Transport Infrastructure, Spatial General Equilibrium and Welfare

Abstract

Large-scale investments in transport infrastructure have been traditionally evaluated assuming the equivalence between direct and indirect economic effects (Jara-Diaz, 1986), which is only correct under -generally non-guaranteed- perfect competition assumptions. Despite this common practice there is still no consensus amongst economists as to how the benefits and costs of large infrastructure projects should be determined. The discussions regarding the desirability, for instance, of the Betuwe railway line, the fifth runway at Schiphol Airport, the North-South underground railway in Amsterdam etc. are illustrative of this. The focus has been, in particular, on the magnitude of ‘indirect’ and ‘strategic’ effects, that is effects on parties other than the direct users of the infrastructure (indirect effects) and those factors that have a favorable effect on the long-term development of the (regional) economy, such as effects relating to firm location and demographics (strategic effects). Focusing on general equilibrium, increasing returns and imperfect competition modeling approach this paper aims to throw light on this subject matter by examining how the social benefits in terms of efficiency resulting from improvements to the infrastructure can be determined in an imperfect regional economy.

Jose C. Melendez-Hidalgo, Piet Rietveld and Erik Verhoef

Free University of Amsterdam, Spatial Economics Department
Tinbergen Institute Amsterdam
Roetersstraat 31
1018 WB Amsterdam

I. Introduction

Until the beginnings of the previous decade, the link between transport infrastructure, space and economic activity in economics was somehow weak, especially in terms of theoretical work. Despite an early recognition of the subject by A.Smith -in relation with the expansion of market size- neither he nor any other classical economist made any sustained reference to the spatial dimension of economic life and its consequences. As has been pointed out recently by Martin (2003), this “space-less” view of the economy was barely challenged by A.Marshall while introducing the concept of industrial districts\(^1\). Despite an interesting potential for further development, Marshall’s contribution was not successful in attracting enough attention from his contemporaries in a way to transform the subject from a curiosity to a formal line of research. As a consequence this important potential branch of economics was basically ignored afterwards.

---

\(^1\) Industrial districts are seen nowadays as the origins of clustering theory. See Porter (1980).
In the middle of the last century the situation started to change. In the context of a post-war world economy and mainly within development economics, infrastructure investments began to acquire a sense of significance as a main determinant of long-run growth, as was claimed by development economists such Rosenstein-Rodan (1943) and Hirschman (1958). In particular, Tinbergen (1957) addressed directly the relation between immediate benefits to traffic—direct effects—arising from a reduction in transport costs and the resulting final change in GDP. While these efforts were more of an isolated nature, the prolific production of evidence after Aschauer (1989) supporting a link between public infrastructure—especially core infrastructure—and productivity in the U.S. were definitely not. The surprisingly high rates of returns on public infrastructure investment implied by Aschauer findings originated a renovated interest on the link between infrastructure and growth at an empirical level and particularly on the spatial structure of the economy with a theoretical emphasis. Concerning transport infrastructure, this interest is reflected in a burgeoning literature on its impacts on the economy (see Rietveld and Bruinsma, 1998 for an overview) as well as a proliferation of a large variety of methods to estimate them (Rietveld and Nijkamp, 2000; Lakshmanan and Anderson, 2002).

Within this wave of research is remarkable the proclivity to use general equilibrium models based on increasing returns, product differentiation and monopolistic competition. The so-called new economic geography (NEG) models are specially positioned to analyze the trade-off between dispersal and agglomeration—or centrifugal and centripetal—forces that arise when increasing returns to scale characterize a proportion of economic activity. In essence this modeling framework provides an economic explanation of the spatial structure of the economy. The link between this kind of modeling and large-scale investment in transport infrastructure seems to arise from the combination of the role of transport cost on the spatial distribution of economic activity under economies of scale and the obvious link between transport infrastructure improvements and transport costs reductions (Fernandez-Texeira, 2002). Nowadays, the use of NEG modeling in location and agglomeration analysis is widespread in theoretical literature, unless is not so in empirical work (Oosterhaven and Knaap 2002; Venables and Gasorek, 1999). Despite that, there is significant lack of work concerning the analysis of efficiency of agglomeration outcomes in general and welfare effects within this modeling approach in particular. In order to clarify the implications of transport improvements on the economy, both at theoretical and empirical levels in this paper we depart from the canonical NEG model to analysis the implications of traditional transport cost-benefit analysis. The rest of the paper is organized as follows. In the second section the description of the basic model is carried out and some extensions both on the demand and supply side are discussed. In section three simulations based on different parameters values are discussed. Both symmetric and asymmetric conditions between regions are taken into consideration. Section forth elaborates over possible extensions and finally conclusions are discussed in a final section.

---

2 For the former see Charlot et. al (2003).
II. Model

In this section we describe the theoretical model used for simulations. This is basically the canonical core-periphery model (Fujita et al. 2000, Baldwin et al. 2003). The discussion is carried out for the general case of \( r \) regions. For simulations we consider a 2 \( \times \) 2 \( \times \) 2 setting, that is, the economy space is composed of two regions, it has two sectors of production (modern and traditional sectors) and there are two production factors (qualified and non-qualified labor). In a second stage of research we would consider extensions concerning firm location and network effects within models of more than two regions.

Consumers

In the general case, we consider an economy of \( n \)-regions with two sectors of production, one producing traditional goods under constant returns and the other producing manufactures (\( n_r \)-varieties in each region) under increasing returns to scale. Production is carried out only based on labor, but this factor can be of a qualified or non-qualified nature. All workers are also final consumers and share the same basic Cobb-Douglas preferences for the two basic types of goods a la Dixit-Stiglitz:

\[
U_r = M_r^\mu T_r^{1-\mu},
\]

where \( M \) represents the composite index of manufacture goods, \( T \) is the consumption of traditional good (e.g. agricultural), and \( \mu \) represents the expenditure share of manufactured good in consumption. The subscript \( r \) reflects the fact that utility is measure at a regional level aggregating all consumers without distinguish between qualified and non-qualified workers. The consumption of manufactures is described by a constant elasticity of substitution sub-utility function defined over a continuum of varieties of manufactured goods, \( m(i) \), with a range of varieties described by \( n \). The preference for variety in manufacturing goods is represented by \( \rho \).

\[
M_r = \left[ \int_0^\rho m(i)^\rho \, di \right]^{1/\rho}
\]

with \( 0 < \rho < 1 \)

For convenience we use \( \rho \) to define the elasticity of substitution between a pair of varieties as \( \sigma \). Whenever \( \sigma \) increases the substitution between a pair of manufactured goods is greater, meaning that the demand conditions for this pair approaches perfect competitive conditions\(^3\).

\(^3\) We assume that this elasticity is the same in all regions. Additionally we rule out for the moment the possibility of strategic interaction between firms.
\[ \sigma \equiv \frac{1}{1-\rho}, \quad \rho \equiv \frac{\sigma-1}{\sigma} \]

In each region consumers maximize utility subject to the budget constraint. Since all consumers are identical in preferences final demand will be the same for all of them differing only in terms of sources of income. We then assume that there is a representative consumer in each region meaning that the relevant income includes all sources of wages in a region, that is, manufacture or traditional production based wages\(^4\). If preferences were quasi-linear aggregation is not a problem but with CES preferences well-behaved aggregation is not assured to hold. Defining income in a region by \( Y \) we have that this can be assigned to traditional goods or different varieties of manufactures:

\[ Y_r = p_r^T + \int_0^n p(i) m(i) \, di \]  

The utility maximization problem can be solved in two steps. First, for any value of the composite \( M \), each \( m(i) \) have to be chosen so as to minimize the cost of attaining it. This is achieved solving the following problem,

\[ \min \int_0^n p_r(i) m_r(i) \, di \quad \text{subject to} \quad M_r = \left[ \int_0^n m_r(i)^\rho \, di \right]^\frac{1}{\rho} \]  

The first order condition for this expenditure minimization problem establish the equality of marginal rates of substitution to price ratios for any pair of varieties and consequently implies an expression for the consumption of a particular variety (e.g. \( m(j) \)), that replaced in the constraint in (4) finally brings an expression for the compensated demand of this particular variety as in (5),

\[ m_r(j) = \frac{p_r(j)^{(\rho-1)}}{\left[ \int_0^n p(i) r^{\rho-\rho-1} \, di \right]^\rho} M_r \]  

The term in the denominator in normally regarded as a price index for the manufactured products consumed in \( r \), denoted here by \( G \) in (6). This index measures the minimum cost of purchasing a unit of the composite index \( M \) of manufacturing goods.

\[ G_r = \left[ \int_0^n p(i)^{\rho} \, di \right]^{(\rho-1)} = \left[ \int_0^n p(i)^{1-\sigma} \, di \right]^{\frac{1}{(1-\sigma)}} \]  

\(^4\) See Mas-Colell et al (1997) Ch. 4 for a discussion on aggregation.
Using (6) the equation for the demand of a particular variety can be simplified to:

$$m_r(j) = \left( \frac{p_r(j)}{G_r} \right)^{1-\sigma} M_r = \left( \frac{p_r(j)}{G_r} \right)^{-\sigma} M_r$$

(7)

In a second stage we can solve the original problem of utility maximization, where consumers divide total income between traditional and composite manufactures. This is a typical Cobb-Douglas maximization problem leading to uncompensated demands in the form of income over price times the expenditure share of the good in total consumption, as in (9).

$$\max U_r = M_r^{\mu} T_r^{1-\mu} \quad \text{subject to} \quad G_r M_r + p_r^T T_r = Y_r$$

(8)

$$T_r = \frac{(1-\mu)Y_r}{p_r^T}; \quad M_r = \frac{\mu Y_r}{G_r}$$

(9)

For each variety of manufactures in the region the demand can be derived as (10), in which the elasticity of demand for every variety is $\sigma$.

$$m_r(j) = \mu Y_r \frac{p_r(j)^{-\sigma}}{G_r^{-(\sigma-1)}} \text{ for } j \in [0, n_r]$$

(10)

Under these conditions an expression for the indirect utility function can be obtained. This expression is the base for welfare analysis within this model. Since all consumers share the same preference structure, this expression is valid for all workers in a region as well.

$$V_r(p_r^T, G_r, Y_r) = \mu^\mu (1-\mu)^{(1-\mu)} Y_r G_r^{-\mu} (p_r^T)^{-(1-\mu)}$$

(11)

Producers

Before describing producer behavior we introduce the spatial dimension in the model explicitly through transport cost. The explicit modeling of a transport sector -with or without distinguishing between different modes- is ruled out at this stage and we stick to the usual indirect way of modeling assuming iceberg type cost of transportation a la Samuelson, in which in order to put one unit of a manufactured good from region r in region s, $\tau_{DrS}$ units of the good should be send, implying that only $1/\tau_{DrS}$ will actually
arrive after traveling $D_{rs}$ units of distance$^5$. In a model of only two regions the price of a variety produced in region $r$ and consumed in $s$ will be:

$$p_{rs} = p_r \tau_{rs}$$

In the case of multiple regions, we can specify in more details the price index for manufactures as:

$$G_r = \left[ \frac{1}{1-\sigma} \int_0^{n_r} (p_{r\tau})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

(12)

Further more, to simplify computations –without compromising results- we can assume that in each region the price for each variety is the same, then (12) become easier to handle:

$$G_r = \left[ n_r p_{r\tau}^{1-\sigma} + n_s (p_{s\tau})^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad r = 1, 2 \quad \text{and} \quad s \neq r$$

Then, aggregate consumption demanded by consumers in location $s$ for a product produced in $r$ now follows from (10):

$$m(f)_s = \mu \tilde{Y}_s (p_{r\tau})^{\sigma-1} G_{s}^{(\sigma-1)}$$

(13)

From the point of view of producers, providing this amount of manufacture for consumption, a greater amount has to be shipped, exactly $\tau_{rs}$ times the amount to be consumed$^6$.

$$q_r = \mu \sum_{s=1}^{n_r} \tilde{Y}_s (p_{r\tau})^{\sigma-1} G_{s}^{\sigma-1} \tau_{rs}$$

(14)

In a two region model the market demand for variety $i$ for producer in region $r$ is given:

$$q(i)_r = m(i)_r + m(i)_s \tau_{rs}$$

(15)

Now we can turn to the specific modeling of producer behavior. The production of the quantities in (16) requires $C_r$ units of labor, where $F_r > 0$ and $c_r > 0$ are respectively the fixed and the marginal labor requirements. Production of any variety exhibits increasing returns to scale internal to the firm$^7$.

---

$^5$ In a network economy with more than two regions it must be recognized that increasing returns to scale are normally present in the transport of goods, as was pointed out by Marshall in the case of maritime transport (McConville, 1999).

$^6$ We are assuming here that no transport cost have to be incurred for trade within regions but this can be modified to incorporated in-region transport infrastructure improvements.

$^7$ It is also possible to model increasing returns to scale external to the firm.
The profit function of each firm located in region \( r \) is therefore:

\[
\Pi_r = p_r q_r - w_r (F_r + c_r q_r)
\]  

(17)

Where the demand faced is taken from (15). From (16) the equilibrium price can be determined using first order conditions, which in this case give the classic result:

\[
p_r \left(1 - \frac{1}{\varepsilon_r}\right) = c_r w_r
\]

(18)

In (17) the price elasticity of demand for variety \( i \) is represented by \( \varepsilon_r \), defined as,

\[
\varepsilon_r = \frac{\partial q_r}{\partial p_r} \frac{p_r}{q_r}
\]

In the particular case of Dixit-Stiglitz monopolistic competition assumption, the “large group” assumption concerning competitors is assumed, and then \( \varepsilon_r \) corresponds to \( \sigma \). In this version of our model we rule out strategic interaction between producers when \( \varepsilon_r = \sigma \). This assumption implies that price for variety \( i \) is above marginal cost just by a constant mark-up, as in (18).

\[
p^*_r = \frac{\sigma}{\sigma - 1} c_r w_r; \quad \sigma = \frac{p^*_r - c_r w_r}{p^*_r}
\]

(18)

Assuming free-entry of competitor’s profits will go to zero and we can derive an expression for the equilibrium level of production for each variety in (19):

\[
\frac{c_r}{\sigma - 1} q_r - F_r = 0
\]

\[
q^*_r = \frac{F_r (\sigma - 1)}{c_r}
\]

(19)

Using (19) we can derive the cost incurred in equilibrium, and use this to find the total demand of labor in each region as in (20)

\[
C^*_r = F_r + c_r q^*_r = F_r \sigma
\]

\[
L^*_r = n C^*_r = n(F_r + c_r q^*)
\]

(20)
The total number of varieties in equilibrium is,

\[ n^* = \frac{L_r}{F_r \sigma} \]  \hfill (21)

Some normalization in parameters can be taken in order to simplify the model and its solution for simulation purposes. Following Baldwin et al. (2003),

\[ F_r \equiv \frac{1}{\sigma}, \quad c_r \equiv \frac{(\sigma - 1)}{\sigma} \]

Using the zero profit condition, this implies that:

\[ p_r = w_r, \quad r = 1,2; \quad q_r = 1, \quad r = 1,2 \]

As mentioned before in this model nominal wages adjust until no firm enters or exits the market, meaning that profits are zero in equilibrium. After some manipulations we arrive to:

\[ w_r = w_r^{1-\sigma} \frac{\mu_Y}{n_r w_r^{1-\sigma} + \tau^{1-\sigma} n_s w_s^{1-\sigma}} + \tau^{1-\sigma} w_r^{1-\sigma} \frac{\mu_Y}{\tau^{1-\sigma} n_r w_r^{1-\sigma} + n_s w_s^{1-\sigma}} \]

This is nonlinear in \(w_r\), meaning that no analytical solutions can be obtained then numerical solutions are the rule in this model. A spatial equilibrium in this case is the final result of the interplay between agglomeration and dispersion forces and is characterized in the literature as the situation where skilled worker have no additional incentives to change location. The equilibrium depends heavily on the transport cost level (Krugman, 1991) and can be formally denoted by an “excess” indirect utility a skilled worker enjoy in region \(i = 1,2\), a spatial equilibrium arises at \(\lambda\) between \((0,1)\) when,

\[ \Delta V(\lambda) \equiv V_1(\lambda) - V_2(\lambda) = 0 \]  \hfill (21)

\textit{Welfare Analysis}^8

Since we are interested in a CBA analysis comparison based traditional evaluation approach and a more general approach that uses broad welfare measures that take care of general equilibrium effects, we should first derived a transport demand to compute the first CBA type and afterwards decide which welfare measure use to compute CBA of the second type.

Expression (22) can be seen as a derived demand for transport services corresponding to variety \(i\). The valid range of values for \(\tau\) in this derived demand is obtained as the range

---

^8 A more extensive explanation of the approach followed to measure welfare change and to compare it to traditional CBA is given in an appendix.
in which the general equilibrium of the model sustains dispersion equilibriums. To account for the total welfare effect in the transport using consumption, we have to multiply the change in consumer surplus by the proportion of varieties consumed in the corresponding region meaning the analysis can be equivalently be carried out over the aggregate derived transport demand for variety \( i \) in region \( r \).

\[
q(j)_{rs} = \mu Y_i (p_r, \tau_r)^{-\sigma} G_i^{(\sigma-1)}
\]

(22)

For the second type of CBA analysis mentioned we make use of Hicksians measures of welfare (e.g equivalent and compensate variations). Corresponding expressions can be derived starting with the indirect utility function in (11). The following two relations implicitly define an expression for the equivalent (EV) and compensate (CV) variations,

\[
V(\tau^0, Y + EV) = U^1
\]

\[
V(\tau^1, Y - CV) = U^0
\]

where \( U^i \) indicate the level of utility depending on the stage, that is, in the original situation before the change in transport cost and in the final situation after the change respectively. Combining these conditions with (11) a pair of explicit expressions for the EV and CV can be derived, for instance, for the equivalent variation,

\[
U^1 = \mu^\mu (1 - \mu)^{(1-\mu)} (Y_r + EV) G_r^{-\mu} (p_r^T)^{-(1-\mu)}
\]

\[
\mu^\mu (1 - \mu)^{(1-\mu)} Y_r G_i^{-\mu} = \mu^\mu (1 - \mu)^{(1-\mu)} (Y_0 + EV) G_0^{-\mu}
\]

\[
(Y_0 + EV) = \frac{Y_i G_i^{-\mu}}{G_0^{-\mu}}
\]

\[
EV = Y_r \left[ \frac{G_i}{G_0} \right]^{-\mu} - Y_0
\]

(28)

and applying a similar procedure we will obtain,

\[
CV = Y_r^1 - Y_r^0 \left[ \frac{G_0}{G_i} \right]^{-\mu}
\]

(29)

The welfare analysis is completed when these two measures of economy-wide welfare are compared with the traditional consumer surplus change arising from a cost-benefit analysis. This comparison is conducted by calculating the ratio of EV over CS. This ratio should be equal or greater than one when considering a reduction in transport costs (by a

---

9 A discussion regarding type and number of equilibriums in core-periphery type models can be found in Baldwin et al (2003).
reduction in $\tau$). A ration greater than one implies the presence of indirect effects not taken into account in traditional CBA.

III. Simulations

As a first set of simulations we run the two region version of the model under two different schemes\(^{10}\). The first one is characterized by symmetry between regions, that is, both regions are exactly the same then trade arise just because of the ‘love of variety’ nature of the model whenever transport cost are high or low enough to generate a dispersion equilibrium. The second type is characterized by the presence of asymmetry between regions arising from a different endowment of non-mobile labor and also keeping the same total amount as in the first simulations but allowing a different distribution of this type of input between regions. This second case can be interpreted as an exogenous differentiation of regions between a core (urban) and a periphery (rural) as compared to an endogenous one arising from equilibrium.

Symmetry

Under symmetry several simulations where carried out changing two main parameters in the model, that is, elasticity of substitution between regions and the share of expenditure in manufactures per region\(^{11}\). Figures 1 and 2 show a representative case. In this case $\sigma = 5$ and $\mu = 0.4$. The change in welfare measured in CS (as in traditional CBA), EV and CV is shown. A first case of significant transport infrastructure improvement (from a $T_{\text{max}} = 3$) is shown in Fig.1. Figure 2 corresponds to the case where transport cost is reduced 10%.

![Fig. 1. Welfare Change ($T_t$ vs. $T_{\text{MAX}}$)](image)

---

\(^{10}\) All simulations are run using Mathematica

\(^{11}\) We assume at this point symmetry in all these parameters.
Results are shown for all the range of transport costs but are especially relevant between T-break (1.63) and T-sustain (1.806). Fig.1 shows that welfare impacts arising from big reductions in transport costs are still well capturing by a traditional CBA for high values of transport costs as well as for intermediate values showing multiple equilibrium. For small changes in these costs the difference between welfare measures is smaller but shows a sudden increase in the multiple equilibrium range. The ratio between EV and CS is shown in Fig.1.b and Fig.2.b. Under an idealized setting such as symmetric economy, CBA practice is still giving appropriate results.

In the range where only agglomeration survives the relevancy of traditional CBA can not be appropriately evaluated since the only trade possible consists in imports of manufactured goods by traditional sector workers in the periphery. In the figures, as can be expected, a significant difference between direct and indirect effects arises.

Fig. 2. Welfare Change (T<sub>t</sub> vs. T<sub>t-1+0.1</sub>)

![Fig. 2. Welfare Change (T<sub>t</sub> vs. T<sub>t-1+0.1</sub>)](image-url)
Asymmetry

A region’s economic size depends on how much qualified and non-qualified labor it has. Since non-qualified labor is mobile and its interregional division is endogenous, intrinsic size asymmetries must come from different endowments of the immobile factor or different distribution of this factor within regions. To this end, we assume either that the two regions are endowed with different stocks of this factor and further more, one region (rural region) is ‘bigger’ in the sense that \( L^* = L + \varepsilon \) with \( \varepsilon > 0 \), or that this factor is distributed different than evenly (phi not 0.5). Introducing asymmetries makes necessary to distinguish the effects between regions.

Under the same parameter values results change considerably. First, the Tomahawk figure for this economy is shown in Fig. 5; now T-break is equal to 1.822 while T-sustain changes to 1.906. Under these more realistic conditions CS is always overestimating economy-wide welfare impacts. Results are also sensitive to the conditions of competition in the market (measured by \( \sigma \)) and the proportion of expenses in manufacturing goods. This result is in line with Puga (1999) results –despite a three region environment is considered there- since suggest that an improved transport link can harm an already disadvantaged region.

Fig. 2. Welfare Change (\( T_t \) vs. \( T_{MAX} \))
Fig. 2. Welfare Change ($T_t$ vs. $T_{t-1}+0.1$)

![Graph showing welfare change over time](image)

Fig. 3b  
Fig. 4b  

Fig. 5. Tomahawk with asymmetry ($T_{break}=1.822$; $T_{sust}=1.908$)

![Graph showing EV/CS ratios and Tomahawk](image)
IV. Some Possible Extensions

The present model is focused on numerical exercises since its structure makes prohibitive to work at an analytical level. Despite that, and as many authors have shown already in the literature (Baldwin et al. 2003), it is possible to work with other versions of NEG models that can be solved analytically and in some cases allow to address in better form questions like the ones raised in this paper. This kind of extensions is left for future work but here we make some suggestions. Extensions to the basic core-periphery model can be classified according to which market and which side of this market affect.

Final Goods Demand

First, we should recognize that utility function does not have to be identical across individuals, especially when comparing agricultural and manufacturing workers. Secondly, even assuming identical preferences these can differ from the usual CES type preferences for variety. As has been suggested by Ottaviano et al. (2000), working with a quasi-linear utility with quadratic sub-utility function can bring better insight and makes things more easily to handle at an analytical level. A very popular specification in industrial organization, international trade and demand literature is the quadratic utility model (Dixit, 1979; Vives, 1990; Anderson et al. 1995; Philips, 1983):

\[
U(q_0; q(i), i \in [0, N]) = \alpha \int_0^N q(i) \, di - \frac{\beta - \gamma}{2} \int_0^N \left[ q(i) \right]^2 \, di - \gamma \left[ \int_0^N q(i) \, di \right]^2 + q_0
\]

This leads to linear demands for varieties \( i \in [0, n] \) in region \( f \) produced in region \( k \):

\[
q(i)_{fk} = a - bp(i)_{fk} + cNp(i)_{fk} + cP_f
\]

where \( P_f \) is an index of prices as in the original core-periphery model. With these functions income effects are ignored and consequently the importance of this effect can be obtained comparing results with the original model.

Inputs Demand

Following Dixit and Stiglitz (1977), production of a quantity \( x(k) \) of any variety \( k \) requires the same fixed \( (\alpha) \) and variable \( (\beta x(k)) \) quantities of the production input in any region. As in Venables(1996) the production input in manufacturing can be modeled as a Cobb-Douglas composite of labor and an aggregate of intermediaries. Following Either(1982), all industrial goods enter symmetrically into the intermediate aggregate with a constant elasticity of substitution across varieties \( \sigma \) (>1). The price index of the aggregate of industrial goods used by firms is region-specific, and is defined by

\[
q_i = \left[ \int_{h \in N} (p_i(h))^{1-\sigma} \, dh + \int_{h \in N} (\tau \cdot p_j(h))^{1-\sigma} \, dh \right]^{1/(1-\sigma)}
\]
where \( p_i(h) \) is the producer price of variety \( h \) in region \( i \). Shipments of the industrial goods incur in ‘iceberg’ trade costs: \( \tau (>1) \) units must be shipped in order that one unit arrives in the other region.

An industrial firm producing quantity \( x(h) \) of variety \( h \) in region \( i \) have a minimum cost function:

\[
C(h) = q_i^\mu w_i^{(1-\mu)} (\alpha + \beta x(h))
\]

where \( \mu \) (with \( 0 \leq \mu < 1 \)) is the share of intermediaries in firms’ costs.

**Final Product Supply**

At the production level, we can also extend the model including more than one type of factor of production. Including capital raises questions about investment and convergence between regions, and its possible relationship with increased quality of transport infrastructure. It can also handle issues as taxes on different types of inputs and public expenditures.

Additionally the way in which producers interact is very important to determine results as already studied in a partial equilibrium framework by Newbery (1998). Dealing with strategic behavior at a production level is a significant extension not only in terms of technical difficulty but also as a way of prospective important source of indirect effects. Departing from (17) –and working in a discrete environment- we can derive more general results –for possible extensions- explicitly computing \( \varepsilon_r \).

\[
\frac{\partial q_r}{\partial p_r} = \frac{-\sigma \ p_r^{-\sigma^{-1}} \sum_j p_j^{(1-\sigma)} - p_r^{(1-\sigma)} p_r^{-\sigma}}{\left( \sum_s p_s^{(1-\sigma)} \right)^2} \mu Y
\]

\[
= \frac{-\sigma \ p_r^{-\sigma^{-1}}}{\left( \sum_j p_j^{(1-\sigma)} \right)^2} \mu Y - \left( \frac{p_r^{(1-\sigma)}}{\sum_s p_s^{(1-\sigma)}} \right)^2 (1-\sigma) \mu Y
\]

\[
= \frac{-\sigma q_r}{p_r} - q_r^2 \frac{(1-\sigma)}{\mu Y}
\]

\[
\varepsilon_r^{Bermand} = \sigma + \frac{(1-\sigma)q_r p_r}{\mu Y} = \sigma + (1-\sigma) s_r
\]

In (30), if we assume a symmetric market when the number of firms increases the elasticity reduces in its second component. In the limit, when we assume “large number”
of monopolistic competitors taking $n$ to infinitive, $s_i$ tends to zero and then we are in the typical case in the literature where $\varepsilon_r = \sigma$.

In producers compete in quantities instead that on prices (Cournot competition) then making use of the inverse demand, as in (31)

$$p_r = \frac{q_r^{-1/\sigma}}{\sum_s q_s^{-1/\sigma}} \mu Y_r$$

(31)

We arrive to a similar expression for the elasticity (32) that also converges to $\sigma$ if we take the limit when $n$ goes to infinity,

$$\frac{1}{\varepsilon_r^{\text{Cournot}}} = \frac{1}{\sigma} + \left(1 - \frac{1}{\sigma}\right) s_r$$

(32)

Equations (30) and (32) are important if we want to model strategic behavior between producers as it was emphasized by Newbery (1998) when addressing the same issue as in this paper but in a partial equilibrium framework. Generally, oligopolistic interaction within the Dixit-Stiglitz framework implies simultaneous solution of mark-ups and market shares, and this can hardly complicated the solution procedure (Venables, et al 2003). One special case that has been studied in the literature of trade and foreign direct investment is when Cournot competition and products are perfect substitutes, so $\sigma \to \infty$, which leads to:

$$\frac{1}{\varepsilon_r^{\text{Cournot}}} = s_r$$

Transport Costs

Urban Cost

The introduction of urban cost is possible mainly by introducing congestion and commuting cost. Congestion costs discourage agglomeration in any one region, and can be introduced explicitly or implicitly. The later is done assuming that due to those costs, the effective labor force as a fraction of the total labor force in a region falls when the size of the labor force grows, that is, the larger the labor force in a region, the less effective labor each worker in that region can supply. Let $Z_j$ be the effective labor force and $L_j$ be the actual force in region $j$, so that $\sum_{n=1}^{N} l_{nj}$, and then the effective labor force is defined as:

$$Z_j = L_j (1 - \gamma L_j)$$

for region $j=1,2$, where $\gamma$ is some positive parameter. Total income in region $j$ will be the wage rate multiplied by the amount of effective labor that is supplied, that is, $Y_j = w_j Z_j$. 

16
Krugman and Elizondo (1996) provide explicit derivation of this equation. They model congestion costs as the cost of commuting from a geographically large city to a central business district where everyone works. The larger the city, the more time workers at the edge of the city must spend getting to and from the center of the city.

Alternatively as stressed by Brakman et al (1996), one can include negative feedbacks (the cost components $\alpha$ and/or $\beta$ rise if $n_j$ increases) because clustering of economic activity causes congestion, long travel time, high cost of establishment, etc. The production structure can be easily adapted to introduce congestion costs in this way. The main idea is that the congestion costs that each firm faces depend on the overall size of the location of production. The size of city $r$ is measured by the total number of manufacturing firms’ $n_r$ in that city. Congestion costs are thus not industry or firm specific, but solely a function of the size of the city as a whole.

$$l_{ir} = n_r^{c/(1-c)}(\alpha + \beta x_{ir}); \quad -1 < c < 1$$

Where $l_{ir}$ is the amount of labor required in city $r$ to produce $x_{ir}$ units of a variety, and the parameter $t$ represents external economies of scale. There are no location-specific external economies of scale if $t = 0$. There are positive location-specific external economies if $-1 < t < 0$. Such a specification could be used to model, for example, learning-by-doing spillovers. For our present purposes, the case of negative location specific external economies arising from congestion are relevant, in which case $0 < t < 1$.

In most of the literature it is assumed that agglomeration of workers into a single region does not involve costs (e.g. congestion). The core-periphery model can be extended by adding urban costs. Following Ottaviano, Tabuchi and Thisse (2000) each agglomeration has a spatial extension that imposes commuting and land costs on the corresponding workers. Space can be then continuous and one-dimensional assuming that each region has a spatial extension and involves a linear city whose center is given but with a variable size. The city center stands for a central business district (CBD) in which all firms locate once they have chosen to set up in the corresponding region. Housing is then a new good described by the amount of land used by workers who became urban residents that must commute to their jobs.

**Modeling Transport Costs**

Transportation costs can be modeled in the familiar “iceberg” way, such that a portion of the good is consumed by the process of transporting it. However, transportation cost can at the same time be subject to increasing returns. Following Mansori (2003) it can be assumed that in each period a fixed cost must be incurred before the transport of goods between two regions can take place (e.g. construction and maintenance of a highway, airport, railway line, or port). Once this periodic fixed cost has been paid, there is then an additional marginal cost to transportation. The constant marginal cost of shipping one unit from region $i$ to region $j$ is $c_{ij} > 0$. Then the total cost of shipping all goods from $i$ to $j$ is:
\[ TC_{ij} = F_{ij} + \tau_{ij} V_{ij} \]

where \( F_{ij} \) is the fixed cost of establishing and maintaining the transportation infrastructure and \( TC_{ij} \) is the total cost of transporting a quantity of goods \( V_{ij} \) between regions \( i \) and \( j \). Since a competitive transportation industry is not possible then it can be assumed that the government runs this industry. Average cost pricing is the charging rule, and this is just total transport cost divided by the volume of trade:

\[ AC_{ij} = \frac{TC_{ij}}{V_{ij}} = \frac{F_{ij}}{V_{ij}} + \tau_{ij} \]

Then iceberg type cost can be represented by:

\[ \tau_{ij}^* = 1 + AC_{ij} \]

In this formulation \( F \) represents the degree of increasing returns to scale in transport.

Another way of extending the modeling of transport cost is following Berhens et al (2003), in a model of two countries with two regions within each country. They argue that “it is a well-established fact that international transport costs are lower on routes processing larger volumes of freight. In this section, we assume that trade costs \( \tau \) are affected by the spatial distribution of firms in the two countries as observed in reality”. More precisely, , it is assume that

\[ \tau(\lambda_H, \lambda_F) = \tau [1 - \xi(\lambda_H + \lambda_F - 1)] \]

where \( \xi \in [0, 1] \) is an indicator of the degree of density economies in transportation and \( \tau \leq \tau_{trade} \) (the level of international transport costs that leads to trade between countries). When both countries are dispersed (\( \lambda_H = \lambda_F = 1/2 \), we have \( \tau(\lambda_H, \lambda_F) = \tau \); while international trade costs drop to \((1-\xi)\tau\), which can be interpreted as the incompressible component of trade costs, when both countries are agglomerated. For \( \xi = 0 \), there are no economies of transport density.

Finally, non-linearities in transport cost can be also captured as in Lanaspa and Sanz (2001). They use a specific function that relates transport cost with population, based on empirical work.

**Multi-region and Networks**

The multi region case can be analyzed in several ways. Starting with an extension to three regions as in Martin and Rogers (1999), infrastructure improvement can be analyzed in a context of intra-regional trade along with inter-regional trade, differentiated between costs incurred in the each of these cases. Brakman et al (2003) also analyze the discrete case with what they call the “package” economy, a discrete version of the “racetrack” model as appeared in Krugman and Venables (1995b).
Turning to the transport cost function in a multi-region setting, unit transport costs from locations $x$ to $y$ can be decompose:

$$\tau(x, y) \equiv \tau \cdot T(x - y)$$

where $\tau$ is the amplitude of transportation costs and where $T(x)$ captures the shape of transportation costs. As in Picard and Tabuchi (2003), the function $T(x) : \mathbb{R} \rightarrow [0,1]$ is a periodic function such that $T(x) = T(l+x) = T(l-x)$ for every $l \in \mathbb{N}$ (set of natural numbers), and such that $T(0) = 0$, $T(1/2) = 1$, and $T'(x) \geq 0$ for every $x \in [0,1/2]$.

The network issue can be also approached within the framework of Ago et al (2004) in the context of three regions located on a line.

V. Conclusions and Policy Recommendations

In this paper we tried to fill a gap in research concerning cost-benefit analysis for transport infrastructure projects. The gap arises when comparing recent theoretical and empirical modeling relating traditional CBA with economy-wide welfare measures arising from transport initiatives. Departing from the canonical core-periphery modeling we construct measures of welfare and linked them with the practice of CBA. Simulations based on symmetry and asymmetry conditions in this model show that a difference exists in both measures and that this measure is negative and increases significantly in the case of asymmetry and more particularly when more realistic features are included in the model. This evidence support the interest in more detailed research on indirect and strategic effects arising from infrastructure projects in the debate that originated studies such as SACTRA (1999) and CPB(2000).

In a future extension of this paper more realistic features are considered such as networks effects (more than two regions), imperfect and heterogeneous labor markets, government intervention, congestion effects and growth.
VI. Appendix

A.I. Welfare Measures

Welfare measures for comparison between a general CBA and a traditional –specific- CBA are the following:

- Consumer Surplus (CS)
- Compensate Variation (CV)
- Equivalent Variation (EV)

CS is measured in two different ways. First it can be measure using “rule of half” type procedure, which means that in both the original and final equilibrium the levels of demand in region i fulfilled with production from region j, that is manufactures goods using transport services, are computed. Next, the following formula is applied using as a measure of “prices” the level of transportation plus one ($\tau$) cost in each equilibrium:

$$CS_{ROH} = s_i (D_g^0 + D_g^1) (\tau^0 - \tau^1)$$  \hspace{1cm} (A.1)

In this formula $s_i$ represents the proportion of manufacture workers staying in region i, or more appropriate the proportion of varieties from region j consumed in region i. It must be used as a weight in the calculation of CS for a particular region since the consumer surplus refers to the total gain in welfare arising from the consumption of transport sector using goods. In this measure no difference between agricultural or manufacture workers in done.

CS can also be measure directly from the expression of demand for variety j consumed in region i, as follows (in the case of i=1, j=2):

$$Dij = \frac{\mu E_j p_j^{-\sigma}}{n_i p_i^{-\sigma} + n_j p_j^{-\sigma}}$$  \hspace{1cm} (A.2)

The integration of this expression over the corresponding range of levels of $\tau$ and subsequently weighted by $s$ gives another measure of CS that should be consistent with the first one is the demand curve is not very far from the linear case. For the purpose of integration I use two numerical integration procedures that in general give close results; these are Trapezoid rule and Simpson rule. These calculations can be carried out for the symmetry case since between equilibriums only $\tau$ is changing. This occurs because of the normalizations employed requires that prices are one in equilibrium (and equal to salaries for manufacture workers) in both scenarios an as a consequence expenditure is equal in both regions and since the equilibriums analyzed are for dispersion we have $n_i = n_j = 0.5$. This means that the movement in over the demand curve and doesn’t affect the position of the demand curve. This is completely different to what happen when asymmetry between regions is allowed. In this case not only $\tau$ changes, but other
elements in the expression for demand change so we can not use anymore the second approach to calculate CS.

CV and EV measures are derived from the expression of indirect utility for the aggregate of consumers in a region. This expression for region r is the following:

\[ V_r(p^T, G, Y) = \mu \mu \mu (1-\mu)^{(1-\mu)} Y G^{-\mu} (p^T)^{-(1-\mu)} \]  
(A.3)

The following two relations implicitly define an expression for the equivalent (EV) and compensate (CV) variations,

\[ V(\tau^0, Y^0 + EV) = U^1 \]
\[ V(\tau^1, Y^1 - CV) = U^0 \]

where \( U^i \) indicate the level of utility depending on the stage, that is, in the original situation before the change in transport cost and in the final situation after the change. Combining these conditions with (3) a pair of explicit expressions for the EV and CV can be derived, for example, for the equivalent variation,

\[ U^1 = \mu \mu \mu (1-\mu)^{(1-\mu)} (Y^0 + EV) \ G^0^{-\mu} (p_r^T)^{-(1-\mu)} \]
\[ \mu \mu \mu (1-\mu)^{(1-\mu)} Y^1 (G^1)^{-\mu} = \mu \mu \mu (1-\mu)^{(1-\mu)} (Y^0 + EV) \ G^0^{-\mu} \]

\[ (Y^0 + EV) = \frac{Y^1 (G^1)^{-\mu}}{G^0^{-\mu}} \]
\[ EV = Y^1 \left( \frac{G^1}{G^0} \right)^{-\mu} - Y^0 \]  
(A.4)

and applying a similar procedure we will obtain,

\[ CV = Y^1 - Y^0 \left( \frac{G^0}{G^1} \right)^{-\mu} \]  
(A.5)

CV and EV measures can also be derived from the compensate demand function for variety \( i \), which can be obtained starting from the FOC of the utility maximization faced by the consumer in this version of the core-periphery model, that is,

\[ \text{Maximization of} \ U_r = M_r^{\mu} T_r^{1-\mu}, \text{ subject to} \ Y_r = p^T_\tau + \int_0^T p(i), m(i), di, \]
where M represents the composite index of manufacture goods, T is the consumption of
traditional good (e.g. agricultural), and μ represents the expenditure share of
manufactured good in consumption. The consumption of manufactures is described by a
constant elasticity of substitution sub utility function defined over a continuum of
varieties of manufactured goods, m(i), with a range of varieties described by n. The
preference of variety in manufacturing goods is represented by ρ.

\[
M_r = \left( \int_0^n m(i) \rho^i di \right)^{\frac{\sigma}{\sigma-1}}
\]

with \(0 > \sigma > 1\) \hfill (A.6)

Since all consumers are identical in preferences, final demand for an specific variety will
be the same for all of them differing only in terms of sources of income. We can also
assume that there is a representative consumer in each region meaning that the relevant
income includes all sources of wages in a region, that is, manufacture or traditional
production based wages. The utility maximization problem can be solved in two steps.
First, for any value of the composite M, each m(i) have to be chosen so as to minimize
the cost of attaining it. This is achieved solving the following problem,

\[
\min \int_0^n p_r(i)m_r(i)di \quad \text{subject to} \quad M_r = \left( \int_0^n m_r(i)\rho^i di \right)^{1/\rho}
\]

with \(M_r \geq 0\) \hfill (A.7)

The first order condition for this expenditure minimization problem establish the equality
of marginal rates of substitution to price ratios for any pair of varieties and consequently
implies an expression for the consumption of a particular variety (e.g. m(j)), that
replaced in the constraint in (4) finally brings an expression for the compensated demand
of this particular variety as in (5),

\[
m_r(j) = \frac{1}{\rho} \left( \int_0^n \frac{p_r(j)}{p(i)} \rho^i di \right)^{1/\rho} M_r
\]

The term in the denominator is normally regarded as a price index for the manufactured
products, denoted here by G in (9). This index measures the minimum cost of purchasing
a unit of the composite index M of manufacturing goods.

\[
G_r = \left[ \int_0^n p(i) \rho^i di \right]^{(\rho-1)} = \left[ \int_0^n p(i) \rho^{i-\sigma} di \right]^{1/(1-\sigma)}
\]

\hfill (A.9)

\hfill 12 See Mas-Colell et al (1997) Ch. 4 for a discussion on aggregation.*
Using (9) the equation for demand of a particular variety can be simplified to:

\[ m_r(j) = \left( \frac{p_r(j)}{G_r} \right)^{\frac{1}{1-\mu}} M_r = \left( \frac{p_r(j)}{G_r} \right)^{-\sigma} M_r \] \hspace{1cm} (A.10)

In a second stage we can solve the original problem of utility maximization, where consumers divide total income between traditional and composite manufactures. This is a typical Cobb-Douglas maximization problem leading to uncompensated demands in the form of income over price times the expenditure share of the good in total consumption, like in (9).

\[ \max U_r = M_r \mu T_r^{1-\mu} \quad \text{subject to} \quad G_r M_r + p_r^T T_r = Y_r \]

\[ T_r = \frac{(1-\mu)Y_r}{p_r^r}, \quad M_r = \frac{\mu Y_r}{G_r} \] \hspace{1cm} (A.11)

For each variety of manufactures uncompensated demand can be derived as in (2), in which the elasticity of demand for every variety is \( \sigma \).

\[ m_r(j) = \mu Y_r \frac{p_r(j)^{-\sigma}}{G_r^{-(\sigma-1)}} \text{ for } j \in [0, n_r] \] \hspace{1cm} (A.12)

To derive compensate demand expression for variety I, instead of using \( M_r \) we should use the corresponding compensate demand expression for the aggregate of manufactures, \( M^c_r \). Since the original consumer maximization problem is a typical Cobb-Douglas, the expression for this type of demand is well-known,

\[ M_r = \frac{\mu E_r}{G_r} \]

where \( E_r \) correspond to the expenditure function that can be derived directly from (3) as,

\[ E_r(p^T, G_Y) = \mu^{-\mu} (1-\mu)^{-(1-\mu)} V G^\mu (p^T)^{(1-\mu)} \] \hspace{1cm} (A.13)

Consequently, a final expression for compensated demand for variety i is as follows,

\[ m_r^c(j) = \left( \frac{p_r(j)}{G_r} \right)^{-\sigma} M_r^c = \left( \frac{p_r(j)}{G_r} \right)^{-\sigma} \frac{\mu E_r}{G_r} = \left( \frac{\mu}{1-\mu} \right)^{(1-\mu)} V_r \frac{p_r(j)^{-\sigma}}{G_r^{-(\sigma-1)}} \] \hspace{1cm} (A.14)

One important property of this function is that it coincides with (9) in the initial equilibrium if the EV is used as a reference of welfare and it coincides with the final equilibrium if CV is used instead. In the case of asymmetry this kind of measure cannot be used since from equilibrium to equilibrium variables other than \( \tau \) change.
VI. References


Bröcker, J., 1999, Trans-European Effects of "Trans-European Networks": A Spatial CGE Analysis, Mimeo, Technical University of Dresden, Dresden


NEI, 2000, Vervoerswaarde Studie Zuiderzeelijn, Eindrapport, NEI Transport i.s.m. Bouwdienst Rijkswaterstaat, Rotterdam.


Standing Advisory Committee on Trunk Road Assessment (SACTRA) (1999), *Transport and the economy*, London.


Schürmann, C., Spiekermann, K., Wegener, M. (1997) *Accessibility Indicators*, Berichte aus dem Institut für Raumplanung 39, Dortmund, IRPUD