean that all surplus income is appropriated by the elite. In the case of the community given as an example in the Introduction above, the Inequality Extraction Ratio would be equal to 1, thus indicating how unequal it really is.\(^4\)

2 The Regional Inequality Frontier

The same question raised by MLW regarding distribution of personal income comes up in the analysis of regional inequality. Regional per capita income cannot be very varied at the lowest levels as there is a minimum limit, and the surplus is insufficient to allow the richer regions to distinguish themselves significantly from the poorer ones. Since this fact is not usually taken into account in the analysis of the evolution of regional inequality, there is a risk of drawing wrong conclusions about the economy being studied.

The transfer of the Inequality Possibility Frontier to the regional sphere is carried out in a very direct way. Instead of analysing families or social classes, the main focus is on spatial units. As was mentioned above, in order to construct the IPF, MLW (2007) created a

\(^4\) MLW emphasised two desirable properties of the Inequality Possibility Frontier: (1) there is no significant variation in G* in relation to reasonable changes in ε; (2) any relaxation of the hypothesis that all the highest earners make the same amount of money does not significantly alter the results of the calculations.
counterfactual which attributes all the surplus income produced to the elite, whilst all the
other earners remain at subsistence level. It is assumed that the elite divide the surplus
income equally amongst themselves. In the regional case, the counterfactual is constructed
in a similar way: for each level of income, all the surplus income produced in the economy
is attributed to a unit (in this study, the state).

2.1 Regional Inequality Frontier (Gini)
Regional G* is calculated in the same way as that proposed for G*, but ε represents the
proportion of the richest unit in the country (or area of reference). While MLW have to
select a certain percentage to represent the elite in the society, the task here is to arbitrate the
richest unit when simulating a maximum spatial concentration of income. The natural
candidate for this role is the place (in this case, the state) which is in fact the richest. In this
way, it is possible to calculate per capita income by assuming that all the other members of
the society are at subsistence level, and that any surplus income is transferred to the richest
unit. Thus ε no longer represents the share of the elite, but the share of the richest unit in the
whole country (or area of reference). Apart from this, the calculation of G* is identical to
that presented in (5) above.

2.2. Regional Inequality Frontier (Williamson’s Vw)
The Vw index is simply a coefficient of variation in population weighted by the share of the
population in each region. In the original version, it is calculated as follows (Williamson,
1965):

\[
V_w = \sqrt{\frac{\sum (y_i - \mu)^2 (p_i / N)}{\mu}}
\]

(6)

In order to calculate Vw*, it is necessary to build a counterfactual similar to the one used for
G*: the population of all the units receives a subsistence income, with the exception of the
richest unit, which appropriates all the surplus income. So equation (6) can be reduced to:

\[
V_w^* = \frac{\sqrt{\left( (s - \mu)^2 (1 - \varepsilon) + (y_i - \mu)^2 \varepsilon \right)}}{\mu}
\]

(7)

It should be noted that all the regional indicators presented here are subject to the well-known modifiable
areal unit problem (see Haining, 2003).
As demonstrated earlier, \( y_h \) can be calculated on the basis of equation (1). If \( V_{w^*} \) is simulated by various values between \( \alpha \) and \( \varepsilon = 0.1 \), the result is Table 2 below.

Table 2 – Simulation of Williamson’s maximum (\( V_{w^*} \)) in accordance with per capita income.

<table>
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<th>( \alpha )</th>
<th>( V_{w^*} )</th>
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<td>2.95</td>
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<tr>
<td>5</td>
<td>3.04</td>
</tr>
</tbody>
</table>

Source: author.

2.3. The regional inequality ratio

In line with the RIF, the Regional Inequality Ratio (RIR) is given by dividing the inequality index observed and the maximum for each level of income. Thus, in the case of the Gini and Vw indexes we have:

\[
RDR_{vw} = \frac{V_w}{V_{w^*}} \\
RDR_{G} = \frac{G}{G^*}
\]  

(8)

(9)

It is worth noting that the trajectory the RIR throughout the process of development is not a trivial one. All that is needed is that the regional disparity observed grow more rapidly than the maximum limit of the inequality indexes for the RIR to increase.

3 RIF and RIR in Brazil from 1872 to 2000

3.1 Procedure

In this paper, per capita incomes were calculated on the basis of the participation of each state in Brazilian GNP. This data was obtained from the following sources:
• Bértola, Castelnovo and Willebald (2009) for the year 1872;  
• For 1920 GNP, the data on municipal GNP calculated by Eustáquio Reis were consolidated for each state. This information is available in IPEADATA (2010);  
• For all subsequent years, IPEADATA is the data source. The data is based on estimates made by the Fundação Getúlio Vargas for 1940, 1950, 1960, and by the IBGE from 1970 onwards.

The annual GNP for each state was arrived at after making the necessary aggregation of the data available, and on the basis of Brazilian GNP as estimated by Maddison (2010) for 1990 (in US dollars, in accordance with Geary-Khamis PPP International). The sum of US$ 400 annually was used as the subsistence income, the same limit as that selected by MLW (2007).

This procedure resulted in per capita incomes lower than subsistence level for some of the northeastern states of Brazil. The series was therefore adjusted so that the poorest state (Ceará with US$ 315 in 1920) would also have a per capita income of US$ 400 (see Appendix). This does not change the validity of the analysis.

It is important to note that the state of Rio de Janeiro had the highest levels of per capita income until 1950. From 1960 onwards, São Paulo has been in pole position, and this change made it necessary to decide which of these cities should relate to over the series. It was decided to maintain the ε for Rio de Janeiro over this period.

3.2. The evolution of inequality in Brazil

The graph below shows the evolution of the Gini indexes (weighted for population) and the Vw for the whole series.

[Graph 1 here]
As was expected, the curves for Vw and G are very similar: the inequality between the states grew until the mid-20th century, after which it began to fall. Grosso modo, Williamson’s inverted U-curve can be considered valid. Only the period from 1960 to 1970 does not follow this trend, as it was a time of growing inequality. Azzoni’s (1997) analysis suggests that the inequality identified in 1970 was the result of a divergence in the last years of the previous decade which was caused, in the case of Brazil, by rapid economic growth.

Graph 2 below shows the G, G* and RIRG, whilst maintaining the ε relating to the participation of the state of Rio de Janeiro in each year of the analysis. The G* grew throughout the series, and the evolution of the G observed was commented on earlier. The RIR index is of particular interest as it remained stable at a high level of around 0.57 from 1872 to 1920. This means that, in spite of the Gini index observed, the RIR was about 57% of the maximum possible. From 1920 onwards, the tendency was for the RIR to fall, apart from the “blip” in 1970 mentioned earlier. When regional inequality increased most (during the “take-off phase” [Rostow, 1959]), the ratio between observed and possible inequality fell. In other words, although economic distribution became more unequal, it receded from its potential maximum level of inequality. Another way of viewing this result would be to say that the inequality at the beginning of the period was not greater simply because Brazil would not have been able to cope with it.

[Graph 2 here]

Graph 3 shows data concerning Vw, Vw* and RIRVw. The results are identical to those for the Gini index, except that the movements of the Vw index are more pronounced due to the greater sensitivity of Williamson’s index to extreme values.

[Graph 3 here]

**Conclusion**

This paper demonstrates the adaptation of MLW’s (2007) concept of the Inequality Possibility Frontier to the regional dimension. Particularly for countries with per capita incomes near to subsistence level, it is recommended that both the Regional Inequality Frontier and the Regional Inequality Ratio be calculated. It is evident that part of the explanation for the upward-moving section of Williamson’s (1965) inverted U-curve stems from the fact that, in the initial stages, higher levels of inequality are simply unachievable. The paper also demonstrates that Williamson’s well-known Vw index is subject to the same distortions as the Gini index for incomes close to subsistence level.
In 1872, the Brazilian economy was closer to the possible regional inequality maximum than at any time in the 20th century. It is also important to note that there was a marked fall in the RIR until 1950. At the beginning of the modern age of economic development, when economic activity was concentrated in the southeast region of Brazil, the country started to move away from the regional inequality frontier. Finally, notwithstanding the correct view that regional disparities increased in the mid-20th century, it should be borne in mind that this happened because of the limits on even greater inequality at the beginning of the series.

Since China and India only recently entered the process of economic growth from the starting point of very low per capita incomes, they also constitute interesting case studies for RIF and RIR. It is in that direction that the next steps in the present research will turn, and it is hoped that the ensuing results will throw light on new aspects of the dynamics of regional inequality in the two countries referred to.
References


Reis, Eustáquio José. 2009. Income per capita of Brazilian municipalities in the 1870s.  

Reis, Eustáquio José, Pimentel, Márcia e Alvarenga, A. Áreas mínimas comparáveis para os períodos intercensitários de 1872 a 2000.


# Appendix

Per capita income in US$ for each Brazilian state: 1872-2000

<table>
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Source: see text.

Note: * indicates the states with which other states were aggregated.
Graph 1: Gini index and Vw for per capita income of Brazilian states between 1872 and 2000
Graph 2: Gini index, Gini* and RIRg for Brazilian states between 1872 and 2000
Graph 3: $V_w$, $V_w^*$ and $RIR_{V_w}$ for Brazilian states between 1872 and 2000

Note: $RIR_{V_w}$ values on the right axis.